C. R. Acad. Sci. Paris, Ser. I 334 (2002) 869–874

Équations aux dérivées partielles/*Partial Differential Equations*
(Probabilités/*Probability Theory*)

Hölder–Sobolev regularity of solutions to a class of SPDE's driven by a spatially colored noise

Marta Sanz-Solé [a], Pierre-A. Vuillermot [b]

[a] Facultat de matemàtiques, Universitat de Barcelona, Gran Via 585, 08007 Barcelona, Spain

[b] I.E.C.N., Université Henri-Poincaré, Nancy 1, BP 239, 54506 Vandoeuvre-lès-Nancy cedex, France

Received 31 January 2002; accepted after revision 15 March 2002

Note presented by Pierre-Louis Lions.

Abstract In this Note we present new results regarding the equivalence, the existence and the joint space–time regularity properties of various notions of solution to a class of non-autonomous, semilinear, stochastic partial differential equations defined on a smooth, bounded, convex domain $D \subset \mathbb{R}^d$ and driven by a spatially colored noise defined from an $\mathrm{L}^2(D)$-valued Wiener process. ***To cite this article: M. Sanz-Solé, P.-A. Vuillermot, C. R. Acad. Sci. Paris, Ser. I 334 (2002) 869–874.*** © 2002 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

Régularité Hölder–Sobolev des solutions d'une classe d'E.D.P.S. dirigées par un bruit coloré

Résumé Dans cette Note nous présentons des résultats nouveaux concernant l'équivalence, l'existence et la régularité spatio–temporelle conjointe de diverses notions de solution relatives à une classe d'équations aux dérivées partielles stochastiques semilinéaires non autonomes définies dans un ouvert régulier borné convexe $D \subset \mathbb{R}^d$ et dirigées par un bruit coloré en la variable spatiale défini à partir d'un processus de Wiener à valeurs dans $\mathrm{L}^2(D)$. ***Pour citer cet article : M. Sanz-Solé, P.-A. Vuillermot, C. R. Acad. Sci. Paris, Ser. I 334 (2002) 869–874.*** © 2002 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

Version française abrégée

La numérotation et la notation que nous utilisons ici ainsi que les hypothèses (K), (L), (I) et (C) se réfèrent directement aux formules, à la notation et aux hypothèses (K), (L), (I) et (C) de la version principale en anglais. Nous nous intéressons ici à l'équivalence, à l'existence et aux propriétés de régularité spatio-temporelle conjointe de diverses notions de solution associées au problème (1). Plus précisément, nous avons le résultat suivant.

THÉORÈME. – *Supposons que* (K), (L), (I) *et* (C) *soient satisfaites. Alors nous avons* $u_\varphi^1(\cdot,t) = u_\varphi^2(\cdot,t) = u_\varphi^3(\cdot,t)$ *p.s. en tant qu'égalités dans* $\mathrm{L}^2(D)$ *pour chaque* $t \in [0,T]$. *De plus, il existe un champ aléatoire unique* $(u_\varphi(\cdot,t))_{t\in[0,T]}$, *solution de* (1) *au sens des définitions* 1, 2 *ou* 3 *ci-dessous, possédant les propriétés suivantes* :

E-mail addresses: sanz@mat.ub.es (M. Sanz-Solé); vuillerm@iecn.u-nancy.fr (P.-A. Vuillermot).

S1631-073X(02)02359-2/FLA

(1) $x \mapsto u_\varphi(x,t) \in \mathrm{H}^1(D)$ *p.s. pour tout* $t \in [0,T]$.
(2) $\sup_{(x,t)\in D\times[0,T]} \mathbb{E}|u_\varphi(x,t)|^r < +\infty$ *pour tout* $r \in [1,+\infty)$.
(3) *Il existe une version de ce champ aléatoire, que nous notons encore* $(u_\varphi(\cdot,t))_{t\in[0,T]}$, *telle que* $u_\varphi(\cdot,\cdot) \in \mathcal{C}^{\beta_1,\beta_2}(D\times[0,T])$ *p.s. pour tout* $\beta_1 \in (0,\alpha)$ *et tout* $\beta_2 \in \left(0, \frac{\alpha}{2} \wedge \frac{2}{d+2}\right)$.

