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A generation theorem for kinetic equations with non-contractive boundary operators

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Abstract In this Note, we present some c_0-semigroup generation results in L_p-spaces for the advection operator submitted to non-contractive boundary conditions covering in particular the classical Maxwell-type boundary conditions. ***To cite this article: B. Lods, C. R. Acad. Sci. Paris, Ser. I 335 (2002) 655–660.***

Un résultat de génération pour des équations cinétiques soumises à des conditions frontières non contractives

Résumé Dans cette Note, on présente quelques résultats de génération de c_0-semigroupe dans les espaces L_p pour l'opérateur d'advection soumis à des conditions aux limites non contractives, couvrant par exemple les conditions frontières de type Maxwell. ***Pour citer cet article : B. Lods, C. R. Acad. Sci. Paris, Ser. I 335 (2002) 655–660.***

Version française abrégée

L'objectif de cette Note est d'énoncer quelques résultats de génération de c_0-semigroupe dans L_p $(1 \leqslant p < \infty)$ pour l'opérateur d'advection

$$T_H\psi(x,v) = -v \cdot \nabla_x \psi(x,v), \quad (x,v) \in \Omega \times V,$$

soumis à des conditions aux limites *non-contractives*. Ici, Ω est un ouvert régulier de $\mathbb{R}^N$ $(N \geqslant 1)$ désignant indifféremment un domaine intérieur ou extérieur, alors que V désigne le support d'une mesure de Radon positivie $d\mu$ sur $\mathbb{R}^N$. L'opérateur T_H décrit le transport de particules (neutrons, molécules gazeuses, etc.) dans le domaine Ω. La fonction $\psi(x,v)$ désigne la densité de particules ayant la position $x \in \Omega$ animées de la vitesse $v \in V$. Les conditions aux limites associées à cet opérateur sont gouvernées par un *opérateur linéaire* H reliant la distribution de particules sur le bord rentrant Γ_- à celle sur le bord sortant Γ_+ :

$$\psi_{|\Gamma_-} = H(\psi_{|\Gamma_+}),$$

où $\Gamma_\pm = \{(x,v) \in \partial\Omega;\ \pm v \cdot n(x) \geqslant 0\}$, $n(x)$ désignant le vecteur normal extérieur à Ω en $x \in \partial\Omega$. L'opérateur frontière H est un opérateur *linéaire et borné* de L_p^+ sur L_p^- où $L_p^\pm$ sont des espaces de traces

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adéquats sur $\Gamma_\pm$ (voir section suivante). Pour la signification physique de l'opérateur H dans le cadre de la cinétique des gaz, on renvoie le lecteur à l'ouvrage de Cercignani et al. [5] (Chapitre 8) (voir aussi [4]).

Il est connu de longue date que, pour un opérateur frontière contractant $\|H\| < 1$, l'opérateur d'advection T_H est générateur d'un c_0-semigroupe de $\mathrm{L}_p(\Omega \times V, \mathrm{d}x\,\mathrm{d}\mu(v))$ $(1 \leqslant p < \infty)$ [6]. L'objectif principal de cette Note est de généraliser ce résultat à des opérateurs frontières *non-contractants*. En effet, de nombreux problèmes de dynamique des populations font intervenir des opérateurs frontières *multiplicatifs*. Par exemple, la *prolifération* d'une population de cellules peut être modélisée à l'aide d'un opérateur frontière H tel que $\|Hu\| = 2\|u\|$ $(u \geqslant 0,\ u \in \mathrm{L}_p^+)$. Ceci indique que, lors de la mitose, chaque cellule mère se divise pour donner naissance à deux cellules filles [11]. Le théorème de génération mentionné plus haut s'avère inopérant pour ce type de problèmes. Récemment Boulanouar [2] a démontré que, dans le cas particulier du transport uni-dimensionnel dans une bande de largeur $2a$ $(a > 0)$, l'opérateur d'advection T_H engendre un c_0-semigroupe de $\mathrm{L}_p(]-a, a[\times[-1, 1], \mathrm{d}x\,\mathrm{d}v)$ pour un opérateur frontière H borné *arbitraire*. En revanche, en dimension quelconque, la situation est totalement différente. Comme il a été souligné dans [3], la principale difficulté mathématique repose sur l'existence de *temps de séjour* des particules à l'intérieur de Ω *arbitrairement petits*. Rappelons que, pour tout $(x, v) \in \overline{\Omega} \times V$, ce temps de séjour est défini par

