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Sufficient statistics for the Brownian sheet

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0. Introduction

Let P denote Wiener measure on $(\Omega, \mathcal{F})$, with $\Omega := C_0[0,\infty) := \{x \in C[0,\infty) : x(0) = 0\}$ and $\mathcal{F} := \sigma(X_t; t \geq 0)$. Then the following statement holds with respect to P:

(I) The process $\tilde{X}$ defined by

$$\tilde{X}_t := X_t - \int_0^t \frac{ds}{s} X_s \qquad (t \geq 0)$$

is a Brownian motion, and, in addition, X_t is independent of $(\tilde{X}_s; s \leq t)$ for all $t \geq 0$.

One may ask whether it is possible to replace P by some other probability measure Q on $(\Omega, \mathcal{F})$ such that this statement remains true with respect to Q. It turns out that the class $\mathcal{J}$ of such measures Q is characterized by the following condition:

(II) There exists a Q-Brownian motion B and a random variable Y such that

$$X_t = B_t + tY \qquad (t \geq 0).$$

In addition, B and Y are Q-independent.

Let $\mathcal{F}_t$, respectively $\hat{\mathcal{F}}_t$, denote the subfield $\sigma(X_s; s \leq t)$, respectively $\sigma(X_s; s \geq t)$, of $\mathcal{F}$. Due to Girsanov's theorem, any $Q \in \mathcal{J}$ is equivalent to P on $\mathcal{F}_t$ for each $t \geq 0$, and the densities are given by

(III)
$$\left.\frac{dQ}{dP}\right|_{\mathcal{F}_t} = h(X_t, t) \qquad (t \geq 0),$$

with h denoting some space-time harmonic function . This implies

$$\text{(III')} \qquad Q[\,.\mid \hat{\mathcal{F}}_t\,] = P[\,.\mid \hat{\mathcal{F}}_t\,] \qquad (t \geq 0).$$

In fact, the conditions (I) to (III') are all equivalent and may thus be viewed as four different characterizations of the convex set $\mathcal{J}$ of probability measures on $(\Omega, \mathcal{F})$; cf. Jeulin-Yor [5] for the equivalence of (I) to (III). Note that (II) implies the integral representation

$$Q = \int_{\mathbb{R}} \nu(dy) P^y$$

of any measure $Q \in \mathcal{J}$, where P^y denotes the distribution of Brownian motion with constant drift $y \in \mathbb{R}$ and ν some probability measure on $\mathbb{R}$, i.e. $\nu(dy) = Q[\,Y \in dy\,]$.

These results admit a generalization to infinite dimensions: Regarding X_t, B_t and Y as E-valued random variables, $E := \{x \in C[0,1] : x(0) = 0\}$, and P on $\Omega := \{x \in C([0,\infty), E) : x(0) = 0\}$ as the distribution of the Brownian sheet, the conditions (I), (II) and (III') (with "Brownian sheet" instead of "Brownian motion") remain equivalent while the equivalence with (III) is lost, cf. [4]. In this context, the formula in condition (II) becomes

$$\text{(1)} \qquad X_{s,t} = B_{s,t} + tY_s \qquad (s \in [0,1], t \geq 0).$$

The equivalence of (II) and (III') was shown by Föllmer [4] using Dynkin's technique of sufficient statistics.

A second approach to a generalization from Brownian motion to Brownian sheet was suggested by Jeulin and Yor in [5]. This approach consists basically in replacing the time parameter t by the pair (s,t) with $s,t \geq 0$ and giving the appropriate generalization of condition (I). Our purpose in this paper is to formulate the analogues of conditions (II) and (III'), and to prove their equivalence with (I). In particular, we obtain the formula

$$X_{s,t} = B_{s,t} + tY_s^1 + sY_t^2 \qquad (s,t \geq 0),$$

which shows the connection with the first approach, cf. formula (1). In fact, the equivalence of the conditions (I), (II) and (III') in the first approach can be shown analogously to our proof of the Theorem below, cf. [1].

