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Chaoticity on a stochastic interval $[0, T]$

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Abstract

The chaotic representation property is given a meaning and established for a class of martingales X defined on some stochastic interval $[0, T]$ and having only finitely many jumps before $T - \varepsilon$.

1. Introduction

Let X be a martingale with predictable bracket $\langle X, X \rangle_t = t$, $(\mathcal{F}_t)$ be its filtration and $\mathcal{F} = \bigcup_{t>0} \mathcal{F}_t$. We say that the martingale X has the chaotic representation property (C.R.P) or is chaotic, if for all $F \in L^2(\Omega, \mathcal{F})$, there exists a sequence (f_k) with $f_k \in L^2(\mathbb{R}_+^k, dt^{\otimes k})$, such that

$$F = \sum_{k=0}^{\infty} F_k,$$

where $F_0 = \mathbb{E}[F]$ and for $k > 0$

$$F_k = \int_{0 < t_1 < \dots < t_k} f_k(t_1, \dots, t_k) dX_{t_1} \dots dX_{t_k}.$$

(For the definition of the latter multiple stochastic integral, see [7].)

The random variables $F_k, k \in N$, are such that

$$\mathbb{E}[F_k F_j] = \delta_j(k) \left(\int_{0 < t_1 < \dots < t_k} f_k^2(t_1, \dots, t_k) dt_1 \dots dt_k \right),$$

where $\delta_j(k) = 0$ if $k \neq j$ and $\delta_k(k) = 1$.

It is interesting to express the chaotic representation property as an isomorphism between $L^2(\Omega, \mathcal{F})$ and the symmetric Fock space over $H = L^2(\mathbb{R}_+, dt)$, defined by

$$Fock(H) = \bigoplus_{k=0}^{\infty} H^{\odot k}.$$

For, $k \in N^*$, the space $H^{\odot k} = L_{sym}^2(\mathbb{R}_+^k, dt_1 \dots dt_k)$ is the set of the class of square integrable functions with respect to $dt_1 \dots dt_k$, which are symmetric with respect to the k parameters $(t_1, \dots, t_k)$. The scalar product over $H^{\odot k}$ is defined by

$$\langle f, g \rangle = \int_{0 < t_1 < \dots < t_k} f(t_1, \dots, t_k) g(t_1, \dots, t_k) dt_1 \dots dt_k,$$

and $H^{\odot 0} = \mathbb{R}$.

The well known examples of martingales having the chaotic representation property are the Brownian motion and the standard Poisson process [6].

Moreover, He and Wang [5] have characterized the Lévy processes which have the predictable representation property but until 1987 we did not know if these processes have the chaotic representation property.

In 1987, the author [2] proved that for the Lévy processes the chaotic representation property and the predictable representation property are equivalent.

In 1988, Emery [3] showed that a martingale earlier discovered by Azéma [1] has the chaotic representation property, introducing at the same time other examples which satisfy the "structure equation" of the form

$$d[X, X]_t = dt + \Phi(t)dX_t, \quad X_0 = x.$$

He later proved in [4] that if the predictable process $\Phi(t)$ is such that the integral $A_t = \int_0^t \Phi^{-2}(s)ds$ is a.s. finite for all t , then the predictable representation property implies the chaotic representation property. This applies to structure equations with Φ of the form

$$\Phi(t) = \phi_1(t)1_{[0, T_1]}(t) + \sum_{n \geq 2} \phi_n(t, T_{n-1}, \dots, T_1)1_{[T_{n-1}, T_n]}(t)$$

where ϕ_n are deterministic and the T_n 's are the successive jumps of the solution X to the structure equation

$$d[X, X]_t = dt + \Phi(t)dX_t, \quad X_0 = x.$$

The hypothesis $A_t < \infty$ implies that there are only finitely many jumps on finite intervals since A_t is the predictable compensator of the number of jumps

