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HE OUYANG

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On isospectral deformations on nilmanifolds

He OUYANG

**Department of Mathematics
Washington University
ST LOUIS MO 63130
U.S.A.**

Suppose G is a simply-connected nilpotent Lie-group and Γ a discrete cocompact subgroup in G .

Suppose g is a left-invariant metric on nilmanifold $M = \Gamma \backslash G$ and $\{\Phi_t\}_{t \in I}$ is a continuous family of almost inner automorphisms relative to Γ (see [G]).

Then Gordon and Wilson proved in [GW] that $\{g_t = \Phi_t^* g\}_{t \in I}$ is an isospectral deformation of metric g on $\Gamma \backslash G$. Conversely we proved following theorem : if G is a 2-step nilpotent Lie-group, $\Gamma < G$ a cocompact lattice and $\{g_t\}_{t \in I}$ is a continuous isospectral family of left-invariant metrics on $\Gamma \backslash G$, then there exists a continuous family of almost inner automorphisms $\{\Phi_t\}_{t \in I}$ of G relative to Γ such that $g_t = \Phi_t^* g_0$. When G is nonsingular, the theorem is proved firstly by H. Pesce. He also independently proved the general theorem by explicitly describing all the irreducible unitary representations appeared in the representation of G on $L^2(\Gamma \backslash G)$.

For arbitrary step nilpotent group case, the same question is still open.

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