1. The result for the Brownian sheet

Let $\Omega := C_0([0,\infty)^2) := \{x \in C([0,\infty)^2) : x_{s,0} = x_{0,t} = 0;\ s,t \geq 0\}$. Using the coordinate mapping $X_{s,t}(\omega) := \omega(s,t)$, we define the fields

$$\mathcal{F} := \sigma(X_{s,t}; s,t \geq 0), \qquad \mathcal{F}_{s,t} := \sigma(X_{u,v}; u \leq s, v \leq t)$$

$$\text{and} \qquad \hat{\mathcal{F}}_{s,t} := \sigma(X_{u,v}; u \geq s \text{ or } v \geq t)$$

on Ω. In order to simplify the notation, we introduce

$$R_{s,t} := [0,s] \times [0,t], \quad \overset{\circ}{R}_{s,t} := [0,s) \times [0,t) \quad \text{and} \quad \partial R_{s,t} := R_{s,t} - \overset{\circ}{R}_{s,t} .$$

Finally, let P on Ω denote the distribution of the Brownian sheet, i.e. X is a continuous gaussian two parameter process with covariance

$$E^P[\, X_{s_1,t_1} X_{s_2,t_2} \,] = (s_1 \wedge s_2)(t_1 \wedge t_2)$$

with respect to P. Now we can state our main result:

Theorem *Let Q be a probability measure on $(\Omega, \mathcal{F})$. Then the following three assertions are equivalent:*

I. (a) With respect to Q, the process $\tilde{X}$ defined by

$$\tilde{X}_{s,t} := \lim_{\varepsilon \to 0} (X_{s,t} - \int_\varepsilon^s \frac{du}{u} X_{u,t} - \int_\varepsilon^t \frac{dv}{v} X_{s,v} + \int_\varepsilon^s \frac{du}{u} \int_\varepsilon^t \frac{dv}{v} X_{u,v})$$

is a Brownian sheet. (We assume the right hand side to be well defined Q-almost surely.)

(b) $(X_{u,v}; (u,v) \in \partial R_{s,t})$ and $(\tilde{X}_{u,v}; (u,v) \in R_{s,t})$ are Q-independent.

II. (a) There exists a Q-Brownian sheet B as well as a pair of $C_0[0,\infty)$-valued random variables (Y^1, Y^2) such that

$$X_{s,t} = B_{s,t} + tY_s^1 + sY_t^s \qquad (s,t \geq 0).$$

(b) B and (Y^1, Y^2) are Q-independent.

III. For all $s,t \geq 0$ and $f \in b\mathcal{F}$ (i.e. f bounded and $\mathcal{F}$-measurable),

$$E^Q[\, f \mid \hat{\mathcal{F}}_{s,t} \,] = \pi_{s,t} f$$

holds Q-almost surely.

In the theorem above, $\pi_{s,t} f$ is defined by

$$\pi_{s,t}^{\omega} f := E^P[\, f(X_{u,v}^{s,t,\omega}, (u,v) \in R_{s,t}; X_{u,v}(\omega), (u,v) \notin \overset{\circ}{R}_{s,t}) \,],$$

where $X^{s,t,\omega}$ denotes the Brownian bridge from 0 to $(X_{u,v}(\omega), (u,v) \in \partial R_{s,t})$, i.e.

$$X_{u,v}^{s,t,g} := X_{u,v} - \frac{u}{s}(X_{s,v} - g_{s,v}) - \frac{v}{t}(X_{u,t} - g_{u,t}) + \frac{uv}{st}(X_{s,t} - g_{s,t})$$

for $(u,v) \in R_{s,t}$ and $g \in C_0([0,\infty)^2)$.

Remark 1 Since $X^{s,t,g}$ is P-independent of $\hat{\mathcal{F}}_{s,t}$, P satisfies *(III)*.

Remark 2 It is easy to see that $\Pi := (\hat{\mathcal{F}}_{s,t}, \pi_{s,t}; s,t \geq 0)$ is a specification in the sense of [2] and [3].

2. Proof of the Theorem

We prove *(I)* $\Rightarrow$ *(II)* $\Rightarrow$ *(III)* $\Rightarrow$ *(I)*.

