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**Équations aux dérivées partielles/*Partial Differential Equations***

# Critical exponents for the Pucci's extremal operators

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**Abstract** In this Note we present some results on the existence of radially symmetric solutions for the nonlinear elliptic equation

$$\mathcal{M}^{+}_{\lambda,\Lambda}(D^2u)+u^p=0,\quad u\geqslant 0\quad \text{in } \mathbb{R}^N. \tag{$*$}$$

Here $N\geqslant 3$, $p>1$ and $\mathcal{M}^{+}_{\lambda,\Lambda}$ denotes the Pucci's extremal operators with parameters $0<\lambda\leqslant\Lambda$. The goal is to describe the solution set as function of the parameter $p$. We find critical exponents $1<p^s_+<p^*_+<p^p_+$, that satisfy: (i) If $1<p<p^*_+$ then there is no nontrivial solution of $(*)$. (ii) If $p=p^*_+$ then there is a unique fast decaying solution of $(*)$. (iii) If $p^*<p\leqslant p^p_+$ then there is a unique pseudo-slow decaying solution to $(*)$. (iv) If $p^p_+<p$ then there is a unique slow decaying solution to $(*)$. Similar results are obtained for the operator $\mathcal{M}^{-}_{\lambda,\Lambda}$. ***To cite this article: P.L. Felmer, A. Quaas, C. R. Acad. Sci. Paris, Ser. I 335 (2002) 909–914.***



### Les exposants critiques pour l'opérateur extrémal de Pucci

**Résumé** Dans cette Note nous présentons des résultats d'existence des solutions radiales pour l'équation elliptique non linéare

$$\mathcal{M}^{+}_{\lambda,\Lambda}(D^2u)+u^p=0,\quad u\geqslant 0\quad \text{dans } \mathbb{R}^N, \tag{$*$}$$

où $N\geqslant 3$, $p>1$ et $\mathcal{M}^{+}_{\lambda,\Lambda}$ est l'opérateur extrémal de Pucci avec les paramètres $0<\lambda\leqslant\Lambda$. L'objectif de cette Note est décrire l'ensemble des solutions en fonction de $p$. On trouve des exposants critiques $1<p^s_+<p^*_+<p^p_+$ tels que : (i) Si $1<p<p^*_+$, alors il n'existe pas de solution non triviale de $(*)$. (ii) Si $p=p^*_+$, il existe une unique solution de $(*)$ à décroisssance rapide. (iii) Si $p^*<p\leqslant p^p_+$, il existe une unique solution de $(*)$ à décroissance pseudo-lente. (iv) Si $p^p_+<p$, il existe une unique solution de $(*)$ à décroissance lente. Un résultat similaire peut se démontrer pour $\mathcal{M}^{-}_{\lambda,\Lambda}$. ***Pour citer cet article : P.L. Felmer, A. Quaas, C. R. Acad. Sci. Paris, Ser. I 335 (2002) 909–914.***



## *Version française abrégée*

Dans cette Note, nous étudions les solutions non négatives de l'équation elliptique non linéaire

$$\mathcal{M}^{\pm}_{\lambda,\Lambda}\big(D^2u\big)+u^p=0,\quad u\geqslant 0\quad \text{dans } \mathbb{R}^N, \tag{1}$$

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où $N \geqslant 3$, $p > 1$ et $\mathcal{M}^{\pm}_{\lambda,\Lambda}$ est l'opérateur extrémal de Pucci avec les paramètres $0 < \lambda \leqslant \Lambda$, défini par :

$$\mathcal{M}^{+}_{\lambda,\Lambda}\big(D^2u\big) = \Lambda \sum_{e_i>0} e_i + \lambda \sum_{e_i<0} e_i \quad \text{et} \quad \mathcal{M}^{-}_{\lambda,\Lambda}\big(D^2u\big) = \lambda \sum_{e_i>0} e_i + \Lambda \sum_{e_i<0} e_i, \tag{2}$$

où $e_i = e_i(D^2u)$, $i = 1,\dots,N$, sont les valeurs propres de $D^2u$ (voir Caffarelli et Cabré [1]). Quand $\lambda = \Lambda = 1$, nous observons que $\mathcal{M}^{\pm}_{\lambda,\Lambda}$ est l'opérateur de Laplace, et donc l'équation (1) devient

$$\Delta u + u^p = 0, \quad u \geqslant 0 \quad \text{dans } \mathbb{R}^N. \tag{3}$$

On sait que les solutions de l'équation (3) dépendent fortement de la valeur de $p$. Quand $1 < p < p^* := (N+2)/(N-2)$, et supposant que $\lim_{|x|\to\infty} u(x) = 0$, il n'y a pas de solution non triviale de (3) (voir [13]). Si $p = p^*$, il y a une unique solution de (3) à changement d'échelle près (voir [2]), cette solution satisfait $u(|x|)|x|^{N-2} \to C > 0$ quand $|x| \to \infty$. Si $p > p^*$, (3) admet une solution qui se comporte comme $C|x|^{-\alpha}$, $C > 0$ quand $|x|$ est grand, où $\alpha = 2/(p-1)$.

