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Some problems arising in connection with the theory of vector measures

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SOME PROBLEMS ARISING IN CONNECTION WITH THE THEORY OF VECTOR MEASJRES

by Joe DIESTEL

Introduction. - This lecture's aim is to discuss some problems arising in connection with the theory of vector measures. The problems discussed come in two varieties : The first class are among those discussed in the recent work "Vector measures" of J. Jerry UHL, Jr. and myself (henceforth referred to as [VM]) ; in discussing problems of this class I've restricted myself to those problems on which I'm aware that there's been some progress. In a sense then the discussion of this class is a progress report. The second class of problems concerns questions that have come to the foreground since the appearance of [VM]. I hope a few of these find interest (and even solution).

A word of admission. At the time of the lecture itself many of the present questions were mentioned without the accompaniment of relevant progress. Ignorance was the main culprit ; I've tried to minorize its presence herein and have been aided in this by discussions with Jean BOURGAIN, Bill JOHNSON and Gilles PISIER. I'd welcome any and all further information regarding examples and relevant news regarding questions contained here.

Incidentally, any unexplained terminology or notation can easily be gleaned from either [VM] or [LT].

1. The basic limit theorems for measures.

The following theorems are classical (or nearly so).

Vitali-Hahn-Saks THEOREM. - *If $\mathfrak{a}$ is a σ-complete Boolean algebra and $\mu_n : \mathfrak{a} \longrightarrow \underset{\sim}{R}$ is a sequence of bounded additive measures each absolutely continuous with respect to a fixed bounded additive measure $\mu : \mathfrak{a} \longrightarrow \underset{\sim}{R}$, then should $\lim_n \mu_n(a)$ exist, for each $a \in \mathfrak{a}$, the absolute continuity of the μ_n's with respect to μ is uniform, i. e., given $\epsilon > 0$, there is $\delta > 0$ such that whenever $a \in \mathfrak{a}$ and $|\mu(b)| \leq \delta$, for all $b \in \mathfrak{a}$, $b \leq a$ then $|\mu_n(b)| \leq \epsilon$, for all n and all $b \leq a$.*

Nikodym boundedness THEOREM. - *If $\mathfrak{a}$ is a σ-complete Boolean algebra and $\{\mu_\alpha ; \alpha \in A\}$ is a family of bounded additive real-valued measures defined on $\mathfrak{a}$ for which*

$$\sup_\alpha |\mu_\alpha(a)| < \infty , \text{ for each } a \in \mathfrak{a} ,$$

then

$$\sup_{\alpha, a} |\mu_\alpha(a)| < \infty .$$

A THEOREM of Grothendieck and Ando. - If Ω is the Stone space of a σ-complete Boolean algebra $\mathfrak{a}$, and X is a separable Banach space, then every bounded linear operator $T : C(\Omega) \longrightarrow X$ is weakly compact.

Each of these theorems finds many applications throughout measure theory and general functional analysis. What's more they are not completely dependent upon the $\mathfrak{a}$ σ-completeness of the Boolean algebra $\mathfrak{a}$ nor do any of them hold for every Boolean algebra. The complete classification of those Boolean algebras $\mathfrak{a}$ for which any of the above theorem's conclusions still hold is open. Several years ago, in a rather futile effort to understand the interrelationships of the above theorems, Barbara FAIRES, Bob HUFF, and myself formulated the natural properties of a Boolean algebra $\mathfrak{a}$ called the Vitali-Hahn-Saks property, the Nikodym boundedness property and the Grothendieck property that insure that the corresponding statements for $\mathfrak{a}$ be true. We were able to show that the Vitali-Hahn-Saks property implies both the Nikodym boundedness property and the Grothendieck property and that conversely the Nikodym boundedness property and the Grothendieck property taken together imply the Vitali-Hahn-Saks property. The arguments were, after the dust had settled a bit, purely formal. The questions of whether or not either the Nikodym boundedness property or the Grothendieck property above suffice to deduce the Vitali-Hahn-Saks theorem was open. After [VM] had been sent away to the publishers, Walter SCHACHEMAYER [22] gave an elegant example of a Boolean algebra having the Nikodym boundedness property but failing the Grothendieck property ; his example is the algebra of Jordan measurable subsets of $[0, 1]$. Recall that $A \subseteq [0, 1]$ is Jordan measurable if the boundary of A, ∂A, has Lebesgue measure zero. However, the following remains open and its solution is probably a function of knowing sufficiently many nontrivial Boolean algebras.

(Q1) Does the Grothendieck property for a Boolean algebra $\mathfrak{a}$ imply the Vitali-Hahn-Saks property for $\mathfrak{a}$ or equivalently, does the Grothendieck property for $\mathfrak{a}$ imply the Nikodym boundedness property ?

The easiest way to see what really is involved here is to give a convenient alternative version of each of the properties involved.

THEOREM. - Let $\mathfrak{a}$ be a Boolean algebra.

(a) $\mathfrak{a}$ has the Grothendieck property if whenever (μ_n) is a uniformly bounded sequence of bounded additive real valued measures defined on $\mathfrak{a}$ such that $\lim_n \mu_n(a) = 0$, for each $a \in \mathfrak{a}$, then the additivity of the μ_n's is uniform, i.e. given $\epsilon > 0$ and any sequence (a_m) of disjoint members of $\mathfrak{a}$, there is an m_0 such that, for any $j \geq k \geq m_0$ and any $b \leq a_k \vee a_{k+1} \vee \cdots \vee a_j$, we have

$$|\mu_n(b)| \leq \epsilon , \text{ for all } n ;$$

(b) $\mathfrak{A}$ has the Nikodym boundedness property if, and only if, given any sequence (μ_n) of bounded additive real valued measures defined on $\mathfrak{A}$ such that $\lim_n \mu_n(a) = 0$, for each $a \in \mathfrak{A}$, then the μ_n's are uniformly bounded on $\mathfrak{A}$.