Ce résultat met ainsi en évidence l'équivalence des trois notions de solution introduites ci-dessous, ainsi que l'existence d'une solution unique de (1) possédant des moments à tous les ordres bornés et jouissant simultanément d'une régularité Sobolev en la variable spatiale et d'une régularité höldérienne spatio-temporelle conjointe. Nous remarquons ici que l'exposant de Hölder relatif à la variable spatiale ne dépend que de l'exposant de Hölder de la condition initiale φ, alors que l'exposant de Hölder relatif à la variable temporelle fait également intervenir la dimension d mais seulement lorsque $d \geqslant 3$; il est possible de montrer que ce phénomène est inhérent à la présence du terme stochastique en (1). Par ailleurs, notre démonstration de l'existence d'une solution de (1) possédant les propriétés de régularité ci-dessus est subordonnée à la propriété d'indiscernabilité des champs aléatoires u_φ^1, u_φ^2 et u_φ^3, un fait non trivial en soi prouvant l'équivalence de deux théories ayant coexisté jusqu'ici indépendamment pour des modèles tels que (1), à savoir la théorie variationnelle de [6] et [9] d'une part, et la théorie basée sur l'existence d'une fonction de Green de [12] d'autre part, cette dernière ayant été récemment considérablement généralisée en [2]. Nous renvoyons le lecteur à la version principale en anglais pour une description plus complète de ces résultats, dont nous donnons les démonstrations détaillées en [11].

In this Note we use the standard notations for the usual spaces of differentiable functions, of Hölder continuous functions and of Lebesgue integrable functions defined on regions of Euclidean space; for $d \in \mathbb{N}^+$ let $D \subset \mathbb{R}^d$ be open, bounded, convex and assume that the boundary ∂D is of class $\mathcal{C}^{2+\alpha}$ for some $\alpha \in (0,1)$. Let C be a linear, self-adjoint, positive, non degenerate trace-class operator in $\mathrm{L}^2(D)$; this implies that C is an integral transform whose generating kernel we denote by κ. In the sequel we write $(e_j)_{j\in\mathbb{N}^+}$ for an orthonormal basis of $\mathrm{L}^2(D)$ consisting of eigenfunctions of the operator C and $(\lambda_j)_{j\in\mathbb{N}^+}$ for the sequence of the corresponding eigenvalues. Let $(W(\cdot,t))_{t\in\mathbb{R}_0^+}$ be an $\mathrm{L}^2(D)$-valued Wiener process defined on a complete stochastic basis $(\Omega,\mathcal{F},(\mathcal{F}_t)_{t\in\mathbb{R}_0^+},\mathbb{P})$, adapted to the filtration $(\mathcal{F}_t)_{t\in\mathbb{R}_0^+}$, starting at the origin and having the covariance operator tC. Let $T \in \mathbb{R}^+$ and let us consider the following class of real, parabolic, Itô initial-boundary value problems:

$$\begin{aligned}
&\mathrm{d}u(x,t) = \big(\mathrm{div}\big(k(x,t)\nabla u(x,t)\big) + g\big(u(x,t)\big)\big)\,\mathrm{d}t + h\big(u(x,t)\big)W(x,\mathrm{d}t), \quad (x,t)\in D\times(0,T],\\
&u(x,0) = \varphi(x), \quad x \in \overline{D},\\
&\frac{\partial u(x,t)}{\partial n(k)} = 0, \quad (x,t) \in \partial D \times (0,T].
\end{aligned} \tag{1}$$

In the preceding equations, the function k is matrix-valued and the last relation stands for the conormal derivative of u relative to k; moreover, we denote by n the unit outer normal vector to ∂D, and we assume that the functions k and n satisfy the following hypothesis.