Il est intéressant de mentionner qu'il n'existe pas de solution de (3) quand $1 < p < p^*$, indépendamment du comportement de $u$ à l'infini (voir [8]), et ceci est un théorème de type Liouville. Quand $1 < p \leqslant N/(N-2) := p^s$, le même théorème de type Liouville est connu pour les sursolutions de (3). Dans un article récent [5], Cutri et Leoni étendent ce résultat au cas de l'opérateur extrémal de Pucci. Ils considèrent l'inégalité

$$\mathcal{M}^{\pm}_{\lambda,\Lambda}\big(D^2u\big) + u^p \leqslant 0, \quad u \geqslant 0 \quad \text{dans } \mathbb{R}^N, \tag{4}$$

et définissent deux pseudo-dimension $\widetilde{N}_+ = \frac{\lambda}{\Lambda}(N-1)+1$ et $\widetilde{N}_- = \frac{\Lambda}{\lambda}(N-1)+1$. Ils concluent que si $1 < p \leqslant p^s_+ := \widetilde{N}_+/(\widetilde{N}_+ - 2)$, alors l'équation (4), avec l'opérateur $\mathcal{M}^{+}_{\lambda,\Lambda}$, n'a pas de solution. Il y a un résultat analogue dans le cas $\mathcal{M}^{-}_{\lambda,\Lambda}$.

Quand on regarde les résultats connus pour les équations semi-linéaires (3) et les résultats décrits ci-dessus, il semble naturel de chercher les exposants critiques pour (1). Il serait également intéresant de comprendre la structure des solutions de (1) en fonction de $p > 1$.

Le but de cette Note est de présenter un résultat de ce type pour le cas des solutions radiales. Le cas général reste ouvert. La version détaillée avec toutes les démonstrations sera inclue dans [7]. Avant d'énoncer nos résultats, nous donnons quelques définitions utiles.

DÉFINITION 0.1. – Supposons que $u$ est une solution radiale de (1). On dit que :

(i) $u$ est une solution de *décroissance pseudo-lente* s'il existe deux constantes $0 < C_1 < C_2$ telles que $C_1 = \liminf_{r\to\infty} r^{\alpha}u(r) < \limsup_{r\to\infty} r^{\alpha}u(r) = C_2$.

(ii) $u$ est une solution de *décroissance lente* s'il existe une constante $0 < c^*$ telle que $\lim_{r\to\infty} r^{\alpha}u(r) = c^*$.

(iii) $u$ est une solution de *décroissance rapide* s'il existe une constante $0 < C$ telle que $\lim_{r\to\infty} r^{\widetilde{N}-2}u(r) = C$, *où* $\widetilde{N} = \widetilde{N}_+$ (resp. $\widetilde{N} = \widetilde{N}_-$), dans le cas de $\mathcal{M}^{+}_{\lambda,\Lambda}$ (resp. $\mathcal{M}^{-}_{\lambda,\Lambda}$).

Voici nos résultats principaux :

THÉORÈME 0.2. – *Soit* $\mathcal{M}^{+}_{\lambda,\Lambda}$ *l'opérateur de l'équation* (1) *et* $\widetilde{N}_+ > 2$. *Alors, il existe des exposants* $1 < p^s_+ < p^*_+ < p^p_+$, *avec* $p^s_+ = \widetilde{N}_+/(\widetilde{N}_+ - 2)$, $p^p_+ = (\widetilde{N}_+ + 2)/(\widetilde{N}_+ - 2)$ *et*, $\max\{p^s_+, p^*\} < p^*_+ < p^p_+$, *tels que* :

(i) *Si* $1 < p < p^*_+$, *alors il n'existe pas de solution radiale non triviale de* (1).
(ii) *Si* $p = p^*_+$, *il existe une unique solution radiale de* (1) *à décroisssace rapide.*
(iii) *Si* $p^* < p \leqslant p^p_+$, *il existe une unique solution radiale de* (1) *à décroissance pseudo-lente.*
(iv) *Si* $p^p_+ < p$, *il existe une unique solution radiale de* (1) *à décroissance lente.*

Ici tout le unique solution est à changement d'echelle près. Un résultat similaire pour $\mathcal{M}^{-}_{\lambda,\Lambda}$ peut être trouvé dans [7].