$$C_t = \sum_{n \geq 1} 1_{[T_n, \infty)}(t).$$

The aim of this work is to study the following problem : Dropping the finiteness assumption for A_t and putting $T_\infty = \sup_n T_n$, we will allow T_∞ to be finite. The above formulas define (in law) the martingale X only on the interval $[0, T_\infty]$. We will prove that X still has the chaotic representation property, in the following sense : If M is a chaotic martingale independent of X (possibly defined on an enlargement of Ω), the martingale

$$Y_t = \begin{cases} X_t & \text{for } t \leq T_\infty \\ X_{T_\infty} + M_{t-T_\infty} - M_0 & \text{for } t \geq T_\infty \end{cases}$$

has the chaotic representation property (we will see in Lemma 2.2. that this does not depend on the choice of M).

2. Chaoticity before a stopping time

This section is devoted to giving a rigorous meaning to the chaotic representation property for a martingale defined only up to some stopping time.

Definition. Let $(X_t)_{t \geq 0}$ be a martingale such that $\langle X, X \rangle_t$ is equal to t , $(\mathcal{F}_t)$ be its filtration and T be a stopping time of $(\mathcal{F}_t)$. We say that X is chaotic on $[0, T]$ if $L^2(\mathcal{F}_T)$ is included in the chaotic space of X , i.e. if each $F \in L^2(\mathcal{F}_T)$ has an expansion $F = \sum_{k=0}^{\infty} F_k$ with $F_0 = \mathbb{E}[F]$ and for $k > 0$

$$F_k = \int_{0 < t_1 < \dots < t_k} f_k(t_1, \dots, t_k) dX_{t_1} \dots dX_{t_k}$$

with $(f_k)_{k \geq 0} \in \text{Fock}(H)$.

Lemma 2.1. *If the martingale $(X_t)_{t \geq 0}$ is chaotic on $[0, T]$ and if a martingale $(Y_t)_{t \geq 0}$ verifies $\langle Y, Y \rangle_t = t$ and $X = Y$ on $[0, T]$, then T is a stopping time for the filtration generated by Y and Y is also chaotic on $[0, T]$.*

Proof. By proposition (1, ii) of [4], each element of $L^2(\mathcal{F}_T)$ is a sum of multiple integrals with respect to Y ; so it only remains to prove that T is a stopping time for Y . For each $t \geq 0$, the indicator of the event $\{T \leq t\}$ is in both $L^2(\mathcal{F}_T)$ and $L^2(\mathcal{F}_t)$, so it is of the form

$$\mathbb{P}(T \leq t) + \sum_{k=1}^{\infty} \int_{0 < t_1 < \dots < t_k < t} f_k(t_1, \dots, t_k) dX_{t_1} \dots dX_{t_k}.$$

By proposition (1, ii) of [4] again, it is also equal to

$$\mathbb{P}(T \leq t) + \sum_{k=1}^{\infty} \int_{0 < t_1 < \dots < t_k < t} f_k(t_1, \dots, t_k) dY_{t_1} \dots dY_{t_k}$$

and this shows that T is a stopping time for Y .

Lemma 2.2. *Let T be a stopping time and X be a martingale defined on the interval $[0, T]$ only and verifying $\langle X, X \rangle_t = t$ on this interval. The following conditions are equivalent.*

1) *For some chaotic martingale M independent of X (and possibly defined on an enlargement of Ω), the martingale*

$$Y_t = \begin{cases} X_t & \text{for } t \leq T \\ X_T + M_{t-T} - M_0 & \text{for } t \geq T \end{cases}$$

has the chaotic representation property.