(K) The entries of k satisfy the symmetry relation $k_{i,j}(\cdot) = k_{j,i}(\cdot)$ for every $i,j \in \{1,\dots,d\}$. Moreover, there exists a constant $\beta \in (\frac{1}{2},1]$ such that $k_{i,j} \in \mathcal{C}^{\alpha,\beta}(\overline{D}\times[0,T])$ for each i,j and, in addition, $k_{i,j,x_l} := \partial k_{i,j}/\partial x_l \in \mathcal{C}^{\alpha,\alpha/2}(\overline{D}\times[0,T])$ for each i, j, l; furthermore, there exists a positive constant $\underline{k}$ such that the inequality $\underline{k}\,|q|^2 \leqslant (k(x,t)q,q)_{\mathbb{R}^d}$ holds for all $q\in\mathbb{R}^d$ and all $(x,t)\in\overline{D}\times[0,T]$, where $|\cdot|$ and $(\cdot,\cdot)_{\mathbb{R}^d}$ denote the Euclidean norm and the Euclidean scalar product in $\mathbb{R}^d$, respectively. Finally, we have $(x,t)\mapsto\sum_{i=1}^d k_{i,j}(x,t)n_i(x) \in \mathcal{C}^{1+\alpha,(1+\alpha)/2}(\partial D\times[0,T])$ for each j and the conormal vector-field $(x,t)\mapsto n(k)(x,t) := k(x,t)n$ is outward pointing, nowhere tangent to ∂D for every $t\in[0,T]$.

To cite this article: M. Sanz-Solé, P.-A. Vuillermot, C. R. Acad. Sci. Paris, Ser. I 334 (2002) 869–874

Regarding the drift-nonlinearity g, the noise-nonlinearity h and the initial condition φ we have the following hypotheses, respectively:

(L) The functions $g, h : \mathbb{R} \mapsto \mathbb{R}$ are Lipschitz continuous.

(I) We have $\varphi \in \mathcal{C}^{2+\alpha}(\overline{D})$; moreover, φ satisfies the conormal boundary condition relative to k.

Relations (1) define a class of nonautonomous, semilinear, stochastic initial-boundary value problems driven by an infinite-dimensional noise which depends on both the space variable x and the time variable t. By virtue of the standard properties of the Wiener process $(W(\cdot,t))_{\mathbb{R}_0^+}$, this noise is colored with respect to x and white with respect to t, the properties of the spatial correlations of the noise being completely encoded in the generating kernel κ. Recall also that we have the Fourier decomposition $W(\cdot,t) = \sum_{j=1}^{+\infty} \lambda_j^{1/2} e_j(\cdot) B_j(t)$ in $\mathrm{L}^2(D)$, where $((B_j(t))_{t\in\mathbb{R}_0^+})_{j\in\mathbb{N}^+}$ denotes a sequence of one-dimensional, independent, standard Brownian motions (*see*, for instance, [3]).

There are many possible ways to define a notion of solution for (1) and it is not *a priori* evident that they should all define indistinguishable random fields. In this note we introduce two notions of *variational solution* along with one notion of *mild solution* to (1). We then formulate our main result in which we state the equivalence of all three notions together with the existence, the uniqueness, the spatial Sobolev regularity, the pointwise boundedness of the moments and the joint space–time Hölder regularity of such solutions. In effect, this result brings together two theories hitherto unrelated for models as general as (1), namely, the variational theory developed in [6,9] and Green's function theory initiated in [12], recently further developed in [2].

