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SCATTERING BY TWO CONVEX BODIES

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1. INTRODUCTION.

Let $\mathcal{O}_1$ be a bounded open convex set in $\mathbb{R}^2$ with smooth boundary Γ_1 such that

(i) $\mathcal{O}_1 \subset \{x = (x_1, x_2) \in \mathbb{R}^2; x_1 < 0\}$,

(ii) $a_1 = (0, 0) \in \Gamma_1$,

(iii) Γ_1 is represented near a_1 as

$$x_1 = -{x_2}^{2l}$$

where l is a positive integer≥ 2,

(iv) the curvature of Γ_1 does not vanish on $\Gamma_1 - \{a_1\}$.

Let $\mathcal{O}_2$ be a bounded open convex set in $\mathbb{R}^2$ with smooth boundary Γ_2 such that

(i) $\mathcal{O}_2 \subset \{x = (x_1, x_2) \in \mathbb{R}^2; x_1 > d\}$ where d is a positive constant,

(ii) $a_2 = (d, 0) \in \Gamma_2$,

(iii) Γ_2 is represented near a_2 as

$$x_1 = d + {x_2}^{2l}.,$$

(iv) the curvature of Γ_2 does not vanish on $\Gamma_1 - \{a_2\}$.

We set

$$\mathcal{O} = \mathcal{O}_1 \cup \mathcal{O}_2, \quad \Gamma = \Gamma_1 \cup \Gamma_2$$

and

$$\Omega = \mathbb{R}^2 - \overline{\mathcal{O}}.$$

Consider the following boundary value problem with parameter $\mu \in \mathbb{C}$

$$\begin{cases} (\Delta + \mu^2)u(x) = 0 & \text{in } \Omega \\ u(x) = g(x) & \text{in } \Gamma \end{cases} \tag{1.1}$$

for $g(x) \in C^\infty(\Gamma)$. For $\operatorname{Im} \mu < 0$, (1.1) has a unique solution in $L^2(\Omega)$. Denote the solution $u(x)$ as

$$u(x) = (U(\mu)g)(x).$$

Then by the regularity theorem for elliptic operators, $U(\mu)$ can be regarded as a continuous operator from $C^\infty(\Gamma)$ into $C^\infty(\overline{\Omega})$. Thus, $U(\mu)$ becomes an $\mathcal{L}(C^\infty(\Gamma), C^\infty(\overline{\Omega}))$−valued holomorphic function in $\{\mu; \operatorname{Im} \mu < 0\}$, where $\mathcal{L}(E, F)$ denotes the set of all the continuous operators from E into F.

We would like to consider the analytic continuation of $U(\mu)$ into $\{\mu; \operatorname{Im} \mu \geq 0\}$. The main result that we shall show is the following theorem:

THEOREM 1. *Assuem that*

$$l \geq 4, \tag{1.2}$$

and set

$$\alpha = (l-1)^{-1}.$$

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Then, for any ε_1, $\varepsilon_2 > 0$, there exists a positive constant $C_{\varepsilon_1 . \varepsilon_2}$ such that $U(\mu)$ can be continued analytically into

$$(1.3) \qquad \{\mu;\ \operatorname{Im}\mu \leq |\operatorname{Re}\mu|^{-(1+2\alpha)^{-1}-\varepsilon_1},\ |\operatorname{Re}\mu| \geq C_{\varepsilon_1,\varepsilon_2}\} \\ - \cup_{r=-\infty}^{\infty}\{\mu; \operatorname{Im}\mu \geq 0 \text{ and } |\frac{\pi}{d}r - \operatorname{Re}\mu| < \varepsilon_2\}.$$

Background of the problem.