2) *Same statement as 1), with "for every M " instead of "for some M ".*

3) *There exists a martingale $(X'_t)_{t \geq 0}$ (possibly defined on an enlargement of Ω), verifying $\langle X', X' \rangle_t = t$, chaotic on $[0, T]$, with restriction X to $[0, T]$.*

4) *Every martingale $(X'_t)_{t \geq 0}$ (possibly defined on an enlargement of Ω), verifying $\langle X', X' \rangle_t = t$, with restriction X to $[0, T]$, is chaotic on $[0, T]$.*

Proof. The implications $2) \Rightarrow 1) \Rightarrow 3)$ are trivial and $3)$ is equivalent to $4)$ by Lemma 2.1. So it suffices to prove $3) \Rightarrow 2)$. The proof is completely similar to the proof of Proposition (1, iii) of [4] and Corollary 2 of [4] except for one detail: With the notations of [4], X is no longer supposed to have the *C.R.P* but only to be chaotic on $[0, T]$. So in the proof of (1, iii), page 14, it is not obvious that there exists an element g in $Fock(H)$ such that

$$U = \int g(A) dX_A := \sum_{k=0}^{\infty} \int_{0 < t_1 < \dots < t_k} g_k(t_1, \dots, t_k) dX_{t_1} \dots dX_{t_k}.$$

But we know that $U = \int_{A \subset [T, \infty[} f(A) dX_A$, so for almost every A , $\mathbb{E}[f^2(A) 1_{A \subset [T, \infty[}]$ is finite, and the chaoticity of X on $[0, T]$ implies that there exists $h(B, A)$ such that $\int h(B, A) dX_B$ is equal to $f(A) 1_{A \subset [T, \infty[}$. Since $f(A) 1_{A \subset [T, \infty[} \in L^2(\mathcal{F}_{\inf A})$, then $h(B, A)$ is null if $\sup B > \inf A$ and the existence of g is obtained by putting

$$g(\{t_1, \dots, t_k\}) = \sum_{i=1}^{k+1} h(\{t_1, \dots, t_{i-1}\}, \{t_i, \dots, t_k\})$$

this proves the lemma.

Definition. Let T be a stopping time and X be a martingale defined only on the interval $[0, T]$ and verifying $\langle X, X \rangle_t = t$ on this interval. We say that X is chaotic on $[0, T]$ if the four conditions of Lemma 2.2 are met.

Lemma 2.3. Let $(T_n)_{n \in \mathbb{N}}$ be a non-decreasing sequence of stopping times and T_∞ its limit.

1) If a martingale $(X_t)_{t \geq 0}$ verifying $\langle X, X \rangle_t = t$ is chaotic on each interval $[0, T_n]$, it is also chaotic on $[0, T_\infty]$.

2) Let X be a martingale defined only on $[0, T_\infty]$ and verifying $\langle X, X \rangle_t = t$. If for each n the restriction of X to $[0, T_n]$ is chaotic on $[0, T_n]$, then X is chaotic on $[0, T_\infty]$.

Proof. 1) For each n , we know that $L^2(\mathcal{F}_{T_n})$ is included in the chaotic space of X . As this chaotic space is closed and as, by the martingale convergence theorem, $\bigcup_n L^2(\mathcal{F}_{T_n})$ is dense in $L^2(\mathcal{F}_{T_\infty})$, the latter is also included in the chaotic space of X .

2) Using Lemma 2.2, it suffices to apply 1) to the martingale

$$Y_t = \begin{cases} X_t & \text{for } t \leq T_\infty \\ X_{T_\infty} + B_{t-T_\infty} - B_0 & \text{for } t \geq T_\infty \end{cases}$$

where B is a Brownian motion independent of X .

3. Construction of the martingale

This section is devoted to constructing the martingale X announced in the introduction.

The set $\Omega = \mathbb{R}_+^{\mathbb{N}}$ is the set of the sequences $\omega = (S_n, n \in \mathbb{N})$ with S_0 equal to zero and $S_n \in \mathbb{R}_+$ for all $n \in \mathbb{N}$.

The sequence ω defines the following increasing sequence :

$$T_n = \sum_{i=0}^n S_i \quad \text{for } n \in \mathbb{N}.$$

Let $T_\infty = \lim_{n \rightarrow \infty} T_n$.