The notation for the remaining part of this Note is the following: we write $(\cdot,\cdot)_2$ for the usual scalar product in $\mathrm{L}^2(D)$, $\|\cdot\|_s$ for $\mathrm{L}^s(D)$-norms, $\mathrm{H}^1(D)$ for the usual Sobolev space of functions on D and $\mathcal{C}([0,T];\mathrm{L}^2(D))$ for the space of all continuous mappings from the interval $[0,T]$ into $\mathrm{L}^2(D)$ endowed with the uniform topology. The first notion we introduce is that of a variational solution tested with functions that depend only on the space variable; it can be traced to the classical works [6] and [9]. In addition to (K), (L) and (I) above, this requires the following hypothesis regarding the basis $(e_j)_{j\in\mathbb{N}^+}$ and the eigenvalues $(\lambda_j)_{j\in\mathbb{N}^+}$ of the operator C:

(C) We have $e_j \in \mathrm{L}^\infty(D)$ for each j and $\sum_{j=1}^{+\infty} \lambda_j \|e_j\|_\infty^2 < +\infty$.

In this context, we remark that hypothesis (C) defines a restricted set of trace-class covariance operators since the preceding condition implies $\sum_{j=1}^{+\infty} \lambda_j := \operatorname{Tr} C < +\infty$ by virtue of the existence of the continuous embedding $\mathrm{L}^\infty(D) \to \mathrm{L}^2(D)$.

DEFINITION 1. – We say that the $\mathrm{L}^2(D)$-valued random field $(u_\varphi^1(\cdot,t))_{t\in[0,T]}$ defined on $(\Omega,\mathcal{F},(\mathcal{F}_t)_{t\in[0,T]},\mathbb{P})$ is a *variational solution of the first kind* to problem (1) if the following conditions hold:
(1) $(u_\varphi^1(\cdot,t))_{t\in[0,T]}$ is progressively measurable on $[0,T]\times\Omega$.
(2) We have $u_\varphi^1 \in \mathrm{L}^2((0,T)\times\Omega;\mathrm{H}^1(D)) \cap \mathrm{L}^2(\Omega;\mathcal{C}([0,T];\mathrm{L}^2(D)))$.
(3) The integral relation

$$\int_D \mathrm{d}x\, v(x) u_\varphi^1(x,t) = \int_D \mathrm{d}x\, v(x)\varphi(x) - \int_0^t \mathrm{d}\tau \int_D \mathrm{d}x \big(\nabla v(x), k(x,\tau)\nabla u_\varphi^1(x,\tau)\big)_{\mathbb{R}^d} + \int_0^t \mathrm{d}\tau \int_D \mathrm{d}x\, v(x) g\big(u_\varphi^1(x,\tau)\big) + \int_0^t\int_D \mathrm{d}x\, v(x) h\big(u_\varphi^1(x,\tau)\big) W(x,\mathrm{d}\tau) \quad (2)$$

holds a.s. for every $v \in \mathrm{H}^1(D)$ and every $t\in[0,T]$, where we have defined the stochastic integral by

$$\int_0^t\int_D \mathrm{d}x\, v(x) h\big(u_\varphi^1(x,\tau)\big) W(x,\mathrm{d}\tau) := \sum_{j=1}^{+\infty} \lambda_j^{1/2} \int_0^t \big(v, h\big(u_\varphi^1(\cdot,\tau)\big)e_j\big)_2 B_j(\mathrm{d}\tau).$$

From the preceding definition and from the above hypotheses, we easily infer that each term in Eq. (2) is well defined and finite a.s.

The second notion we introduce is that of a variational solution tested by means of functions that depend on *both* the space- and the time variable. For every $t \in (0, T]$ let $\mathrm{H}^1(D \times (0,t))$ be the Sobolev space of all real-valued functions $v \in \mathrm{L}^2(D \times (0,t))$ that possess distributional derivatives $v_{x_j} \in \mathrm{L}^2(D \times (0,t))$ for every $j \in \{1,\dots,d\}$ along with a distributional time-derivative $v_\tau \in \mathrm{L}^2(D \times (0,t))$, the norm of $\mathrm{H}^1(D \times (0,t))$ being defined in the usual way. The following definition requires exactly the same four hypotheses as above.