I would like to mention about the background of the above theorem. In the study of scattering by obstacles, the problem to know relationships between the geometry of obstacles and the distribution of poles of scattering matrices is one of the most important and interesting problems. As to the distribution of poles of scattering matrices for trapping obstacles, Bardos-Guillot-Ralston[BGR] first made consideration on the following example in $\mathbb{R}^n$, $n \geq 3$ odd :

$$\mathcal{O} = \mathcal{O}_1 \cup \mathcal{O}_2$$

where

$$\mathcal{O}_j,\ j = 1,2 \text{ are strictly convex and } \overline{\mathcal{O}_1} \cap \overline{\mathcal{O}_2} = \phi.$$

Denote by $\mathcal{S}(\mu)$ the scattering matrix. They showed that, for any $\varepsilon > 0$, $\mathcal{S}(\mu)$ has an infinite number of poles in the logarithmic domain

$$\{\mu;\ \operatorname{Im}\mu \leq \varepsilon \log(|\mu| + 1)\}.$$

Next Ikawa[Ik 1] considered the same example and showed that there exists a positive constant c_0 such that, in the strip $\{\mu; 0 < \operatorname{Im}\mu < \frac{2}{3}c_0\}$ the poles of $\mathcal{S}(\mu)$ distribute asymptotically at the points $\frac{\pi}{d}j + \sqrt{-1}c_0$, $j = 0, \pm1, \pm2, \cdots$, where $d = \text{distance}(\mathcal{O}_1, \mathcal{O}_2)$.

After that C.Gérard[Gé] proved that, for any $a > 0$, the poles of $\mathcal{S}(\mu)$ in the strip $\{\mu;\ 0 < \operatorname{Im}\mu < a\}$ distribute asymptotically on the points

$$\frac{\pi}{d}j + \sqrt{-1}c_m, \quad j = 0, \pm1, \pm2, \cdots, \quad m = 0, 2, \cdots, l_0$$

where

$$0 < c_0 \leq c_1 \leq c_2 \leq \cdots \leq c_{l_0} < a.$$

The constants c_m are determined by d and the geometry of Γ_j near a_j, j=1,2 where a_1 and a_2 are the point $a_j \in \Gamma_j$, $j = 1, 2$ such that

$$\text{distance}(\mathcal{O}_1, \mathcal{O}_2) = |a_1 - a_2|.$$

The formula which gives c_m indicates that, if the all the principal curvatures of Γ_j at a_j, $j = 1, 2$, become small, the constants c_m beocme also small, and if the all the principal curvatures vanishes at a_j, $j = 1, 2$, all the c_m determined by the formula are equal to 0. Remark that, even though the formula determining c_m is valid for the case that all the principal curvatures vanish at a_j, the reasoning for the distribution of poles of [Ik 1] or of [Gé] is no more valid.

Thus it is natural to question whether there exist an infinite number of poles of $\mathcal{S}(\mu)$ in $\{\mu; \operatorname{Im}\mu < \varepsilon\}$ for any $\varepsilon > 0$ in the case that all the principal curvatures vanish at a_j. Concerning this question, we considered in [Ik 2] an example in $\mathbb{R}^3$ such that Γ_1 and Γ_2 are represented as

$$x_1 = -(x_2^2 + x_3^2)^{2l} \quad \text{near } a_1$$

and

$$x_1 = d + (x_2^2 + x_3^2)^{2l} \quad \text{near } a_2$$

respectively, and showd that, if $l \geq 2$, $\mathcal{S}(\mu)$ has infinitely many poles in $\{\mu; \operatorname{Im}\mu \leq |\operatorname{Re}\mu|^{-\gamma}\}$ for some positive constant γ.

By taking account of the results of [Ik 1] and [Gé], it seems very likely that the poles of $\mathcal{S}(\mu)$ in the domain $\{\mu; \operatorname{Im}\mu \leq |\operatorname{Re}\mu|^{-\gamma}\}$ exist only near the points $\frac{\pi}{d}j$, $j = \pm 1, \pm 2, \cdots$.

Recall that the poles of $\mathcal{S}(\mu)$ coincide with those of $U(\mu)$. Therefore, even though Theorem 1 in this note is of the analytic continuation of $U(\mu)$ for an obstacle in $\mathbb{R}^2$, it gives us a partial answer to the above conjecture.