$$t(x, v) = \sup\{t > 0;\ x - sv \in \Omega\ \forall 0 < s < t\}.$$

Si la vitesse $v \in V$ d'une particule pénétrant le domaine en un point $x \in \partial\Omega$ est tangente à Ω alors celle-ci ressort instantanément du domaine (au même point x) et $t(x, v) = 0$. Dans un tel cas, il est possible que (pour un certain opérateur frontière H) T_H n'engendre pas de c_0-semigroupe de $\mathrm{L}_p(\Omega \times V, \mathrm{d}x\,\mathrm{d}\mu(v))$ (voir contre-exemple 1). Ainsi, pour que T_H soit générateur, il suffit que l'opérateur frontière « tienne peu compte » des vitesses tangentielles au sens du théorème suivant.

THÉORÈME 0.1. – *Soit* χ_ε *l'opérateur de multiplication par la fonction caractéristique de* $\{(x, v) \in \Gamma_+;\ t(x, v) \leqslant \varepsilon\}$ $(\varepsilon > 0)$. *Soit* $H \in \mathcal{L}(\mathrm{L}_p^+, \mathrm{L}_p^-)$ *tel que* $\limsup_{\varepsilon \to 0} \|H\chi_\varepsilon\| < 1$. *Alors,* T_H *est générateur d'un* c_0*-semigroupe de* $\mathrm{L}_p(\Omega \times V, \mathrm{d}x\,\mathrm{d}\mu(v))$ $(1 \leqslant p < \infty)$.

Dans la dernière section de cette Note, nous appliquons ce théorème aux cas modèles des conditions de réflexion de type Maxwell couvrant la majorité des conditions aux frontières [5] (voir Proposition 2.2 et Proposition 2.1 pour un analogue lorsque $p = 1$).

COROLLAIRE 0.2. – *Supposons* $1 < p < \infty$. *Soit* $H \in \mathcal{L}(\mathrm{L}_p^+, \mathrm{L}_p^-)$ *défini par*

$$\begin{aligned} H(\psi_{|\Gamma_+})(x, v) = {} & \alpha(x)\, \psi_{|\Gamma_+}\big(x, v - 2\big(v \cdot n(x)\big)n(x)\big) \\ & + \beta(x) \int_{\{v' \cdot n(x) > 0\}} k(v, v')\psi_{|\Gamma_+}(x, v')\big|v' \cdot n(x)\big|\,\mathrm{d}v', \quad (x, v) \in \Gamma_-\ ; \end{aligned}$$

où $\|\alpha(\cdot)\|_{\mathrm{L}^\infty(\partial\Omega)} < 1$ *et* $\beta(\cdot) \in \mathrm{L}_+^\infty(\partial\Omega)$. *Si*

$$\sup_{x \in \partial\Omega} \int_{\{v \cdot n(x) < 0\}} \big|v \cdot n(x)\big|\,\mathrm{d}v \left(\int_{\{v' \cdot n(x) \geqslant 0\}} \big|k(v, v')\big|^q \big|v' \cdot n(x)\big|\,\mathrm{d}v' \right)^{p/q} < \infty \quad \left(\frac{1}{p} + \frac{1}{q} = 1\right),$$

alors T_H *engendre un* c_0*-semigroupe de* $\mathrm{L}_p(\Omega \times V, \mathrm{d}x\,\mathrm{d}v)$ $(1 < p < \infty)$.

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1. Introduction and preliminaries