2.1 *(I)* $\Rightarrow$ *(II)*

The key to this part of the proof is the following

Lemma 1 *Let B denote a Brownian sheet and X a process satisfying the following stochastic differential equation:*

$$(2) \qquad X_{s,t} = B_{s,t} + \int_0^s \frac{du}{u} X_{u,t} + \int_0^t \frac{dv}{v} X_{s,v} - \int_0^s \frac{du}{u} \int_0^t \frac{dv}{v} X_{u,v}$$

(We assume the right hand side to be well defined almost surely.)
Then

$$\frac{X_{s_2,t_2}}{s_2 t_2} - \frac{X_{s_2,t_1}}{s_2 t_1} - \frac{X_{s_1,t_2}}{s_1 t_2} + \frac{X_{s_1,t_1}}{s_1 t_1} = \int_{s_1}^{s_2} \int_{t_1}^{t_2} \frac{dB_{s,t}}{st}$$

holds for all s_i, t_i satisfying $0 < s_1 \leq s_2$ and $0 < t_1 \leq t_2$, or, shorter but less precisely,

$$d\frac{X_{s,t}}{st} = \frac{dB_{s,t}}{st}.$$

Proof: The proof is straightforward but involves some computation:

$$\begin{aligned}
\int_{s_1}^{s_2} \int_{t_1}^{t_2} \frac{dB_{s,t}}{st} &= \frac{B_{s_2,t_2}}{s_2 t_2} - \frac{B_{s_2,t_1}}{s_2 t_1} - \frac{B_{s_1,t_2}}{s_1 t_2} + \frac{B_{s_1,t_1}}{s_1 t_1} \\
&\quad + \int_{t_1}^{t_2} \frac{dt}{t} \left(\frac{B_{s_2,t}}{s_2 t} - \frac{B_{s_1,t}}{s_1 t} \right) + \int_{s_1}^{s_2} \frac{ds}{s} \left(\frac{B_{s,t_2}}{s t_2} - \frac{B_{s,t_1}}{s t_1} \right) \\
&\quad + \int_{s_1}^{s_2} \frac{ds}{s} \int_{t_1}^{t_2} \frac{dt}{t} \frac{B_{s,t}}{st} \\
&= \frac{X_{s_2,t_2}}{s_2 t_2} - \frac{X_{s_2,t_1}}{s_2 t_1} - \frac{X_{s_1,t_2}}{s_1 t_2} + \frac{X_{s_1,t_1}}{s_1 t_1}.
\end{aligned}$$

The first equation follows by considering the corresponding indicator functions. To conclude, replace B by X using (2) and simplify according to the classical product rule. □

Since Q satisfies *(I.a)*, we may apply Lemma 1. The right hand side of

$$\frac{X_{u,v}}{uv} - \frac{X_{u,t}}{ut} - \frac{X_{s,v}}{sv} + \frac{X_{s,t}}{st} = \int_s^u \int_t^v \frac{d\tilde{X}_{a,b}}{ab}$$

converges in $L^2(\Omega, \mathcal{F}, Q)$ for $u, v \to \infty$. Therefore,

$$Y_{s,t} := \lim_{u,v\to\infty} \left(\frac{X_{s,v}}{sv} + \frac{X_{u,t}}{ut} - \frac{X_{u,v}}{uv}\right)$$

exists Q-a.s. for all $s, t > 0$, i.e.

(3) $$X_{s,t} = B_{s,t} + stY_{s,t} =: B_{s,t} + H_{s,t}.$$

Obviously, B defined by

$$B_{s,t} := st \int_s^\infty \int_t^\infty \frac{d\tilde{X}_{a,b}}{ab}$$

is a Brownian sheet.

We investigate the drift H. We see from (3) that H lives on $C_0([0,\infty)^2)$ Q-almost surely. Since

$$Y_{s,t_2} - Y_{s,t_1} = \lim_{u\to\infty} \left(\frac{X_{u,t_2}}{ut_2} - \frac{X_{u,t_1}}{ut_1}\right)$$

is independent of $s > 0$, we may introduce

$$Y_s^1 := sY_{s,1} \quad \text{and} \quad Y_t^2 := tY_{1,t} - tY_{1,1} = tY_{s,t} - tY_{s,1}$$

and decompose H as

$$H_{s,t} = tY_s^1 + sY_t^2 \qquad (s, t > 0).$$

Now, since $H \in C_0([0,\infty)^2)$ holds, $Y^1, Y^2 \in C_0[0,\infty)$.