2. CONSTRUCTION OF ASYMPTOTIC SOLUTIONS.

For the construction of asymptotic solutions, it is essential to consider the behavior of solutions near the periodic rays, which is the ray going and returning between a_1 and a_2. Denote by $\Omega(\delta)$ $(\delta > 0)$ the domain surrounded by the four curves

$$x_2 = \delta, \quad x_2 = -\delta, \quad x_1 = -x_2^{2l}, \quad x_1 = d + x_2^{2l},$$

and set

$$S_j(\delta) = \Omega(\delta) \cap \Gamma_j, \quad j = 1, 2.$$

We fix a small $\delta > 0$. Let $\omega \in \{\omega \in \mathbb{R}^2; |\omega| = 1\}$, and let $f(x) \in C_0^\infty(S_1(\delta))$. We shall construct asymptotic solutions to boundary value problem (1.1) for an oscillatory boundary data on $S_1(\delta)$

$$g(x, \mu) = e^{-i\mu x \cdot \omega} f(x). \tag{2.1}$$

We set

$$D_{r,\beta,\varepsilon} = \{\mu = ik + \sigma; \, 2r\pi + \varepsilon \leq |k| \leq 2(r+1)\pi - \varepsilon, \, \sigma \leq r^{-\beta}\}$$

and

$$D_{\beta,\varepsilon} = \cup_{r=1}^{\infty} D_{r,\beta,\varepsilon}.$$

We shall show the following

THEOREM 2.1. *For any fixed constants $\beta > (1+2\alpha)^{-1}$, $\varepsilon > 0$ and N positive integer, there is an asymptotic solution $u(x,\mu)$ of the boundary value problem (1.2) for the oscillatory data (2.1) with the following properties:*

(i) $u(\cdot,\mu)$ *is* $C^\infty(\overline{\Omega})$ – *valued holomorphic function in* $D_{\beta,\varepsilon}$,

(ii) $(\Delta+\mu^2)u(x,\mu) = 0$ *in* Ω *for all* $\mu \in D_{\beta,\varepsilon}$,

(iii) $|u(x,\mu) - g(x,\mu)| \le C_N\,|\mu|^{-N}$ *for all* $x \in \Gamma_1$ *and* $\mu \in D_{\beta,\varepsilon}$,

(iv) $|u(x,\mu)| \le C_N\,|\mu|^{-N}$ *for all* $x \in \Gamma_2$ *and* $\mu \in D_{\beta,\varepsilon}$.

Theorem 1 is derived from Theorem 2.1 by the standard argument. For the proof of Theorem 2.1, the following proposition is crucial. Its proof of the following proposition is fairly long, and we omit it.

PROPOSITION 2.2. *Let ω be an element of S^1 near $(1,0)$, and set*

$$\varphi_1(x) = x\cdot\omega.$$

For any positive integer N, there is a sequence of real valued smooth functions defined in a neighborhood of $\Omega(\delta)$ having the following expansions in $n^{-\alpha}$:

$$\frac{\partial\varphi_n}{\partial x_2}(x) = b_0(x)n^{-1-\alpha} + b_1(x)n^{-1-2\alpha} + \cdots + b_M(x)n^{-1-(M+1)\alpha},$$

$$\varphi_{2n}(x) = c_0(x) + 2nd + c_1(x)n^{-1-2\alpha} + \cdots + c_M(x)n^{-1-(M+1)\alpha},$$

$$\varphi_{2n+1}(x) = \tilde{c}_0(x) + (2n+1)d + \tilde{c}_1(x)n^{-1-2\alpha} + \tilde{c}_2(x)n^{-1-3\alpha} + \cdots + \tilde{c}_M(x)n^{-1-(M+1)\alpha},$$

where M is a positive integer and $b_j(x)$, $c_j(x)$, $\tilde{c}_j(x)$, $j = 1,2,\cdots, M$ are smooth functions.
Moreover, $\varphi_j(x)$, $j = 1,2,\cdots$, satisfy the eikonal equation

$$|\nabla\varphi_j(x)| = 1 \quad \text{in } \Omega(\delta)$$

and the difference $\varphi_{j+1} - \varphi_j$ on the boundary satisfies

$$(\varphi_{2n} - \varphi_{2n-1})(x) = e_0(x) + e_{N-1}(x)n^{-1-N\alpha} + e_N(x)n^{-1-(N+1)\alpha} + \cdots + e_M(x)n^{-1-(M+1)\alpha} \quad \text{for all } x \in S_1(\delta),$$

$$(\varphi_{2n+1} - \varphi_{2n})(x) = \tilde{e}_0(x) + \tilde{e}_{N-1}(x)n^{-1-N\alpha} + \tilde{e}_N(x)n^{-1-(N+1)\alpha} + \cdots + \tilde{e}_M(x)n^{-1-(M+1)\alpha} \quad \text{for all } x \in S_2(\delta),$$

