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SOME APPLICATIONS OF LINEAR FORMS IN LOGARITHMS

by T. N. SHOREY

I shall mention some of the recent applications of linear forms in logarithms. All of them depend on the powerful results of BAKER [1] and van der POORTEN [3] on linear forms.

Denote by $P[r]$ the greatest prime factor of the integer r. Let a and b be non zero fixed integers. Van der POORTEN [2] proved that $P[ax^n + by^n]$ tends to infinity with n (> 1) uniformly in integers x, y with $(x, y) = 1$ and $\max(|x|, |y|) > 1$ (See also [4]). STEWART ([7], ch. 3) strengthened this to

$$P[ax^n + by^n] \gg \left(\frac{n}{\log n}\right)^{\frac{1}{2}} .$$

Here the constant implied by $\gg$ depends only on a and b.

Let $m \geqslant 2$ be a fixed integer. In [5], it was shown that $P[ax^n + by^m]$ tends to infinity with n uniformly in integers x, y with $|x| > 1$ and $(x, y) = 1$. An explicit lower bound for $P[ax^n + by^m]$ was given recently by the author [6], namely

$$P[ax^n + by^m] \gg ((\log n)(\log \log n))^{\frac{1}{2}} .$$

Here $n \geqslant e^e$ and the constant implied by $\gg$ depends only on a, b and m.

All the results mentioned above are effective. One can refer to [5] ; it contains a survey of earlier results in this direction.

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