In this Note we consider the study of solutions to the nonlinear elliptic equation

$$\mathcal{M}^{\pm}_{\lambda,\Lambda}\big(D^2u\big)+u^p=0,\quad u\geqslant 0\quad \text{in } \mathbb{R}^N, \tag{5}$$

where $N\geqslant 3$, $p>1$ and $\mathcal{M}^{\pm}_{\lambda,\Lambda}$ denotes the Pucci's extremal operators with parameters $0<\lambda\leqslant\Lambda$. These operators are defined as follows: let $e_i=e_i(D^2u)$, $i=1,\dots,N$, be the eigenvalues of $D^2u$, then

$$\mathcal{M}^{+}_{\lambda,\Lambda}\big(D^2u\big)=\Lambda\sum_{e_i>0}e_i+\lambda\sum_{e_i<0}e_i\quad\text{and}\quad \mathcal{M}^{-}_{\lambda,\Lambda}\big(D^2u\big)=\lambda\sum_{e_i>0}e_i+\Lambda\sum_{e_i<0}e_i. \tag{6}$$

When $\lambda=\Lambda=1$ we observe that the operators $\mathcal{M}^{\pm}_{\lambda,\Lambda}$ simply reduce to the Laplace operator, so (5) becomes

$$\Delta u+u^p=0,\quad u\geqslant 0\quad \text{in } \mathbb{R}^N. \tag{7}$$

This very well known equation has a solution set whose structure strongly depends on the exponent $p$. When $1<p<p^*:=(N+2)/(N-2)$ then Eq. (7) has no nontrivial solution vanishing at infinity, as can be proved using the celebrated Pohozaev identity [13]. If $p=p^*$ then it is shown by Caffarelli, Gidas and Spruck in [2] that, up to scaling, Eq. (7) possesses exactly one solution. This solution behaves like $C|x|^{2-N}$ near infinity. When $p>p^*$ then Eq. (7) admits radial solutions behaving like $C|x|^{-\alpha}$ near infinity, where $\alpha=2/(p-1)$. The critical character of $p^*$ is enhanced by the fact that it intervenes in compactness properties of Sobolev spaces, a reason for being known as critical Sobolev exponent.

It is interesting to mention that the nonexistence of solutions to (7) when $1<p<p^*$ holds even if we do not assume a given behavior at infinity. This result is known as Liouville type theorem and it was proved by Gidas and Spruck [8]. When $1<p\leqslant N/(N-2):=p^s$, then a Liouville type theorem is known for supersolutions of (7), that is solutions of the inequality

$$\Delta u+u^p\leqslant 0,\quad u\geqslant 0\quad \text{in } \mathbb{R}^N. \tag{8}$$

This number $p^s$ is called sometimes the second critical exponent for (7). In a recent paper [5], Cutri and Leoni extend this result for the Pucci's extremal operators. They consider the inequality

$$\mathcal{M}^{\pm}_{\lambda,\Lambda}\big(D^2u\big)+u^p\leqslant 0,\quad u\geqslant 0\quad \text{in } \mathbb{R}^N, \tag{9}$$

and define the dimension-like numbers $\widetilde{N}_+=\frac{\lambda}{\Lambda}(N-1)+1$ and $\widetilde{N}_-=\frac{\Lambda}{\lambda}(N-1)+1$. Then they prove that for $1<p\leqslant p^s_\pm:=\widetilde{N}_\pm/(\widetilde{N}_\pm-2)$ Eq. (9) has only the trivial solution.

In view of the results for the semilinear Eq. (7) that we have discussed above and the new results for inequality (9) just mentioned, it is natural to ask about the existence of critical exponents of the Sobolev type for (5). In particular it would be interesting to understand the structure of solutions for Eq. (5) in terms for different values of $p>1$. It would also be interesting to prove Liouville type theorems for positive solutions in $\mathbb{R}^N$ and to understand the mechanisms for existence of positive solutions in general bounded domains.

It is the purpose of this Note to announce some results in the case of radially symmetric solutions. The general problem seems to be too difficult at this point, and we expect that our results will shed some light on it. The detailed version with complete proofs will be published in a forthcoming paper [7]. Before we state our results we give a definition to classify the possible radial solutions of Eq. (7).