For $i \in \mathbb{N}$, let ϕ_{i+1} be a measurable $\mathbb{R}_*$ valued function defined on $\mathbb{R}_+^{i+1}$. We define the point process p_t by

$$p_t = \begin{cases} 0 & \text{for } t \in [0, T_1[\\ \sum_{j=1}^i \phi_j(T_j, \dots, T_1) & \text{for } t \in [T_i, T_{i+1}[. \end{cases}$$

The process (p_t) generates the increasing family of σ -fields $\mathcal{F}_t^0$ defined by

$$\mathcal{F}_t^0 = \sigma(p_s, s \leq t), \quad \mathcal{F}^0 = \sigma(p_s, s > 0).$$

We use the following notations:

$$\Phi_{i+1}(t) = \phi_{i+1}(t, T_i, \dots, T_1) \quad \text{for } i \geq 1,$$

$$\Phi(t) = \Phi_{i+1}(t) \quad \text{if } t \in [T_i, T_{i+1}].$$

We suppose that, for all $i \in \mathbb{N}$, there exists a $\mathcal{F}_{T_i}^0$ measurable positive function $\tau_{i+1} > T_i$, such that

$$\int_{T_i}^t \Phi_{i+1}^{-2}(s) ds < +\infty \quad \text{for } t \in [T_i, \tau_{i+1}[\quad \text{and} \quad \int_{T_i}^{\tau_{i+1}} \Phi_{i+1}^{-2}(s) ds = \infty.$$

The probability measure $\mathbb{P}$ on $(\Omega, \mathcal{F}^0)$ is defined by the law of T_1 , with density

$$\Phi_1^{-2}(t) \exp \left\{ - \int_0^t \Phi_1^{-2}(s) ds \right\} 1_{]0, \tau_1[}(t) dt$$

and the conditional law of T_{i+1} , with density

$$\Phi_{i+1}^{-2}(t) \exp \left\{ - \int_{T_i}^t \Phi_{i+1}^{-2}(s) ds \right\} 1_{]T_i, \tau_{i+1}[}(t) dt.$$

The σ -fields $\mathcal{F}_i^0$ are augmented with all subsets of $\mathbb{P}$ -null sets of $\mathcal{F}^0$ and denoted by $\mathcal{F}_i$. For all $i \in \mathbb{N}$, T_i is a stopping time of $(\mathcal{F}_i)$.

Proposition 3.1. Let $N(dt, dx)$ be the random measure on $\mathbb{R}_+ \times \mathbb{R}_*$ defined for $t > 0$ and A a measurable set of $\mathbb{R}_*$ by

$$N([0, t] \times A) = \sum_{T_n \leq t} 1_A(\Phi_n(T_n)).$$

The predictable projection of $N(dt, dx)$ with respect to the probability $\mathbb{P}$ is given by

$$\nu(dt, dx) = \Phi^{-2}(t) 1_{[0, T_\infty[}(t) dt \delta_{\Phi(t)}(dx).$$

Proof. Let $n \in \mathbb{N}_*$, f be a bounded measurable function on $\mathbb{R}_+^n$ and g be a bounded measurable function on $\mathbb{R}$.

Let us consider the predictable process

$$Z(t, x) = 1_{]T_n, T_{n+1}]}(t) f(T_1, \dots, T_n) g(x).$$

We have to prove that

$$\mathbb{E} \left[\int_0^\infty \int_{\mathbb{R}_*} Z(t, x) N(dt, dx) \right] = \mathbb{E} \left[\int_0^\infty Z(t, \Phi(t)) \Phi^{-2}(t) dt \right].$$