DEFINITION 2. – We say that the $\mathrm{L}^2(D)$-valued random field $(u^2_\varphi(\cdot,t))_{t\in[0,T]}$ defined on $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, \mathbb{P})$ is a *variational solution of the second kind* to problem (1) if the first two conditions of Definition 1 hold, and if the integral relation

$$\begin{aligned}\int_D \mathrm{d}x\, v(x,t)u^2_\varphi(x,t) &= \int_D \mathrm{d}x\, v(x,0)\varphi(x) + \int_0^t \mathrm{d}\tau \int_D \mathrm{d}x\, v_\tau(x,\tau)u^2_\varphi(x,\tau)\\ &\quad - \int_0^t \mathrm{d}\tau \int_D \mathrm{d}x\, \big(\nabla v(x,\tau), k(x,\tau)\nabla u^2_\varphi(x,\tau)\big)_{\mathbb{R}^d}\\ &\quad + \int_0^t \mathrm{d}\tau \int_D \mathrm{d}x\, v(x,\tau) g\big(u^2_\varphi(x,\tau)\big)\\ &\quad + \int_0^t \int_D \mathrm{d}x\, v(x,\tau) h\big(u^2_\varphi(x,\tau)\big) W(x,\mathrm{d}\tau) \qquad (3)\end{aligned}$$

holds a.s. for every $v \in \mathrm{H}^1(D \times (0,t))$ and every $t \in [0,T]$, where $x \mapsto v(x,0) \in \mathrm{L}^2(D)$ and $x \mapsto v(x,t) \in \mathrm{L}^2(D)$ denote the Sobolev traces of v on D and $D \times \{\tau \in \mathbb{R} : \tau = t\}$, respectively, and where we have defined the stochastic integral as in Definition 1.

Again, we see that every term in Eq. (3) is well defined and finite a.s., and that the structure of (3) is identical to that of (2) up to the appearance of the term that involves the partial derivative v_τ.

The third notion we need is one of mild solution defined from the parabolic Green's function G associated with the principal part of (1). Recall that under hypotheses (K) and (I), the function $G : \overline{D} \times [0,T] \times \overline{D} \times [0,T] \setminus \{s,t \in [0,T] : s \geqslant t\} \mapsto \mathbb{R}$ is continuous, twice continuously differentiable in x, once continuously differentiable in t and satisfies the fundamental heat kernel estimates

$$\big|\partial_x^\mu \partial_t^\nu G(x,t;y,s)\big| \leqslant c(t-s)^{-(d+|\mu|+2\nu)/2} \exp\left[-c\frac{|x-y|^2}{t-s}\right] \qquad (4)$$

where $\mu = (\mu_1,\dots,\mu_d) \in \mathbb{N}^d$, $\nu \in \mathbb{N}$ and $|\mu| + 2\nu \leqslant 2$ with $|\mu| = \sum_{j=1}^d \mu_j$ (*see*, for instance, [5]). The following definition still requires the same hypotheses as above.

DEFINITION 3. – We say that the $\mathrm{L}^2(D)$-valued random field $(u^3_\varphi(\cdot,t))_{t\in[0,T]}$ defined on $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, \mathbb{P})$ is a *mild solution* to problem (1) if the first two conditions of Definition 1 hold, and if the relation

$$\begin{aligned}u^3_\varphi(\cdot,t) &= \int_D \mathrm{d}y\, G(\cdot,t;y,0)\varphi(y) + \int_0^t \mathrm{d}\tau \int_D \mathrm{d}y\, G(\cdot,t;y,\tau) g\big(u^3_\varphi(y,\tau)\big)\\ &\quad + \int_0^t \int_D \mathrm{d}y\, G(\cdot,t;y,\tau) h\big(u^3_\varphi(y,\tau)\big) W(y,\mathrm{d}\tau) \qquad (5)\end{aligned}$$

holds a.s. for every $t \in [0,T]$ as an equality in $\mathrm{L}^2(D)$, where for $t = 0$ we have $\int_D \mathrm{d}y\, G(\cdot,0;y,0)\varphi(y) := \lim_{t\searrow 0} \int_D \mathrm{d}y\, G(\cdot,t;y,0)\varphi(y) = \varphi(\cdot)$ and where we have defined the stochastic integral as above.