In this Note, we show that the advection operator in L_p-spaces

$$T_H\psi(x, v) = -v \cdot \nabla_x \psi(x, v), \quad (x, v) \in \Omega \times V,$$

generates a c_0-semigroup for a (large) class of *non-contractive* boundary operators H. Here, Ω is a smooth open subset of $\mathbb{R}^N$ $(N \geqslant 1)$, which can be an interior domain or an exterior one, and V is the support of a positive Radon measure $\mathrm{d}\mu$ on $\mathbb{R}^N$. This operator describes the transport of particles (neutrons, molecules of gas, etc.) in the domain Ω. The function $\psi(x,v)$ represents the density of particles having the position $x \in \Omega$ and the velocity $v \in V$. The behavior of the density ψ at the wall of the domain Ω is governed by an abstract boundary operator H relating the incoming flux to the outgoing one. More precisely, let us denote by Γ_- and Γ_+ the incoming and outgoing part, respectively, of the boundary of the phase space $\Omega \times V$

$$\Gamma_\pm = \big\{(x,v) \in \partial\Omega \times V;\ \pm v \cdot n(x) \geqslant 0\big\},$$

where $n(x)$ stands for the outward normal unit at $x \in \partial\Omega$. If we note $\psi_{|\Gamma_\pm}$ the density of particles at $\Gamma_\pm$ then it is assumed that

$$\psi_{|\Gamma_-} = H\psi_{|\Gamma_+},$$

where H is a linear bounded operator acting on some suitable trace spaces.

It is well known that, for contractive boundary conditions (i.e., $\|H\| < 1$), T_H generates a c_0-semigroup in $\mathrm{L}_p(\Omega \times V, \mathrm{d}x\,\mathrm{d}\mu(v))$ $(1 \leqslant p < \infty)$ [6]. It is the main business of this Note to generalize this result to *non-contractive* boundary operators. Indeed, in population dynamics, the boundary conditions are naturally governed by a *multiplicative* operator H. Typically, for a *proliferating population of cells*, mother cells undergo fission to give birth to two daughter cells, i.e., $\|Hu\| = 2\|u\|$ $(u \geqslant 0)$ [11].

Recently, Boulanouar has shown that the transport operator T_H in a slab with thickness $2a$ $(a > 0)$ is a generator of a c_0-semigroup in $\mathrm{L}_p(]-a,a[\times[-1,1], \mathrm{d}x\,\mathrm{d}v)$ for *arbitrary boundary operator H* [2]. The case of multi-dimensionnal geometry is more complicated. As it was mentionned in [3] (see also the recent contribution [10]), the main difficulty relies on the fact that, for a convex domain $\Omega \subset \mathbb{R}^N$ with $N > 1$, the *"time of sojourn"* of particles in Ω may be *arbitrary small*. More precisely, let us introduce

$$t(x,v) = \sup\big\{t > 0;\ x - sv \in \Omega\ \forall 0 < s < t\big\}, \quad (x,v) \in \overline{\Omega} \times V.$$

For the reader convenience, we will note $\tau(x,v) = t(x,v)$ for any $(x,v) \in \partial\Omega \times V$. Then, if $\inf\{\tau(x,v), (x,v) \in \Gamma_+ \times V\} = 0$, a particle incoming at $x \in \partial\Omega$ may outgo at the same point instantaneously. This means that the velocity of such a particle is tangential at $x \in \partial\Omega$. In such a case, one can exhibit a boundary operator H for which T_H is not a generator of a c_0-semigroup in $\mathrm{L}_p(\Omega \times V, \mathrm{d}x\,\mathrm{d}\mu(v))$ $(1 \leqslant p < \infty)$ (see counterexample 1). Roughly speaking, the suitable assumption on the boundary operator H is that such tangential velocities are weakly-taken into account by H, regardless of its size (see Theorem 1.3). We will also show how this assumption is fullfilled by Maxwell-type boundary conditions (see Section 2).

Let us introduce

$$X_p = \mathrm{L}_p\big(\Omega \times V, \mathrm{d}x\,\mathrm{d}\mu(v)\big), \quad 1 \leqslant p < \infty,$$

where Ω is a smooth interior (respectively exterior) domain of $\mathbb{R}^N$ $(N \geqslant 1)$, i.e., Ω is bounded (respectively, $\mathbb{R}^N \setminus \Omega$ is bounded). We define the partial Sobolev space

$$\mathrm{W}_p = \{\psi \in X_p;\ v \cdot \nabla_x \psi \in X_p\}.$$

Suitable L_p-spaces for the traces on $\Gamma_\pm$ are defined as

$$\mathrm{L}^\pm_p = \mathrm{L}_p\big(\Gamma_\pm; \big|v \cdot n(x)\big|\,\mathrm{d}\gamma(x)\,\mathrm{d}\mu(v)\big),$$