It remains to show the Q-independence of B and (Y^1, Y^2), or, equivalently, the Q-independence of B and H: Let $Z \in L^2(\Omega, \tilde{\mathcal{F}}, Q)$, $\tilde{\mathcal{F}} := \sigma(\tilde{X}_{s,t}; s, t \geq 0)$ and let $\varphi \in C([0,\infty)^{n^2})$ be bounded. Then *(I.b)* implies

$$\begin{aligned} &E^Q[\, E^Q[\, Z \mid \tilde{\mathcal{F}}_{u,v}\,]\, \varphi(s_j t_k (\frac{X_{s_j,v}}{s_j v} + \frac{X_{u,t_k}}{ut_k} - \frac{X_{u,v}}{uv}); j,k \leq n)\,] \\ = \;\; & E^Q[\, Z\,]\, E^Q[\, \varphi(s_j t_k (\frac{X_{s_j,v}}{s_j v} + \frac{X_{u,t_k}}{ut_k} - \frac{X_{u,v}}{uv}); j,k \leq n)\,]. \end{aligned}$$

The independence follows by taking limits $u, v \to \infty$. □

2.2 *(II) ⇒ (III)*

We assume Q to satisfy *(II)*. Since H and B are Q-independent, *(II)* translates into the integral representation

$$Q = \int_M \nu(dh) P^h, \tag{4}$$

where $\nu(dh) := Q[\, H \in dh \,]$, $P^h := \tau_h(P)$, τ_h denotes translation by h and the "Martin boundary" M is given by

$$M := \{h \in C_0([0,\infty)^2) : \exists\, y^1, y^2 \in C_0[0,\infty) : h_{s,t} = t y^1_s + s y^2_t\}.$$

Therefore we only have to show that $P^h[\,.\mid \hat{\mathcal{F}}_{s,t}\,] = \pi_{s,t}$ for $h \in M$. We prove this assertion by observing that $\tau_h(X^{s,t,g}) = X^{s,t,(\tau_h g)}$ holds on $R_{s,t}$ for any $h \in M$, as a short computation shows, and thus

$$\begin{aligned}
& E^{P^h}[\,(\pi_t f)\, g\,] \\
&= \int_\Omega P^h(d\omega) E^P[\, f(X^{s,t,\omega}_{u,v}, (u,v) \in R_{s,t}; X_{u,v}(\omega), (u,v) \notin R^\circ_{s,t})\,]\, g(\omega) \\
&= \int_\Omega P(d\omega) E^P[\, f(X^{s,t,\tau_h\omega}_{u,v}, \text{—}; (\tau_h X)_{u,v}(\omega), \text{—})\,]\, (g \circ \tau_h)(\omega) \\
&= \int_\Omega P(d\omega) E^P[\, (f \circ \tau_h)(X^{s,t,\omega}_{u,v}, \text{—}; X_{u,v}(\omega), \text{—})\,]\, (g \circ \tau_h)(\omega) \\
&= E^P[\,(\pi_{s,t}(f \circ \tau_h))\,(g \circ \tau_h)\,] \\
&= E^{P^h}[\, f\, g\,]
\end{aligned}$$

for all $f \in b\mathcal{F}$ and $g \in b\hat{\mathcal{F}}_{s,t}$. □

2.3 *(III)* $\Rightarrow$ *(I)*

This part of the proof is based on the fact that P satisfies *(I)*, cf. Jeulin-Yor [5]. We include an argument for part *(I.b)*:

Lemma 2 *Let X denote a Brownian sheet. Then, for all $a, b \geq 0$,*

$$\begin{aligned}
& \sigma(\tilde{X}_{s,t}; (s,t) \in R_{a,b}) \\
&= \sigma(X^{s,t,0}_{u,v}, (u,v) \in R_{s,t}; (s,t) \in R_{a,b}) \\
&= \sigma(X^{a,b,0}_{s,t}; (s,t) \in R_{a,b}).
\end{aligned}$$