where $e_0(x)$ and $\tilde{e}_0(x)$ satisfy the following estimate

$$|e_0(x)|,\ |\tilde{e}_0(x)| \leq |x_2|^N.$$

By using the sequence $\{\varphi_j\}_{j=1}^{\infty}$ of phase functions in the above proposition, we construct a sequence of asymptotic solutions by the standard procedure. For $\mu = k + i\sigma, \sigma < 0$ set

$$u_j(x,\mu) = \exp(-i\mu\varphi_j(x))\, v_j(x,\mu),$$
$$v_j(x,\mu) = \sum_{p=0}^{P} v_{jp}(x)(i\mu)^{-p},$$

and we shall construt v_{jp} successively by the following procedure:
Set

$$T_j = \nabla\varphi_j \cdot \nabla + \Delta\varphi_j.$$

Let $v_{00}(x)$ be solution of

$$\begin{cases} T_0\, v_{00} = 0 & \text{in } \Omega(\delta), \\ v_{00}(x) = f(x) & \text{on } S_1(\delta) \end{cases}$$

and $v_{0p}(x),\ p = 1, 2, \cdots, P$ be solutions of

$$\begin{cases} T_0\, v_{0p} = -\Delta\, v_{0,p-1} & \text{in } \Omega(\delta), \\ v_{0p}(x) = 0 & \text{on } S_1(\delta). \end{cases}$$

Let $j \geq 1$ and suppose that $v_{j-1,p}(x)$ are defined. Define v_{jp} as the solutions of

$$\begin{cases} T_j\, v_{jp} = \Delta\, v_{j,p-1} & \text{in } \Omega(\delta), \\ v_{jp}(x) = v_{j-1,p} & \text{on } S_{\epsilon(j)}(\delta) \end{cases}$$

where we take $v_{j,-1} \equiv 0$ and

$$\epsilon(j) = \begin{cases} 1 & \text{for } j \text{ even}, \\ 2 & \text{for } j \text{ odd}. \end{cases}$$

By using the properties of φ_j mentioned in Proposition, we get the following asymptotic expansion of $v_{np}(x)$ in $n^{-\alpha}$:

$$v_{np}(x) \sim w_{p0}(x)n^p + w_{p1}(x)n^{p-\alpha} + w_{p2}(x)n^{p-2\alpha} + \cdots + w_{pK}(x)n^{p-K\alpha}.$$

Now we set

$$u(x,\mu) = \sum_{n=0}^{\infty} (-1)^n\, u_n(x,\mu).$$

Evidently $u(x,\mu)$ converges absolutely for $\operatorname{Re}\mu = \sigma < 0$, and we have the following relations:

(2.2) $$(\Delta + \mu^2)\, u(x,\mu) = (i\mu)^{-P} \sum_{n=0}^{\infty} \exp(-i\mu\varphi_n(x))\, \Delta v_{nP}(x) \quad \text{in } \Omega(\delta),$$

(2.3) $$u(x,\mu)|_{S_1(\delta)} = \exp(-i\mu\varphi_1(x))\, f(x) + \sum_{n=0}^{\infty} \{\exp(-i\mu\varphi_{2n}(x)) - \exp(-i\mu\varphi_{2n-1}(x))\}\, v_{2n}(x,\mu) \quad \text{on} \quad S_1(\delta)$$

and

(2.4) $$u(x,\mu)|_{S_2(\delta)} = \sum_{n=0}^{\infty} \{\exp(-i\mu\varphi_{2n+1}(x)) - \exp(-i\mu\varphi_{2n}(x))\}\, v_{2n+1}(x,\mu) \quad \text{on} \quad S_2(\delta).$$

Now let η and ε be arbtrary positive constants. By using Proposition 2.2 we have from (2.2)

(2.5) $$|(\Delta + \mu^2)u(x,\mu)| \le C|\mu|^{-P} \quad \text{for } x \in \Omega(\delta) \text{and } \operatorname{Im}\mu < -\varepsilon.$$

We have from (2.3)