$\mathrm{d}\gamma(\cdot)$ being the Lebesgue measure on $\partial\Omega$. For any $\psi \in \mathrm{W}_p$, one can define the traces $\psi_{|\Gamma_\pm}$ on $\Gamma_\pm$, however these traces do not belong to $\mathrm{L}^\pm_p$ but to a certain weighted space [6]. Let us define

$$\widetilde{\mathrm{W}}_p = \big\{\psi \in \mathrm{W}_p;\ \psi_{|\Gamma_\pm} \in \mathrm{L}^\pm_p\big\}.$$

Let H be a bounded linear operator from L_p^+ to L_p^-

$$H \in \mathcal{L}\big(\mathrm{L}_p^+, \mathrm{L}_p^-\big), \quad 1 \leqslant p < \infty.$$

The free streaming operator associated with the boundary condition H is

$$\begin{cases} T_H : D(T_H) \subset X_p \to X_p, \\ \varphi \mapsto T_H\varphi(x,v) := -v \cdot \nabla_x \varphi(x,v), \end{cases}$$

with domain $D(T_H) := \{\psi \in \widetilde{W}_p$ such that $\psi_{|\Gamma_-} = H(\psi_{|\Gamma_+})\}$. We have in mind the classical examples of boundary operators (see Cercignani et al. [5]).

1.1. Generation theorem

We begin this section by recalling the classical generation result for contractive boundary operators (see, for instance, [6, Theorem 2.2, Chapter XII]).

THEOREM 1.1. – *Let $H \in \mathcal{L}(\mathrm{L}_p^+, \mathrm{L}_p^-)$ $(1 \leqslant p < \infty)$. If $\|H\|_{\mathcal{L}(\mathrm{L}_p^+, \mathrm{L}_p^-)} < 1$ then T_H generates a c_0-semigroup in X_p.*

Remark 1. – When $H \geqslant 0$, by using monotone convergence arguments, one can show that the closure of T_H generates a c_0-semigroup in X_p whenever $\|H\| = 1$ [6, Theorem 2.3, Chapter XII].

For *non-contractive* boundary operator we follow our strategy for population dynamics [9]: to solve the evolution problem

$$\begin{cases} \dfrac{\partial \psi}{\partial t}(x,v,t) + v \cdot \nabla_x \psi(x,v,t) = 0, \\ \psi_{|\Gamma_-} = H(\psi_{|\Gamma_+}), \\ \psi(x,v,0) = \psi_0(x,v), \end{cases}$$

with $\psi_0 \in X_p$, we first make a suitable change of unknown in the spirit of the one used in [9] (see also [6, Chapter XIII]). This leads to an equivalent evolution equation involving a contractive boundary operator provided that H is *"small enough" in the neighbourhood of the tangential velocities.*

More precisely, for any $0 < q < 1$ let us define

$$M_q : u \in \mathrm{L}_p^+ \mapsto M_q u(x,v) = q^{\tau_k(x,v)} u(x,v) \in \mathrm{L}_p^+,$$

with $\tau_k(x,v) = \max\{\tau(x,v); k\}$, $(x,v) \in \Gamma_+$, k being a *fixed* positive real number. Let B_q be defined as

$$B_q : \varphi \in X_p \mapsto B_q\varphi(x,v) = q^{t_k(x,v)} \varphi(x,v) \in X_p,$$

where $t_k(x,v) = \max\{t(x,v);\ k\}$, $(x,v) \in \Omega \times V$. Let us introduce the streaming operator associated to the boundary operator $HM_q \in \mathcal{L}(\mathrm{L}_p^+, \mathrm{L}_p^-)$

$$\begin{cases} T_{H_q} : D(T_{H_q}) \subset X_p \to X_p, \\ \varphi \mapsto T_{H_q}\varphi(x,v) := -v \cdot \nabla_x \varphi(x,v) - \ln q\, \varphi(x,v) \end{cases}$$

with $D(T_{H_q}) = \{\psi \in \widetilde{W}_p;\ \psi_{|\Gamma_-} = HM_q(\psi_{|\Gamma_+})\}$. The operators T_H and T_{H_q} are related as follows.