Proof: We have

$$\tilde{X}_{s,t} = \int_0^s \frac{du}{u} \int_0^t \frac{dv}{v} X^{s,t,0}_{u,v}, \qquad X^{s,t,0}_{u,v} = (X^{a,b,0})^{s,t,0}_{u,v}$$

$$\text{and} \qquad X^{a,b,0}_{s,t} = st \int_s^a \int_t^b \frac{d\tilde{X}_{u,v}}{uv}.$$

The third equation is a consequence of Lemma 1. □

Now we assume that Q satisfies *(III)*. It is easy to see that $\tilde{X}_{s,t}$ is well defined Q-almost surely: Denote the set in question by A. Then

$$\begin{aligned} Q[\,\{\tilde{X}_{s,t} \text{ is well defined}\}\,] &= E^Q[\,I_A\,] = E^Q[\,\pi_{a,b} I_A\,] \\ &= \int_\Omega Q(d\omega)\; E^P[\,\lim_{\varepsilon\to 0}(X^{a,b,\omega}_{s,t} - \int_\varepsilon^s \frac{du}{u} X^{a,b,\omega}_{u,t} - \ldots + \ldots) \text{ is well defined}\,] \\ &= 1, \end{aligned}$$

since

$$\begin{aligned} &(\widetilde{X^{a,b,g}})_{s,t} \\ &= \lim_{\varepsilon\to 0}(X^{a,b,g}_{s,t} - \int_\varepsilon^s \frac{du}{u} X^{a,b,g}_{u,t} - \int_\varepsilon^t \frac{dv}{v} X^{a,b,g}_{s,v} + \int_\varepsilon^s \frac{du}{u}\int_\varepsilon^t \frac{dv}{v} X^{a,b,g}_{u,v}) \\ &= \tilde{X}_{s,t} + \lim_{\varepsilon\to 0}(\int_\varepsilon^s \frac{du}{u}\frac{t}{b}(X_{u,b} - g_{u,b}) + \int_\varepsilon^t \frac{dv}{v}\frac{s}{a}(X_{a,v} - g_{a,v}) \\ &\qquad - \int_\varepsilon^s \frac{du}{u}\int_\varepsilon^t \frac{dv}{v}[\frac{u}{a}(X_{a,v} - g_{a,v}) + \frac{v}{b}(X_{u,b} - g_{u,b})]) \\ &= \tilde{X}_{s,t} + \lim_{\varepsilon\to 0}(\frac{\varepsilon}{b}\int_\varepsilon^s \frac{du}{u}(X_{u,b} - g_{u,b}) + \frac{\varepsilon}{a}\int_\varepsilon^t \frac{dv}{v}(X_{a,v} - g_{a,v})) \\ &= \tilde{X}_{s,t}, \end{aligned}$$

whenever $\tilde{X}_{s,t}$ is well defined. In order to show that Q satisfies *(I)*, we consider complex-valued functions which depend on $\tilde{X}$: Let $s_{j-1} \le s_j < s$, $t_{j-1} \le t_j < t$. Then for Q-almost every ω, one has

$$\begin{aligned} &E^Q[\,\exp\{i \sum_{j,k=1}^n \lambda_{j,k}(\tilde{X}_{s_j,t_k} - \tilde{X}_{s_j,t_{k-1}} - \tilde{X}_{s_j,t_{k-1}} - \tilde{X}_{s_{j-1},t_{k-1}})\} \mid \hat{\mathcal{F}}_{s,t}\,](\omega) \\ &= E^P[\,\exp\{i \sum_{j,k=1}^n \lambda_{j,k}((\widetilde{X^{s,t,\omega}})_{s_j,t_k} - \ldots - \ldots + \ldots)\}\,] \\ &= E^P[\,\exp\{i \sum_{j,k=1}^n \lambda_{j,k}(\tilde{X}_{s_j,t_k} - \tilde{X}_{s_j,t_{k-1}} - \tilde{X}_{s_j,t_{k-1}} - \tilde{X}_{s_{j-1},t_{k-1}})\}\,] \\ &= \exp\{-\frac{1}{2}\sum_{j,k=1}^n \lambda_{j,k}^2 (s_j - s_{j-1})(t_k - t_{k-1})\}. \end{aligned}$$