The proof that each term on the right-hand side of (5) defines an $\mathrm{L}^2(D)$-valued function a.s. is complicated by the existence of the singularity on the time-diagonal in G but we can carry it out successfully by means of suitable applications of the estimates (4).

Our main result is the following.

THEOREM. – *Assume that hypotheses* (K), (L), (I) *and* (C) *hold. Then we have* $u^1_\varphi(\cdot,t) = u^2_\varphi(\cdot,t) = u^3_\varphi(\cdot,t)$ *a.s. as equalities in* $\mathrm{L}^2(D)$ *for every* $t \in [0,T]$. *Furthermore, there exists a unique random field* $(u_\varphi(\cdot,t))_{t\in[0,T]}$ *that solves problem* (1) *in the sense of any of the above three definitions, with the following properties*:

(1) $x \mapsto u_\varphi(x,t) \in \mathrm{H}^1(D)$ *a.s. for every* $t \in [0,T]$.

(2) $\sup_{(x,t)\in D\times[0,T]} \mathbb{E}|u_\varphi(x,t)|^r < +\infty$ *for every* $r \in [1,+\infty)$.

(3) *There exists a version of this random field, still written* $(u_\varphi(\cdot,t))_{t\in[0,T]}$, *such that* $u_\varphi(\cdot,\cdot) \in \mathcal{C}^{\beta_1,\beta_2}(D\times[0,T])$ *a.s. for every* $\beta_1 \in (0,\alpha)$ *and every* $\beta_2 \in \left(0, \frac{\alpha}{2} \wedge \frac{2}{d+2}\right)$.

Remarks and brief sketch of the proof. – (1) The very first statement of the theorem has to be interpreted as stating the equivalence of the above three definitions: every variational solution of the first kind is a variational solution of the second kind, every variational solution of the second kind is a mild solution, while every mild solution is, in turn, a variational solution of the first kind.

(2) There exist several results in various contexts that establish relationships between different kinds of variational solutions and their mild formulations, both in the deterministic and in the stochastic case (*see*, for instance, [1,3,4,7,12]). However, none of the above works deals with nonautonomous, stochastic reaction–diffusion equations such as (1).

(3) In order to prove that $u^2_\varphi(\cdot,t) = u^3_\varphi(\cdot,t)$ a.s. in $\mathrm{L}^2(D)$ for every $t \in [0,T]$, it is sufficient to prove that every variational solution of the second kind is a mild solution to (1), for then an argument based on Gronwall's inequality allows one to conclude; the fact that $(u^2_\varphi(\cdot,t))_{t\in[0,T]}$ is indeed a mild solution to (1) follows from the choice of a very special class of test functions in (3). Consequently, in order to prove the indistinguishability of all three random fields, it is sufficient to show that every variational solution of the first kind is a variational solution of the second kind to (1). This can be achieved, for instance, by first proving that $(u^1_\varphi(\cdot,t))_{t\in[0,T]}$ satisfies (3) for test functions that are polynomials in x and t, and then by extending the result thus obtained to all $v \in \mathrm{H}^1(D\times(0,t))$ by suitable density arguments.

(4) Whereas Sobolev regularity in the space variable is built into the above definitions and hence can follow from standard existence proofs for variational solutions of the first kind, the joint space–time Hölder regularity is not; our proof of the latter property is based on a multidimensional version of Kolmogorov's continuity theorem through very fine estimates regarding Green's function G, and is in fact subordinated to the fact that $\mathbb{E}|u_\varphi(x,t)|^r < +\infty$ uniformly in $(x,t) \in D\times[0,T]$ for every $r \in [1,+\infty)$. Our proof of this property, in turn, follows from a fixed point argument based on (5) which we carry out in a suitable Banach space of random fields.