LEMMA 1.2. – *For any $0 < q < 1$, $B_q^{-1} D(T_H) = D(T_{H_q})$ and $T_H = B_q T_{H_q} B_q^{-1}$.*

Remark 2. – As a consequence of Lemma 1.2, for a given $0 < q < 1$, T_{H_q} is a generator of a c_0-semigroup in X_p if and only if T_H is.

For any $\varepsilon > 0$, let us note χ_ε the multiplication operator in L_p^+ by the characteristic function of the set $\{(x,v) \in \Gamma_+;\ \tau(x,v) \leqslant \varepsilon\}$. We are in position to prove the main result of this Note.

THEOREM 1.3. – *Let* $H \in \mathcal{L}(\mathrm{L}_p^+, \mathrm{L}_p^-)$. *Assume that* $\limsup_{\varepsilon\to 0} \|H\chi_\varepsilon\|_{\mathcal{L}(\mathrm{L}_p^+,\mathrm{L}_p^-)} < 1$. *Then* T_H *generates a* c_0*-semigroup in* X_p $(1 \leqslant p < \infty)$.

Proof. – Clearly, for any $0 < q < 1$ and $0 < \varepsilon < k$,

$$\begin{aligned}\|HM_q\| &\leqslant \|H\chi_\varepsilon M_q\| + \big\|H(I-\chi_\varepsilon)M_q\big\| \\ &\leqslant \|H\chi_\varepsilon\| + \|H\|\big\|(I-\chi_\varepsilon)M_q\big\|.\end{aligned}$$

Since for any *fixed* $0 < \varepsilon < k$,

$$\begin{aligned}\big\|(I-\chi_\varepsilon)M_q\big\| &= \sup\big\{\mathrm{e}^{\tau_k(x,v)\ln q};\ (x,v) \in \Gamma_+ \text{ et } \tau_k(x,v) \geqslant \varepsilon\big\} \\ &= \mathrm{e}^{\varepsilon \ln q} \longrightarrow 0 \quad (q \to 0)\end{aligned}$$

we get that $\limsup_{q\to 0} \|HM_q\| < 1$. Thus, there exists a real number $0 < q_0 < 1$ such that $\|HM_{q_0}\| < 1$ and $T_{H_{q_0}}$ is a generator of a c_0-semigroup in X_p. We conclude thanks to Remark 2. □

Remark 3. – It is possible to show that the type of the c_0-semigroup generated by T_H is less than (or equal to) $\max\{\frac{1}{\varepsilon_0}\ln\|H\|, 0\}$ where $\varepsilon_0 = \sup\{\varepsilon > 0;\ \|H\chi_\varepsilon\| < 1\}$.

Remark 4. – In the case $p = 1$ a completely different proof was given by Latrach et Mokhtar-Kharroubi [7] by using a theorem of Batty and Robinson [1].

Remark 5. – Roughly speaking, the assumption on Theorem 1.3 is a smallness assumption of H in the neighbourhood of $\{(x,v) \in \Gamma_+;\ \tau(x,v) = 0\} = \{(x,v) \in \Gamma_+;\ v \cdot n(x) = 0\}$. This means that the tangential velocities are weakly taken into account by H *regardless* of its norm.

Remark 6. – Let us note that for transport equations in a slab with thickness $2a$ $(a > 0)$, $\inf\{\tau(x,\mu);\ (x,\mu) \in \Gamma_+\} = 2a > 0$. An immediate consequence is that T_H generates a c_0-semigroup for any boundary operator $H \in \mathcal{L}(\mathrm{L}_p^+, \mathrm{L}_p^-)$. Thus we find again Boulanouar result [2].

Transport equation arising in population dynamics [9] provides us with the following counterexample showing the optimality of Theorem 1.3.

Example 1. – Let $\Omega = \{(a,l) \in \mathbb{R}^2;\ 0 < a < l,\ 0 < l < 1\}$ and assume that V is reduced to $V = \{(1,0)\}$. The measure we consider on V is the Dirac mass supported by $(1,0)$. The free streaming operator writes $T_H\varphi(a,l) := -\frac{\partial\varphi}{\partial a}(a,l)$. Assume that $H \in \mathcal{L}(\mathrm{L}_p((0,1);\mathrm{d}l))$ $(1 < p < \infty)$ is given by

$$H(\varphi_{|\Gamma_+})(l) = c\varphi_{|\Gamma_-}(l), \quad 0 < l < 1,$$

with $c > 1$. One can check that $\|H\chi_\varepsilon\|_{\mathrm{L}_p((0,1);\mathrm{d}l)} = c > 1$ $(\forall\varepsilon > 0)$. On the other hand, the spectrum of T_H is not included in a left half-plane. Thus, T_H is not a generator of a c_0-semigroup in X_p $(1 \leqslant p < \infty)$ (see [8] for details).