DEFINITION 0.1. – Assume $u$ is a radial solution of (5) then we say that:

(i) $u$ is a *pseudo-slow decaying solution* if there exist constants $0<C_1<C_2$ such that $C_1=\liminf_{r\to\infty}r^\alpha u(r)<\limsup_{r\to\infty}r^\alpha u(r)=C_2$.

(ii) $u$ is a *slow decaying solution* if there exists $0<c^*$ such that $\lim_{r\to\infty}r^\alpha u(r)=c^*$.

(iii) $u$ is a *fast decaying solution* if there exists $0<C$ such that $\lim_{r\to\infty}r^{\widetilde{N}-2}u(r)=C$, where $\widetilde{N}=\widetilde{N}_+$ or $\widetilde{N}=\widetilde{N}_-$, depending if $\mathcal{M}^{+}_{\lambda,\Lambda}$ or $\mathcal{M}^{-}_{\lambda,\Lambda}$ appears in (5).

Our main results are summarized in the following theorem.

THEOREM 0.2. – *Suppose we consider the Pucci's extremal operator* $\mathcal{M}^+_{\lambda,\Lambda}$ *in Eq.* (5). *Suppose in addition that* $\widetilde{N}_+ > 2$. *Then there are critical exponents* $1 < p^s_+ < p^*_+ < p^p_+$, *with* $p^s_+ = \widetilde{N}_+/(\widetilde{N}_+ - 2)$, $p^p_+ = (\widetilde{N}_+ + 2)/(\widetilde{N}_+ - 2)$ *and* $\max\{p^s_+, p^*\} < p^*_+ < p^p_+$, *that satisfy*:
(i) *If* $1 < p < p^*_+$ *then there is no nontrivial radial solution of* (5).
(ii) *If* $p = p^*_+$ *then there is a unique fast decaying radial solution of* (5).
(iii) *If* $p^* < p \leqslant p^p_+$ *then there is a unique pseudo-slow decaying radial solution to* (5).
(iv) *If* $p^p_+ < p$ *then there is a unique slow decaying radial solution to* (5).

Here the uniqueness is understood up to scaling. Regarding the operator $\mathcal{M}^-_{\lambda,\Lambda}$ we have a similar result. See [7] Theorem 1.2. In the rest of this Note we will deal only with $\mathcal{M}^+_{\lambda,\Lambda}$.

Our approach consists in a combination of the Emden–Fowler phase plane analysis with the Coffman–Kolodner technique. We start considering the classical Emden–Fowler transformation that allows us to view the problem in the phase plane. With the aid of suitable energy functions we understand much of the behavior of the solutions. Their asymptotic behavior is obtained in some cases using the Poincaré–Bendixon theorem. This phase plane analysis has been used in related problems by Clemons and Jones [3], Kajikiya [9] and Erbe and Tang [6] among many others.

On the other hand we use the Coffman–Kolodner technique. The idea is to differentiate the solution with respect to the initial value, see [4] and [10]. The function so obtained possesses valuable information on the problem. This idea has been used by several authors in dealing with uniqueness questions, in particular by Kwong [11], Kwong and Zhang [12] and Erbe and Tang [6]. In our case though we do not differentiate with respect to the initial value, which is kept fixed, but with respect to the power $p$. Thus the variation function satisfies a non-homogeneous equation, in contrast with the situations treated earlier.

## 1. Preliminaries

Since we are dealing only with radially symmetric functions, the Pucci's extremal operators take a very simple form. We consider the initial value problem

$$u'' = M\left(\frac{-\lambda(N-1)}{r}u' - u^p\right) \quad \text{in } (0,+\infty), \quad u(0) = \gamma, \quad u'(0) = 0, \tag{10}$$

where $\gamma > 0$ and $M(s) = s/\Lambda$ if $s \geqslant 0$ and $M(s) = s/\lambda$ if $s < 0$. We note that this equation possesses a unique solution we denote by $u(r,p,\gamma)$ and that non-negative solutions of (10) correspond to radially symmetric solutions of (5). It can be proved that the solutions of (10) are decreasing, while they remain positive and that they have the following scaling property: $\gamma u(\gamma^{1/\alpha} r, p, \gamma_0) = u(r, p, \gamma_0\gamma)$, for all $\gamma_0, \gamma > 0$.