(5) In the last statement of the theorem we remark that the dimension plays no rôle regarding the Hölder continuity property of $(u_\varphi(\cdot,t))_{t\in[0,T]}$ with respect to the space variable, whereas it appears explicitly in the expression of β_2 but only for $d \geqslant 3$. Our proof shows that this phenomenon is inherent in the presence of the stochastic term in (1).

(6) Hypothesis (C) is crucial for the proof of the above theorem; however, if we weaken the notion of solution further by requiring only the boundedness of the moments along with relation (5), we can relax hypothesis (C) a bit by replacing, for instance, the condition $\sum_{j=1}^{+\infty} \lambda_j \|e_j\|_\infty^2 < +\infty$ by $\sum_{j=1}^{+\infty} \lambda_j \|e_j\|_s^2 < +\infty$ for $s \in \left(\frac{d}{1-2\eta}, +\infty\right)$ and some $\eta \in (0,\frac{1}{2})$; in this case, we can still prove the existence of a jointly Hölder continuous version of a solution; we can also show that conditions such as $\sum_{j=1}^{+\infty} \lambda_j \|e_j\|_s^2 < +\infty$ play the same rôle in our analysis of (1) as certain *spectral measure conditions* play in the recent analyses of some *autonomous* stochastic partial differential equations defined on the whole of $\mathbb{R}^d$ (*see*, for instance, [8,10] and their references).

We refer the reader to [11] for more details and for the complete proofs of all of the above results.

References

[1] A. Chojnowska-Michalik, Stochastic differential equations in Hilbert spaces, Banach Center Publ. 5 (1979) 53–73.

[2] R.C. Dalang, Extending martingale measure stochastic integral with applications to spatially homogeneous S.P.D. E's, Electronic J. Probab. 4 (1999) 1–29.

[3] G. Da Prato, J. Zabczyk, Stochastic Equations in Infinite Dimensions, Encyclopedia of Mathematics and its Applications, Vol. 44, Cambridge University Press, Cambridge, 1992.

[4] D.A. Dawson, L.G. Gorostiza, Solutions of evolution equations in Hilbert space, J. Differential Equations 68 (1987) 299–319.

[5] S.D. Eidelman, S.D. Ivasišen, Investigation of the Green matrix for a homogeneous parabolic boundary value problem, Trans. Moscow Math. Soc. 23 (1970) 179–242.

[6] N.V. Krylov, B.L. Rozovskii, Stochastic evolution equations, J. Soviet Math. 16 (1981) 1233–1277.

[7] J.A. León, Stochastic evolution equations with respect to semimartingales in Hilbert space, Stochastics 27 (1989) 1–21.

[8] O. Lévêque, Hyperbolic stochastic partial differential equations driven by boundary noises, Thèse EPFL 2452, Lausanne, 2001.

[9] E. Pardoux, Équations aux dérivées partielles stochastiques nonlinéaires monotones : Étude de solutions fortes de type Itô, Thèse de l'Université Paris–Orsay 1556, Paris, 1975.

[10] S. Peszat, J. Zabczyk, Nonlinear stochastic wave and heat equations, Probab. Theory Related Fields 116 (2000) 421–443.

[11] M. Sanz-Solé, P.-A. Vuillermot, Equivalence and Hölder–Sobolev regularity of solutions for a class of non-autonomous stochastic partial differential equations, 2002, in preparation.

[12] J.B. Walsh, An introduction to stochastic partial differential equations, in: École d'Été de Probabilités de Saint-Flour XIV, Lecture Notes in Math., Vol. 1180, Springer, New York, 1986, pp. 265–439.