(2.6) $$|u(x,\mu) - g(x,\mu)| \le C_{N,\eta}|\mu|^{-\eta N} \text{for all } x \in S_2(|\mu|^{-\eta}) \text{ and } \operatorname{Im}\mu < -\varepsilon.$$

Similarly we have from (2.4)

(2.7) $$|u(x,\mu)| \le C_{N,\eta}|\mu|^{-\eta N} \text{for all } x \in S_2(|\mu|^{-\eta}) \text{ and } \operatorname{Im}\mu < -\varepsilon.$$

If we use the argment used in [Ik 2] and that of [V] jointly, we can derive from the estimates (2.5), (2.6) and (2.7) the assertion of Theorem 2.1 for $\operatorname{Re}\mu < 0$.

Next, consider the anlytic continuation of $u(x,\mu)$ and the above estimates. By applying Lemma 3.2 to each term of $u(x,\mu)$, we see that $u(x,\mu)$ and above estimates can be prolonged analytically into $D_{\beta,\varepsilon}$.

3. ANALYTIC CONTIMUATION OF THE ZETA FUNCTION AND ITS GENERALIZATION.

Let m be a positive integer and let z and s be complex numbers. For $|z| < 1$ we define the function $F(z,s:m)$ by

(3.1) $$F(z,s:m) = \sum_{n \ge m} z^n n^{-s}.$$

Obviously, the right hand side of (3.1) converges absolutely, which implies that the function $F(z, s : m)$ is holomorphic in $z \in \{z; |z| < 1\}$ for any $s \in \mathbb{C}$.

We consider the analytic continuation of F. First assume $\mathrm{Re}\, s > 0$, and set

$$I(z, s : m) = \int_0^\infty \frac{z^m\, e^{-mx}\, x^{s-1}}{1 - ze^{-x}}\, dz.$$

We see that, for each $\mathrm{Re}\, s > 0$, $I(z, s : m)$ is holomorphic in $z \in \mathbb{C} - [1, \infty)$. By the standard way we have the following integral representation of F:

$$F(z, s : m) = \frac{1}{\Gamma(s)}\, I(z, s : m) \qquad \text{for } |z| < 1. \tag{3.2}$$

On the other hand, the definition (1) gives us that

$$z \frac{\partial F}{\partial z}(z, s : m) = F(z, s - 1 : m) \qquad \text{for all } |z| < 1.$$

Let a be a positive integer. Then we have for $\mathrm{Re}\, s > 0$ and $|z| < 1$ the expression

$$F(z, s - a : m) = \frac{1}{\Gamma(s)} \left(z \frac{\partial}{\partial z}\right)^a I(z, s : m), \tag{3.3}$$

from which we derive the following

LEMMA 3.1. *For any $s \in \mathbb{C}$, the function $F(z, s : m)$ can be continued holomorphically into the domain $D = \mathbb{C} - [1, \infty)$. Moreover, we have the following estimate:*

$$|F(z, s : m)| \leq C_{K,a} \frac{\Gamma(\mathrm{Re} z + a)}{|\Gamma(s + a)|}\, m^{-\mathrm{Re}\, s}\, |z|^m\, (1 + |z|)^a \tag{3.4}$$
$$\textit{for all } \mathrm{Re} s > -a \textit{ and } z \in K,$$

where K is arbitrary compact set in D, a is an arbitrary positive integer and $C_{K,a}$ is a constant independent of m.

PROOF: By using the fact that $I(z, s : m)$ is holomorphic in $z \in D$ for any $s \in \mathbb{C}$, the expression (3) proves Lemma 3 except the estimate (4). It is easy to show by the induction that

$$\left(z \frac{\partial}{\partial z}\right)^a \frac{z^m}{1 - ze^{-x}} = \frac{m^a\, z^m}{(1 - ze^{-x})^{a+1}} \{1 + c_{a.1}(m)\, ze^{-x} +$$
$$+ c_{a.2}(m)(ze^{-x})^2 + \cdots + c_{a.a}(m)(ze^{-x})^a\},$$

where the coefficients $c_{a,l}(m)$, $l = 1, 2, \ldots, a$ are polynomials of m^{-1} of order less than a, and they satisfy

$$|c_{a.l}(m)| \leq C_a \qquad \text{for all } m.$$

Thus, if we set

$$\max_{\substack{x\geq 0\\ z\in K}} |1 - ze^{-x}| = c_K,$$

we have for all $\operatorname{Re} s > 0$

$$\begin{aligned}&\left|\left(z\frac{\partial}{\partial z}\right)^a I(z,s:m)\right|\\ &\leq m^a\,|z|^m\,(c_K)^{-(a+1)}\,C_a(1+|z|)^a \int_0^\infty e^{-mx}|x^{s-1}|\,dx.\end{aligned}$$