From the equality

$$\int_0^\infty \int_{\mathbb{R}_*} Z(t, x) N(dt, dx) = f(T_1, \dots, T_n) g(\Phi_{n+1}(T_{n+1})),$$

and using the conditional law of T_{n+1} , with respect to $(T_1, \dots, T_n)$, we obtain

$$\begin{aligned} & \mathbb{E} \left[\int_0^\infty \int_{\mathbb{R}_*} Z(t, x) N(dt, dx) \right] \\ &= \mathbb{E} \left[f(T_1, \dots, T_n) \int_{T_n}^{T_{n+1}} g(\Phi_{n+1}(t_{n+1})) \Phi_{n+1}^{-2}(t_{n+1}) \right. \\ & \quad \left. \exp \left(- \int_{T_n}^{t_{n+1}} \Phi_{n+1}^{-2}(s) ds \right) dt_{n+1} \right]. \end{aligned}$$

An integration by parts gives

$$\begin{aligned}
& \int_{T_n}^{\tau_{n+1}} \int_{T_n}^{t_{n+1}} g(\Phi_{n+1}(t)) \Phi_{n+1}^{-2}(t) dt \\
& \left\{ \Phi_{n+1}^{-2}(t_{n+1}) \exp \left(- \int_{T_n}^{t_{n+1}} \Phi_{n+1}^{-2}(s) ds \right) \right\} dt_{n+1} \\
& = \int_{T_n}^{\tau_{n+1}} g(\Phi_{n+1}(t_{n+1})) \Phi_{n+1}^{-2}(t_{n+1}) \exp \left(- \int_{T_n}^{t_{n+1}} \Phi_{n+1}^{-2}(s) ds \right) dt_{n+1}.
\end{aligned}$$

Thus,

$$\begin{aligned}
& \mathbb{E}[f(T_1, \dots, T_n) \int_{T_n}^{\tau_{n+1}} g(\Phi_{n+1}(t)) \Phi_{n+1}^{-2}(t) dt] \\
& = \mathbb{E}[f(T_1, \dots, T_n) g(\Phi_{n+1}(T_{n+1}))],
\end{aligned}$$

which is exactly what was to be proved.

Proposition 3.2. *Let $m \in \mathbb{R}$.*

1) *The process (X_t) defined on the predictable interval $[0, T_\infty[$ by*

$$X_t = m + p_t - \int_0^t \Phi(s)^{-1} ds$$

is a $(\mathcal{F}_t)$ square integrable martingale with $\langle X, X \rangle_t = t$, it verifies the structure equation

$$d[X, X]_t = dt + \Phi(t) dX_t, X_0 = m.$$

2) *When the definition of X is extended to $[0, T_\infty]$ by*

$X_{T_\infty} = \lim_{n \rightarrow \infty} X_{T_n}$ on $T_\infty < \infty$, the martingale X has the chaotic representation property on $[0, T_\infty]$.

In the case when $T_\infty = \infty$ a.s., the chaotic property 2) is a consequence of the Theorem 5 of [4].

Proof. 1) Since X_t is also equal to

$$X_t = m + \int_0^t \int_{\mathbf{R}_*} xp(dx, dt) - \int_0^t \int_{\mathbf{R}_*} x\nu(dx, dt)$$

by Proposition 3.1, X is a martingale with predictable bracket $\langle X, X \rangle_{t \wedge T_\infty} = t \wedge T_\infty$ and satisfies the structure equation

$$d[X, X]_t = 1_{[t < T_\infty]} dt + \Phi(t) dX_t, X_0 = m.$$

2) By Lemma 2.3, it suffices to verify that, for each finite n , X is chaotic on $[0, T_n]$. Define a martingale X^n by the same construction as X , but with $\phi_i \equiv 1$ for $i > n$. The martingale M^n is identical in law to X on $[0, T_n]$ and is a compensated standard Poisson process after T_n . It has the chaotic representation property by Theorem 5 of [4]; this implies in particular that it is chaotic on $[0, T_n]$. So the restriction of X to $[0, T_n]$ is chaotic by Lemma 2.2, and X is chaotic on $[0, T_\infty]$ by Lemma 2.3.