In the next definition we classify the exponent $p$ according to the behavior of the solution of the initial value problem (10). We define:

$$\mathcal{C} = \{p \mid p > 1,\ u(r,p,\gamma) \text{ has a finite zero}\},$$
$$\mathcal{P} = \{p \mid p > 1,\ u(r,p,\gamma) \text{ is positive and is pseudo-slow decaying}\},$$
$$\mathcal{S} = \{p \mid p > 1,\ u(r,p,\gamma) \text{ is positive and is slow decaying}\},$$
$$\mathcal{F} = \{p \mid p > 1,\ u(r,p,\gamma) \text{ is positive is and fast decaying}\}.$$

In view of the scaling property, we notice that these sets do not depend on the particular value of $\gamma > 0$.

## 2. Emden–Fowler analysis

An important step in the proof is to perform the classical Emden–Fowler change of variables $x(t) = r^\alpha u(r)$, $r = e^t$. This allows us to use phase plane analysis. We have that the initial value problem (10) reduces to the autonomous differential equation

$$x'' = -\alpha(\alpha+1)x + (1+2\alpha)x' + M\big(\lambda(N-1)(\alpha x - x') - x^p\big), \quad x(-\infty) = 0, \quad x'(-\infty) = 0. \tag{11}$$

Studying this dynamical system we obtain the following two propositions:

PROPOSITION 2.1. – (a) *If* $p > \frac{\widetilde{N}+2}{\widetilde{N}-2}$ *then* $p \in \mathcal{S}$; (b) *If* $p \leqslant \max\{\frac{\widetilde{N}}{\widetilde{N}-2}, \frac{N+2}{N-2}\}$ *then* $p \in \mathcal{C}$.

PROPOSITION 2.2. – *We have*: (i) $\frac{\widetilde{N}+2}{\widetilde{N}-2} \in \mathcal{P}$; (ii) $\mathcal{P} \setminus \{\frac{\widetilde{N}+2}{\widetilde{N}-2}\}$ *is open, and* (iii) *If* $p \leqslant \frac{\widetilde{N}+2}{\widetilde{N}-2}$, *then* $p \notin \mathcal{S}$.

In the proof of these propositions we use two energy like functions

$$e(t) = \frac{(x')^2}{2} + \frac{\alpha x^{p+1}}{2\lambda(N-1)} - \frac{(\alpha x)^2}{2} \quad \text{and} \quad E(t) = \frac{(x')^2}{2} + \frac{x^{p+1}}{\Lambda(p+1)} - \frac{\tilde{b}x^2}{2}. \tag{12}$$

These energy functions allow us to understand the behavior of the trajectories. The Poincaré–Bendixon theorem is also used. It is interesting to note that in the range of $p$ where the solution is pseudo-slow decaying, the periodic orbit of the dynamical system corresponds to a singular solution to $\mathcal{M}^+_{\lambda,\Lambda}(D^2u) + u^p = 0$, which change infinitely many times its concavity. These solutions are not present in the case of the Laplacian and appear in trying to compensate the fact that $\lambda < \Lambda$.

## 3. Coffman–Kolodner analysis

The second main step in our proof is to understand the nature of the solutions obtained near a fast decaying solution. Our final goal is to prove that $\mathcal{F}$ is a Singleton. The idea is to differentiate the solution of (10) with respect to $p$. The resulting function $\varphi$ has valuable information on the solutions near the fast decaying one. By analyzing $\varphi$ we prove the following two propositions, that are crucial in the proof of our main results.

PROPOSITION 3.1. – *If* $p^* \in \mathcal{F}$, *then for* $p < p^*$ *close to* $p^*$ *we have* $p \in \mathcal{C}$.

PROPOSITION 3.2. – *If* $p^* \in \mathcal{F}$, *then for* $p > p^*$ *close to* $p^*$ *we have* $p \in \mathcal{S} \cup \mathcal{P}$.

We prove that the function $\varphi(\cdot, p)$ is a $C^1$ function in $\mathbb{R}^+$, for $p$ near $p^*$. In order to understand the asymptotic behavior of $\varphi$ we study the function $w = w_\theta(r) = r^\theta u(r, p^*)$, for $\theta > 0$ chosen so that $\theta = (\widetilde{N}-1)/2$ if $\widetilde{N} > 3$ and $\theta = (\widetilde{N}-2)/2$ if $2 < \widetilde{N} \leqslant 3$. This function was introduced by Erbe and Tang in [6], for a related problem. We also define $y(r) = \partial w(r)/\partial p = r^\theta \varphi$. When $\widetilde{N} > 3$, $y$ satisfies the equation

$$y'' + \left(\frac{(\widetilde{N}-1)(3-\widetilde{N})}{4r^2} + \frac{p^* u^{p^*-1}}{\Lambda}\right) y + r^\theta \frac{u^{p^*}}{\Lambda} \log u = 0 \quad \text{if } r > r_0. \tag{13}$$

Using that $u$ is a fast decaying solution we find that the coefficient in the second term of (13) is negative for $r$ large. A similar situation occurs when $2 < \widetilde{N} \leqslant 3$. The following lemma on the asymptotic behavior of $y$ is crucial.

