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Erratum

Correction to the paper “Growth fluctuations in a class of deposition models” Annales de l’Institut Henri Poincaré Probabilités et statistiques 39 (2003) 639–685 ☆

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The proof of Lemma 5.1 is incorrect, as it is based on the false statement in the paragraph following Eq. (40) on the product property of the distribution of $(\underline{\eta}''(0),\ \underline{\zeta}''(0))$. This paragraph corrected and Lemma 5.1 with a correct proof can be found below.

Recall that $S(0) = U^{(s(0))}(0) = U^{(0)}(0)$ is the first site on the right-hand side of the origin initially with second class particles. We introduce the notation $(\underline{\eta}''(t),\ \underline{\zeta}''(t)) := (\boldsymbol{\tau}_{S(0)}\underline{\eta}(t),\ \boldsymbol{\tau}_{S(0)}\underline{\zeta}(t))$, which is the $(\underline{\eta}(t),\ \underline{\zeta}(t))$ process shifted to this initial position $S(0)$ of the $\overline{S}$-particle. We also introduce its S''-particle: $S''(t) := \overline{S}(t) - \overline{S}(0)$. Hence the initial distribution of $(\underline{\eta}''(0),\ \underline{\zeta}''(0))$ is modified according to this random shifting-procedure; we show the details in the proof of the next lemma.

Using the Palm measures, we show that the expected rates for S to jump are bounded in time.

Lemma 5.1. *Let $n \in \mathbb{Z}^+,\ k \in \mathbb{Z}$, and*

$$c_i(t) := f\bigl(\zeta_i(t)\bigr) - f\bigl(\eta_i(t)\bigr) + f\bigl(-\eta_i(t)\bigr) - f\bigl(-\zeta_i(t)\bigr) \tag{42}$$

the rate for any second class particle to jump from site i. Then

$$\mathbf{E}\bigl(\bigl[c_{S(t)}(t)\bigr]^n \cdot \bigl[\zeta_{S(t)}(t) - \eta_{S(t)}(t)\bigr]^k\bigr) \leqslant K(n,\ k)$$

uniformly in time.

Proof. First we consider the pair $(\underline{\eta}'(0),\ \underline{\zeta}'(0))$ defined following (38). As described there, this is in fact the pair $(\underline{\eta}(0),\ \underline{\zeta}(0))$ at time $t = 0$, as seen from “a typical second class particle”, or equivalently, as seen from “a typical $\overline{S}$-particle”. In this pair, we have at least one second class particle at the origin, which we call S'. We let our process $(\underline{\eta}',\ \underline{\zeta}')$ evolve, and we follow this “typical” S'-particle. Started from the Palm-distribution, this tagged S'-particle keeps on “being typical” (see [1]), i.e., for a function g of the process as seen by S',

$$\mathbf{E}\bigl(g\bigl(\boldsymbol{\tau}_{S'(t)}\underline{\eta}'(t),\ \boldsymbol{\tau}_{S'(t)}\underline{\zeta}'(t)\bigr)\bigr) = \widehat{\mathbf{E}}\bigl(g\bigl(\underline{\eta}(t),\ \underline{\zeta}(t)\bigr)\bigr)$$

with definition (37).

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Now we first show the desired result for the S'-particle of $(\underline{\eta}', \underline{\zeta}')$ instead of the S-particle of $(\underline{\eta}, \underline{\zeta})$. In the previous display, we put the function

$$g\big(\underline{\eta}(t), \underline{\zeta}(t)\big) := \big[c_0(t)\big]^n \cdot \big[\zeta_0(t) - \eta_0(t)\big]^k,$$

and we denote by k^+ the positive part of k. We know that $\zeta_0(t) - \eta_0(t) \geqslant 1$ holds $\widehat{\mathbf{P}}$-a.s., hence

$$\begin{aligned}\mathbf{E}\big(\big[c_{S'(t)}(t)\big]^n \cdot \big[\zeta'_{S'(t)}(t) - \eta'_{S'(t)}(t)\big]^k\big) &= \widehat{\mathbf{E}}\big(\big[c_0(t)\big]^n \cdot \big[\zeta_0(t) - \eta_0(t)\big]^k\big)\\ &\leqslant \widehat{\mathbf{E}}\big(\big[c_0(t)\big]^n \cdot \big[\zeta_0(t) - \eta_0(t)\big]^{k^+}\big)\\ &= \frac{\mathbf{E}([c_0(t)]^n \cdot [\zeta_0(t) - \eta_0(t)]^{k^+ + 1})}{\mathbf{E}(\zeta_0(t) - \eta_0(t))}\end{aligned}$$