4. Examples

Let $(\lambda_n, n \in \mathbb{N}^*)$ be a sequence of strictly positive real numbers and $(T_n, n \in \mathbb{N}^*)$ be the successive jumps such that the sojourn times $(T_{n+1} - T_n, n \in \mathbb{N})$ being independent exponentially distributed variables. The density of $T_n - T_{n-1}$ is

$$\lambda_n e^{-\lambda_n t}.$$

When

$$\sum_{n=1}^{\infty} \lambda_n^{-1} < \infty,$$

T_{∞} is finite almost surely; or else, it is infinite almost surely. For $t \in [T_{n-1}, T_n[$ The predictable process Φ is given by

$\Phi(t) = \sqrt{\lambda_n^{-1}}$ and the martingale X by

$$X_t = -\sqrt{\lambda_n}(t - T_{n-1}) + \sum_{i=1}^{n-1} \left(\sqrt{\lambda_i^{-1}} - \sqrt{\lambda_i}(T_i - T_{i-1}) \right).$$

It is chaotic on $[0, T_{\infty}]$ by the preceding proposition.

Another example is given by the structure equation

$$d[X, X]_t = dt + f(X_{t-})dX_t, \quad X_0 = m$$

with m is such that $f(m) \neq 0$ and f is a deterministic continuous function. Let $T_{\infty} = \inf\{t > 0, X_t = 0\}$, for $t < T_{\infty}$, X_t can be constructed as follows: let $(T_n, n \in \mathbb{N}^*)$ be the jump times of X_t , and suppose that the integral equation

$$x_t = f(X_{T_n} - \int_{T_n}^t x_s^{-1} ds), \quad t > T_n,$$

has a unique solution $t \rightarrow \Phi_{n+1}(t, X_{T_n}, \tau_{n+1})$ on the widest interval $[T_n, \tau_{n+1}[$ of $[T_n, \infty[$ where x_t is defined.

If x_t is such that

$$\int_{T_n}^t x_s^{-2} ds < \infty, \quad \text{for } t \in [T_n, \tau_{n+1}[\quad \text{and} \quad \int_{T_n}^{\tau_{n+1}} x_s^{-2} ds = \infty,$$

then we can see that $x_{T_{n+1}} = \Delta X_{T_{n+1}}$ is the jump size at T_{n+1} ,

$$X_{T_{n+1}} = X_{T_n} + \Phi_{n+1}(T_{n+1}, X_{T_n}, \tau_{n+1}) - \int_{T_n}^{T_{n+1}} \Phi_{n+1}^{-1}(s, X_{T_n}, \tau_{n+1}) ds,$$

and for $t \in [T_n, T_{n+1}[$,

$$X_t = X_{T_n} - \int_{T_n}^t \Phi_{n+1}^{-1}(s, X_{T_n}, \tau_{n+1}) ds.$$

If we put $T_0 = 0$, then for all $n \in \mathbb{N}$ the law of T_{n+1} , with respect to $(T_0, \dots, T_n)$, is supported by $]T_n, \tau_{n+1}[$ and has the density

$$\Phi_{n+1}^{-2}(t, X_{T_n}, \tau_{n+1}) \exp \left\{ - \int_{T_n}^t \Phi_{n+1}^{-2}(s, X_{T_n}, \tau_{n+1}) ds \right\}.$$

By Proposition 3.2, X is chaotic on $[0, T_{\infty}]$.

If $f(x) = \beta x$ we find again the Azéma martingale with parameter $\beta \notin \{-1, 0\}$ on the interval $[0, T_{\infty}]$, where T_{∞} is the first time when $X = 0$ (T_{∞} is also the first accumulation point of jump times of X).

Remark.

The solution of the differential equation $x_t = f(a - \int_0^t x_s^{-1} ds)$ allows us to construct the martingale X on $[0, T_\infty]$; the existence and the uniqueness of the solution of this equation implies the existence and the uniqueness in law of X on $[0, T_\infty]$.

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