2. Application to Maxwell-type boundary conditions

We show here how Theorem 1.3 can be successfully applied to Maxwell-type boundary conditions. A first result deals with the case $p = 1$.

PROPOSITION 2.1. – *Assume* $p = 1$. *Let* $H \in \mathcal{L}(\mathrm{L}_1^+, \mathrm{L}_1^-)$ *be given by*

$$\begin{aligned}H(\psi_{|\Gamma_+})(x,v) &= \alpha(x)\psi_{|\Gamma_+}\big(x, v - 2\big(v\cdot n(x)\big)n(x)\big) \\ &\quad + \int_{\{v'\cdot n(x)\geqslant 0\}} h(x,v,v')\psi_{|\Gamma_+}(x,v')\big|v'\cdot n(x)\big|\,\mathrm{d}v', \quad (x,v) \in \Gamma_-;\end{aligned}$$

where $\|\alpha(\cdot)\|_{L^\infty(\partial\Omega)} < 1$ *and the kernel* $h(\cdot,\cdot,\cdot)$ *is non-negative. Then,* T_H *generates a* c_0*-semigroup in* X_1 *if*

$$\lim_{\varepsilon\to 0} \operatorname*{ess\,sup}_{\tau(x,v')\leqslant\varepsilon} \int_{\{v'\cdot n(x)>0\}} h(x,v,v')\left|v'\cdot n(x)\right| \mathrm{d}v' < 1 - \left\|\alpha(\cdot)\right\|_{L^\infty(\partial\Omega)}. \tag{1}$$

Proof. – Eq. (1) is equivalent to $\limsup_{\varepsilon\to 0}\|H\chi_\varepsilon\| < 1$. □

For $1 < p < \infty$, we have the following.

PROPOSITION 2.2. – *Assume* $1 < p < \infty$ *and let* $H \in \mathcal{L}(L_p^+, L_p^-)$ *be given by*

$$\begin{aligned} H(\psi_{|\Gamma_+})(x,v) &= \alpha(x)\,\psi_{|\Gamma_+}\big(x, v - 2\big(v\cdot n(x)\big)n(x)\big) \\ &\quad + \beta(x)\int_{\{v'\cdot n(x)\geqslant 0\}} k(v,v')\psi_{|\Gamma_+}(x,v')\left|v'\cdot n(x)\right|\mathrm{d}v', \quad (x,v)\in\Gamma_-; \end{aligned}$$

where $\|\alpha(\cdot)\|_{L^\infty(\partial\Omega)} < 1$ *and* $\beta(\cdot)\in L_+^\infty(\partial\Omega)$. *Assume that*

$$\sup_{x\in\partial\Omega}\int_{\{v\cdot n(x)<0\}}\left|v\cdot n(x)\right|\mathrm{d}v\left(\int_{\{v'\cdot n(x)\geqslant 0\}}\left|k(v,v')\right|^q\left|v'\cdot n(x)\right|\mathrm{d}v'\right)^{p/q} < \infty \quad \left(\frac{1}{p}+\frac{1}{q}=1\right). \tag{2}$$

Then, T_H *is a generator of a* c_0*-semigroup in* X_p.

Proof. – By using continuity arguments and Eq. (2), one proves that $\lim_{\varepsilon\to 0}\|K\chi_\varepsilon\| = 0$ where

$$K(\psi_{|\Gamma_+})(x,v) = \beta(x)\int_{\{v'\cdot n(x)\geqslant 0\}} k(v,v')\psi_{|\Gamma_+}(x,v')\left|v'\cdot n(x)\right|\mathrm{d}v', \quad (x,v)\in\Gamma_-.$$

The proof follows then immediately from Theorem 1.3 (see [8] for details). □

Remark 7. – The two previous result cover the classical boundary conditions described in Section 1.

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