by (37). The function $c_0(t)$ consists of sums of $f(\pm\eta_0(t))$ and $f(\pm\zeta_0(t))$, hence the numerator is an $n + k^+ + 1$-order polinom of these functions and of $\zeta_0(t)$, $\eta_0(t)$. These are all random variables with all moments finite. Therefore, using Cauchy's inequality, the numerator can be bounded from above by products of moments of either $f(\eta_0(t))$ or $f(\zeta_0(t))$ or $\eta_0(t)$, or $\zeta_0(t)$. The models $\underline{\eta}$ and $\underline{\zeta}$ are both separately in their stationary distributions, hence these bounds are constants in time. The denominator is a positive number due to $\theta_2 > \theta_1$ and strict monotonicity of $\mathbf{E}_\theta(z)$ in θ. We see that we found a bound, uniform in time for the function g of $(\underline{\eta}', \underline{\zeta}')$ as seen from S'.

We need to find similar bound for a function g of the original pair $(\underline{\eta}, \underline{\zeta})$, as seen from S. This is equivalent to finding a bound for g of $(\underline{\eta}'', \underline{\zeta}'')$ defined above, as seen from S'' of this pair. Let us consider first the initial distribution of $(\underline{\eta}'', \underline{\zeta}'')$, which we shall call $\underline{\mu}''$. By definition, it is clear that this distribution is the product of the original marginals μ for sites $i > 0$. Fix a K positive integer and two vectors $\underline{x}, \underline{y} \in \mathbb{Z}^{\mathbb{Z}}$. For simplicity we introduce the notations

$$\begin{aligned}\underline{\eta}_{[a,\, b]} &:= (\eta_a, \ldots, \eta_b) \quad \text{and} \quad \underline{\zeta}_{[a,\, b]} := (\zeta_a, \ldots, \zeta_b),\\ \underline{x}_{[a,\, b]} &:= (x_a, \ldots, x_b) \quad \text{and} \quad \underline{y}_{[a,\, b]} := (y_a, \ldots, y_b)\end{aligned}$$

and, where not written, we consider our models at time zero. We break the events according to the initial position $S(0)$ of the S-particle in the original pair $(\underline{\eta}, \underline{\zeta})$:

$$\begin{aligned}&\mathbf{P}\big(\underline{\eta}''_{[-K,\,0]} = \underline{x}_{[-K,\,0]},\ \underline{\zeta}''_{[-K,\,0]} = \underline{y}_{[-K,\,0]}\big)\\ &\quad = \mathbf{P}\big(\underline{\eta}_{[S(0)-K,\,S(0)]} = \underline{x}_{[-K,\,0]},\ \underline{\zeta}_{[S(0)-K,\,S(0)]} = \underline{y}_{[-K,\,0]}\big)\\ &\quad = \sum_{n=0}^{K} \mathbf{P}\big(\underline{\eta}_{[n-K,\,n]} = \underline{x}_{[-K,\,0]},\ \underline{\zeta}_{[n-K,\,n]} = \underline{y}_{[-K,\,0]},\ S(0) = n\big)\\ &\qquad + \sum_{n=K+1}^{\infty} \mathbf{P}\big(\underline{\eta}_{[n-K,\,n]} = \underline{x}_{[-K,\,0]},\ \underline{\zeta}_{[n-K,\,n]} = \underline{y}_{[-K,\,0]},\ S(0) = n\big)\\ &\quad = \sum_{n=0}^{K} \mathbf{P}\big(\underline{\eta}_{[n-K,\,n]} = \underline{x}_{[-K,\,0]},\ \underline{\zeta}_{[n-K,\,n]} = \underline{y}_{[-K,\,0]}\big) \cdot E_n(\underline{x}, \underline{y})\\ &\qquad + \sum_{n=K+1}^{\infty} \mathbf{P}\big(\underline{\eta}_{[n-K,\,n]} = \underline{x}_{[-K,\,0]},\ \underline{\zeta}_{[n-K,\,n]} = \underline{y}_{[-K,\,0]}\big) \cdot E_K(\underline{x}, \underline{y}) \cdot \mathbf{P}\{F_{n-K}\},\end{aligned}$$

where the function E_n of $\underline{x}$ and $\underline{y}$ is an indicator defined by

$$E_n(\underline{x}, \underline{y}) := \mathbf{1}\{x_{-n} = y_{-n},\ x_{-n+1} = y_{-n+1}, \ldots, x_{-1} = y_{-1},\ x_0 < y_0\},$$

and the event F_{n-K} is

$$F_{n-K} := \{\eta_0 = \zeta_0,\ \eta_1 = \zeta_1, \ldots, \eta_{n-K-1} = \zeta_{n-K-1}\}.$$