□

3. Another proof using Dynkin's technique of Sufficient Statistics

In the following, we give an alternative direct proof of the implication *(III)* $\Rightarrow$ *(II)* in the Theorem. This is analogous to Föllmer's proof in [4], cf. Introduction. We assume familiarity with the notions and results in [2]. As mentioned in Remark 2, $\Pi := (\mathcal{F}_{s,t}, \pi_{s,t}, s,t \geq 0)$ is a local specification. Therefore,

$$\hat{\mathcal{F}}_\infty := \bigcap_{s,t\geq 0} \hat{\mathcal{F}}_{s,t}$$

is sufficient for the set $G(\Pi)$ of Gibbs-states specified by Π, i.e. the set of probability measures Q satisfying *(III)*. Furthermore, the integral representation

$$Q = \int_{G(\Pi)^e} \mu(d\tilde{P})\, \tilde{P}$$

holds where μ denotes a probability measure on the set $G(\Pi)^e$ of extreme points of $G(\Pi)$. In order to prove *(II)* or equivalently

$$Q = \int_M \nu(dh) P^h$$

for some probability measure ν on M, cf. formula (4), it suffices to show

$$G(\Pi)^e \subset \{P^h : h \in M\}$$

since $P^h \mapsto h$ is a measurable mapping from $G(\Pi)^e$ to M.

Now we assume $Q \in G(\Pi)^e$ and choose two sequences (s_k) and (t_k) with $s_k, t_k \overset{k\to\infty}{\longrightarrow} \infty$. Then

$$\pi^\omega_{s_k,t_k} \overset{k\to\infty}{\Rightarrow} Q,$$

i.e. weak convergence holds for some $\omega \in \Omega$, cf. [2]. In particular, the marginal distributions at a fixed parameter (s,t) converge, and this implies the existence of

$$(5) \qquad h_{s,t} := \lim_{k\to\infty} \left(\frac{s}{s_k} X_{s_k,t}(\omega) + \frac{t}{t_k} X_{s,t_k}(\omega) - \frac{st}{s_k t_k} X_{s_k,t_k}(\omega)\right) \in \mathbb{R}$$

for any $s,t \geq 0$.

We claim $Q = P^h$. Indeed, we may regard Q as well as P^h as measures on the set of all real-valued functions on $[0,\infty)^2$. Then, for any continuous, bounded function $f = g(X_{s_1,t_1}, \ldots, X_{s_n,t_n})$, we obtain

$$\begin{aligned} Q[f] &= \lim_{k\to\infty} \pi^{\omega}_{s_k,t_k} f \\ &= \lim_{k\to\infty} P[\, g(X^{s_k,t_k}_{s_i,t_i}, 0 \le i \le n)\,] \\ &= P[\, g(X_{s_i,t_i} + h_{s_i,t_i}, 0 \le i \le n)\,] \\ &= P^h[f]. \end{aligned}$$

The function h in formula (5) is continuous on $[0,\infty)^2$ since, choosing sequences (s_k) and (t_k) with $s_k \to s$ and $t_k \to t$, one has

$$\begin{aligned} 1 &= Q[\,\liminf_{k\to\infty} X_{s_k,t_k} = \limsup_{k\to\infty} X_{s_k,t_k}\,] \\ &= P[\,\liminf_{k\to\infty}(X_{s_k,t_k} + h_{s_k,t_k}) = \limsup_{k\to\infty}(X_{s_k,t_k} + h_{s_k,t_k})\,] \\ &= P[\,\liminf_{k\to\infty} h_{s_k,t_k} = \limsup_{k\to\infty} h_{s_k,t_k}\,]. \end{aligned}$$

Finally, $h \in M$ follows as in section 2.1. □

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