LEMMA 3.3. – *The function* $y$ *defined above satisfies* $y(r) > 0$ *and* $y'(r) > 0$ *for* $r$ *large.*

*Sketch of proof of Proposition* 3.1. – Let $p^* \in \mathcal{F}$ and $p < p^*$ sufficiently close to $p^*$. Suppose first that $p \in \mathcal{P} \cup \mathcal{S}$. Then, comparing the growth of $u(r, p^*)$ and $u(r, p)$ we find that for $r$ large $u(r, p^*) < u(r, p)$. But, from Lemma 3.3 we have $y(\bar{r}) > 0$ for large $\bar{r}$, which implies $(u(\bar{r}, p) - u(\bar{r}, p^*))/(p - p^*) > 0$ for $p$ close to $p^*$. Thus $u(\bar{r}, p) - u(\bar{r}, p^*) < 0$.

Suppose next that $p \in \mathcal{F}$ and $\widetilde{N} > 3$. Let us define $w(r) = r^{(\widetilde{N}-1)/2} u(r, p)$, $w_*(r) = r^{(\widetilde{N}-1)/2} u(r, p^*)$ and $v = w_* - w$. We see that for $r$ large $v$ satisfies the equation

$$v''(r) + \frac{(\widetilde{N}-1)(3-\widetilde{N})}{4r^2} v(r) + r^{(\widetilde{N}-1)/2} \frac{(u(r, p^*)^{p^*} - u(r, p)^p)}{\Lambda} = 0. \tag{14}$$

We use the mean value theorem and the fact that $u'(r,p) < 0$, to find $\bar r$ and $\varepsilon > 0$ such that $u(r,p) < 1$, for all $r \geqslant \bar r$ and for all $p \in (p^* - \varepsilon, p^*)$. Then, for an appropriate $\xi(r)$ we find that $v$ satisfies

$$v'' + \left(\frac{(\widetilde N - 1)(3 - \widetilde N)}{4r^2} + \frac{p^*(\xi(r))^{p^*-1}}{\Lambda}\right) v \geqslant 0, \quad \text{for all } r \geqslant \bar r. \tag{15}$$

Using the growth of $u(r, p^*)$ and that $p > \widetilde N/(\widetilde N - 2)$ we conclude the existence of $r^*$ such that

$$\frac{(\widetilde N - 1)(3 - \widetilde N)}{4r^2} + \frac{p^*(u(r,p^*))^{p^*-1}}{\Lambda} < 0, \quad \text{for all } r \geqslant r^*. \tag{16}$$

On the other hand, by Lemma 3.3, there exists $\tilde r$ such that $y(\tilde r) > 0$ and $y'(\tilde r) > 0$, for $\tilde r > \max\{r^*, \bar r\}$. Thus $v(\tilde r) > 0$ and $v'(\tilde r) > 0$ for a fix $p \in (p^* - \varepsilon, p^*)$ close to $p^*$. But since $p \in \mathcal{F}$ and $\widetilde N > 3$, we have $v(r) \to 0$ as $r \to \infty$. Thus $v$ has a positive maximum, let us say in $\hat r$. Since $y(\hat r) > 0$, we get $u(\hat r, p) < u(\hat r, p^*)$, hence $u(\hat r, p^*) > \xi(\hat r)$. Thus, from (16) we have $(\widetilde N - 1)(3 - \widetilde N)/(4\hat r^2) + p^*(\xi(\hat r))^{p^*-1}/\Lambda < 0$. But then we get a contradiction from (15), and the fact that $\hat r$ is a maximum of $v$. The case $2 < \widetilde N \leqslant 3$ is proved in a similar way. □

The proof of Theorem 0.2 is a direct consequence of previous propositions, the openness of $\mathcal{C}$ and $\mathcal{P} \setminus \{(\widetilde N_+ + 2)/(\widetilde N_+ - 2)\}$.

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