The last equality follows from the product structure of $\underline{\mu}$ and from the fact that $S(0)$ is the first site to the right of the origin where $\eta_i \neq \zeta_i$. Continuing the computation results in

$$\begin{aligned}
&\mathbf{P}\big(\underline{\eta}''_{[-K,\,0]} = \underline{x}_{[-K,\,0]},\ \underline{\zeta}''_{[-K,\,0]} = \underline{y}_{[-K,\,0]}\big)\\
&\quad = \prod_{i=-K}^{0} \mu(x_i,\ y_i) \cdot \left[\sum_{n=0}^{K} E_n(\underline{x},\ \underline{y}) + E_K(\underline{x},\ \underline{y}) \cdot \sum_{n=K+1}^{\infty} \mu\{\eta_0 = \zeta_0\}^{n-K}\right]\\
&\quad = \prod_{i=-K}^{0} \mu(x_i,\ y_i) \cdot \left[\sum_{n=0}^{K} E_n(\underline{x},\ \underline{y}) + E_K(\underline{x},\ \underline{y}) \cdot \frac{\mu\{\eta_0 = \zeta_0\}}{\mu\{\eta_0 < \zeta_0\}}\right]
\end{aligned}$$

using translation-invariance.

For later purposes, we are interested in the Radon–Nikodym derivative of the distribution $\underline{\mu}''$ of $(\underline{\eta}'',\ \underline{\zeta}'')$ w.r.t. the Palm distribution $\underline{\hat{\mu}}$ of $(\underline{\eta}',\ \underline{\zeta}')$. Since both have product of marginals μ for sites $i > 0$, we only have to deal with the left part of the origin. Passing to the limit $K \to \infty$, we have

$$\begin{aligned}
\frac{\mathrm{d}\underline{\mu}''}{\mathrm{d}\underline{\hat{\mu}}}(\underline{x},\ \underline{y}) &= \lim_{K\to\infty} \frac{\mathbf{P}(\underline{\eta}''_{[-K,\,0]} = \underline{x}_{[-K,\,0]},\ \underline{\zeta}''_{[-K,\,0]} = \underline{y}_{[-K,\,0]})}{\mathbf{P}(\underline{\eta}'_{[-K,\,0]} = \underline{x}_{[-K,\,0]},\ \underline{\zeta}'_{[-K,\,0]} = \underline{y}_{[-K,\,0]})}\\
&= \lim_{K\to\infty} \frac{\prod_{i=-K}^{0} \mu(x_i,\ y_i)}{\prod_{i=-K}^{-1} \mu(x_i,\ y_i)\hat{\mu}(x_0,\ y_0)} \cdot \left[\sum_{n=0}^{K} E_n(\underline{x},\ \underline{y}) + E_K(\underline{x},\ \underline{y}) \cdot \frac{\mu\{\eta_0 = \zeta_0\}}{\mu\{\eta_0 < \zeta_0\}}\right]\\
&= \frac{\mu(x_0,\ y_0)}{\hat{\mu}(x_0,\ y_0)} \cdot \left[\sum_{n=0}^{\infty} E_n(\underline{x},\ \underline{y}) + \lim_{K\to\infty} E_K(\underline{x},\ \underline{y}) \cdot \frac{\mu\{\eta_0 = \zeta_0\}}{\mu\{\eta_0 < \zeta_0\}}\right]\\
&= \frac{\mu(x_0,\ y_0)}{\hat{\mu}(x_0,\ y_0)} \cdot \sum_{n=0}^{\infty} E_n(\underline{x},\ \underline{y})
\end{aligned}$$

for $\underline{\hat{\mu}}$-almost all configurations $(\underline{x},\ \underline{y})$. Note that the sum on the right-hand side gives exactly the distance between the origin and the first position i to the left of the origin with $x_i \neq y_i$. Hence this sum is finite for $\underline{\hat{\mu}}$-almost all configurations $(\underline{x},\ \underline{y})$.

In view of this result, we can now obtain our estimates. The main idea here is that the pairs $(\underline{\eta}',\ \underline{\zeta}')$ and $(\underline{\eta}'',\ \underline{\zeta}'')$ only differ in their initial distribution, hence their behavior conditioned on the same initial configuration agree. This is used for obtaining the third expression, and Cauchy's inequality is used for the fourth one below.