Substituting this estimate into (3.3) we get immediately for all $\operatorname{Re} s > 0$

$$|F(z,s-a:m)| \leq (C_K)^{-(a+1)}\,c_a\,\frac{\Gamma(\operatorname{Re} z)}{|\Gamma(s)|}\,m^{a-\operatorname{Re} s}\,|z|^m\,(1+|z|)^a.$$

Denoting $s-a$ in the above inequality by s anew, we get (3.4). Q.E.D.

Set

$$R_\beta(\mu:q) = \sum_{n\geq|k|^\beta} \exp\left(-i\mu(n + c_2 n^{-1-2\alpha} + c_3 n^{-1-3\alpha} + \cdots + c_M n^{-1-M\alpha})\right) n^q.$$

For $\sigma < 0$, it is evident that the right hand side converges absolutely. Now we consider the holomorphic extension of $R_\beta(\mu, q)$ in to $\sigma > 0$.

LEMMA 3.2. *Let $\beta > (1+2\alpha)^{-1}$ and let $\varepsilon > 0$. For any positive integer r, $R_\beta(\mu:q)$ can be prolonged analytically into $D_{r,\beta,\varepsilon}$. Moreover, we have the following estimates:*

$$|R_\beta(\mu:q)| \leq C_{\beta,\varepsilon}\, r^{q\beta} \qquad \text{for all } \mu\in D_{r,\beta,\varepsilon}$$

and

$$\left|R_\beta(\mu:q) - F(e^{-i\mu}, -q:[r^\beta])\right| \leq C_{K,q}\,c_1\,r^{q\beta-\gamma} \quad \text{for all } \mu\in D_{r,\beta,\varepsilon}.$$

PROOF: Here we proof only the case where $c_j = 0$ for all $j \geq 2$. For each $n \geq 0$ we have

$$\exp\left(-i\mu(n + c_1 n^{-1-2\alpha})\right) n^q = z^n \sum_{l=0}^\infty \frac{(-i\mu)^l}{l!}\,{c_1}^l\, n^{-(1+2\alpha)l}\, n^q,$$

where we set $z = \exp(-i\mu)$. Evidently, it holds that $\{z = \exp(-i\mu);\ \mu\in D_{r,\beta,\varepsilon}\}$ is contained in a compact set K in $D = \mathbb{C} - [1,\infty)$ for all r. Note that

$$\sum_{n\geq|k|^\beta} z^n\, n^{-(1+2\alpha)l}\, n^q = F(z, (1+2\alpha)l - q : m),$$

where $m = [|k|^\beta]$. Thus, by applying the previous lemma we have for all $\mu \in D_{r,\beta,\varepsilon}$

$$|\frac{(-i\mu)^l}{l!} c_1{}^l F(z, (1+2\alpha)l - q, m)| \leq C_{K,q}|z|^m(1+|z|)^q \frac{|k|^l}{l!} |c_1|^l m^{-(1+2\alpha)l+q}.$$

By using the facts that

$$|z^m| = |e^{-ikm+m\sigma}| = e^{m\sigma} \leq Ce^{r^{-\beta}\cdot r^\beta} = C$$

and

$$m^{-(1+2\alpha)l}|k|^l \leq |k|^{-(1+2\alpha)\beta+1} = C|k|^{-\gamma}, \quad \gamma = (1+2\alpha)\beta - 1 > 0.$$

we have

$$\begin{aligned} &|R_\beta(\mu) - F(z, -q : m)| \\ \leq &C_{K,q}(1+|z|)^q \, |k|^{q\beta} \sum_{l=1}^{\infty} \frac{1}{l!}(c_1 k^\gamma)^l \leq C_{K,q}\, c_1 |k|^{q\beta-\gamma}. \end{aligned}$$

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