$$\begin{aligned}
&\mathbf{E}\big(\big[c_{S''(t)}(t)\big]^n \cdot \big[\zeta_{S''(t)}(t) - \eta_{S''(t)}(t)\big]^k\big)\\
&\quad = \int\limits_{\tilde{\Omega}\cap\{x_0<y_0\}} \mathbf{E}\big(\big[c_{S''(t)}(t)\big]^n \cdot \big[\zeta_{S''(t)}(t) - \eta_{S''(t)}(t)\big]^k \mid \underline{\eta}''(0) = \underline{x},\ \underline{\zeta}''(0) = \underline{y}\big)\, \mathrm{d}\underline{\mu}''(\underline{x},\ \underline{y})\\
&\quad = \int\limits_{\tilde{\Omega}\cap\{x_0<y_0\}} \mathbf{E}\big(\big[c_{S'(t)}(t)\big]^n \cdot \big[\zeta_{S'(t)}(t) - \eta_{S'(t)}(t)\big]^k \mid \underline{\eta}'(0) = \underline{x},\ \underline{\zeta}'(0) = \underline{y}\big)\\
&\qquad \times \frac{\mu(x_0,\ y_0)}{\hat{\mu}(x_0,\ y_0)} \cdot \sum_{n=0}^{\infty} E_n(\underline{x},\ \underline{y})\, \mathrm{d}\underline{\hat{\mu}}(\underline{x},\ \underline{y})
\end{aligned}$$

$$\leqslant \Bigg[\int\limits_{\widetilde{\Omega}\cap\{x_0<y_0\}} \Big[\mathbf{E}\big(\big[c_{S'(t)}(t)\big]^n \cdot \big[\zeta_{S'(t)}(t) - \eta_{S'(t)}(t)\big]^k \mid \underline{\eta}'(0) = \underline{x},\ \underline{\zeta}'(0) = \underline{y}\big)\Big]^2 \,\mathrm{d}\underline{\hat{\mu}}(\underline{x},\ \underline{y})\Bigg]^{1/2}$$

$$\times \Bigg[\int\limits_{\widetilde{\Omega}\cap\{x_0<y_0\}} \Bigg[\frac{\mu(x_0,\ y_0)}{\hat{\mu}(x_0,\ y_0)} \cdot \sum_{n=0}^{\infty} E_n(\underline{x},\ \underline{y})\Bigg]^2 \cdot \mathrm{d}\underline{\hat{\mu}}(\underline{x},\ \underline{y})\Bigg]^{1/2}$$

$$\leqslant \Bigg[\int\limits_{\widetilde{\Omega}\cap\{x_0<y_0\}} \mathbf{E}\big(\big[c_{S'(t)}(t)\big]^{2n} \cdot \big[\zeta_{S'(t)}(t) - \eta_{S'(t)}(t)\big]^{2k} \mid \underline{\eta}'(0) = \underline{x},\ \underline{\zeta}'(0) = \underline{y}\big) \,\mathrm{d}\underline{\hat{\mu}}(\underline{x},\ \underline{y})\Bigg]^{1/2}$$

$$\times \Bigg[\int\limits_{\widetilde{\Omega}\cap\{x_0<y_0\}} \frac{\mu(x_0,\ y_0)}{\hat{\mu}(x_0,\ y_0)} \cdot \Bigg[\sum_{n=0}^{\infty} E_n(\underline{x},\ \underline{y})\Bigg]^2 \cdot \mathrm{d}\underline{\mu}(\underline{x},\ \underline{y})\Bigg]^{1/2}$$

$$= \Big[\mathbf{E}\big(\big[c_{S'(t)}(t)\big]^{2n} \cdot \big[\zeta_{S'(t)}(t) - \eta_{S'(t)}(t)\big]^{2k}\big)\Big]^{1/2}$$

$$\times \Bigg[\int\limits_{\widetilde{\Omega}\cap\{x_0<y_0\}} \frac{\mathbf{E}(\zeta_0 - \eta_0)}{y_0 - x_0} \cdot \Bigg[\sum_{n=0}^{\infty} E_n(\underline{x},\ \underline{y})\Bigg]^2 \cdot \mathrm{d}\underline{\mu}(\underline{x},\ \underline{y})\Bigg]^{1/2}$$

by (38). The first factor of the last display is finite by the first part of the proof. Using the definition of the indicator E_n, the second factor can be bounded from above by

$$\big[\mathbf{E}(\zeta_0 - \eta_0)\big]^{1/2} \cdot \Bigg[\int\limits_{\widetilde{\Omega}\cap\{x_0<y_0\}} \Bigg[\sum_{n=0}^{\infty}(2n+1) \cdot E_n(\underline{x},\ \underline{y})\Bigg] \cdot \mathrm{d}\underline{\mu}(\underline{x},\ \underline{y})\Bigg]^{1/2}$$

$$= \big[\mathbf{E}(\zeta_0 - \eta_0)\big]^{1/2} \cdot \Bigg[\sum_{n=0}^{\infty}(2n+1) \cdot \mu\{\eta_0 = \zeta_0\}^n \cdot \mu\{\eta_0 < \zeta_0\}\Bigg]^{1/2}$$

using the product property of $\underline{\mu}$, and is again finite. $\square$

References

[1] S.C. Port, C.J. Stone, Infinite particle systems, Trans. Amer. Math. Soc. 178 (1973) 307–340.