Another not-yet-classical result, due to H. P. ROSENTHAL, concerning σ-complete Boolean algebras must be mentioned here.

THEOREM (ROSENTHAL). - If $\mathfrak{A}$ is a σ-complete Boolean algebra and Ω is the Stone space of $\mathfrak{A}$, then any nonweakly compact linear operator $T : C(\Omega) \longrightarrow X$ fixes a copy of ℓ_∞. [An operator fixes a copy of Z if there's an isomorph of Z in the operator's domain on which the operator is an isomorphism.]

This result is a consequence of a remarquable lemma (called aptly enough "Rosenthal's lemma") which is closely tied to σ-complete objects.

If one assumes just the Grothendieck property, then one still has the following theorem.

THEOREM (DIESTEL-SEIFERT). - If $\mathfrak{A}$ is a Boolean algebra with the Grothendieck property and Ω is the Stone space of $\mathfrak{A}$, then any nonweakly compact linear operator $T : C(\Omega) \longrightarrow X$ fixes a copy of $C[0, 1]$.

(Q2) If $\mathfrak{A}$ is a Boolean algebra with the Grothendieck property (or the Vitali-Hahn-Saks property) and Ω is its Stone space, need each nonweakly compact operator $T : C(\Omega) \longrightarrow X$ fix a copy of ℓ_∞ ?

Actually there's a more general form of (Q2) having to do with a class of Banach spaces having the so-called "Grothendieck property", i. e., spaces in whose dual weak* null sequences are weakly [illegible]. A reasonable objective for this class would be a response to the following property.

(Q3) Need every nonreflexive Grothendieck space have ℓ_∞ as a quotient ?

In trying to answer the above questions and relatives of them a constant bugaboo was the desire to obtain boundedness from countable additivity ; recall that a measure μ defined on the Boolean algebra $\mathfrak{A}$ with real values is countably additive if given any sequence (a_n) in $\mathfrak{A}$ which is decreasing to 0, then $\mu(a_n)$ tends to zero as well. Curiously a complete answer to the following is still unknown.

(Q4) For what Boolean algebras $\mathfrak{A}$ are countably additive real-valued measures on $\mathfrak{A}$ bounded ?

Actually, when this lecture was delivered there were no examples other than σ-complete Boolean algebras for which boundedness could be derived from countable additivity. Michel TALAGRAND [26] has given several relatively easy and elucidating examples regarding this problem. In particular, he has shown there are non-σ-complete Boolean algebras on which every countably additive measure is bounded and there are nearly σ-complete Boolean algebras that admit unbounded countably additive measures.

2. The Radon-Nikodym property.

At the time of the publication of [VM] and indeed at the time this lecture was actually given, the most basic problem in the theory of vector measures was UHL's question : If X is a separable Banach space with the Radon-Nikodym property (RNP), need X imbed in a separable dual ? Almost simultaneously (with the lecture's delivery unbeknownst) to the lecturer, J. BOURGAIN and F. DELBAEN were devilishly establishing the RNP for certain separable $\mathcal{L}_\infty$-spaces constructed by BOURGAIN for other devious purposes ; it follows from the work of LINDENSTRAUSS, PELCZYNSKI and ROSENTHAL ([13], [14]) that separable $\mathcal{L}_\infty$-spaces are not allowed in separable duals. Therefore, UHL's question was answered in the negative. However, a close relative of UHL's question remains open and, particularly in light of the BOURGAIN-DELBAEN result, should be enlightening as regards to the relationship of RNP and separable dual spaces. It comes, by way of PELCZYNSKI, the following property.

(Q1) Does every space with the RNP contain a separable dual subspace ?

Noteworthy, here is the fact that BOURGAIN's example is hereditarily ℓ_1 , i. e., each subspace contains an isomorph of ℓ_1 .

Frequently, isomorphic concepts (such as reflexivity, weak sequential completeness and superreflexivity) can be determined to hold or not hold for a given space by checking the subspaces of the space that have Schauder bases.

(Q2) Need X have the RNP if every subspace of X having Schauder basis has the RNP ?

Here all the progress has been due to Jean BOURGAIN. For general X 's, he has shown that, if every subspace Y having a finite dimensional Schauder decomposition has the RNP, then X has the RNP. If X is a dual space, then BOURGAIN has shown X has the RNP provided each subspace of X having Schauder basis has the RNP. Neither of these results is yet published and neither is easy. In tandem, they make a final response to (Q2) appear difficult.

Part of the difficulty in resolving questions relating ideas from basis theory and Radon-Nikodym theory might well lie in the fact that they are not natural bedfellows. Bob HUFF has suggested that perhaps the kind of relationship that ought to be investigated is in the line of finding a class $\mathcal{H}$ (my lettering !) of basic sequences such that X has the RNP if, and only if, each basic sequence in X belongs to $\mathcal{H}$. The existence of such a class is, of course, in doubt ; its identification (should it exist) would be welcomed news.

The Radon-Nikodym property can be localized in the sense that one can define what it means for a set to have the property. One way of doing this is to say that a set $B \subseteq X$ has the RNP if each subset of B is dentable. Once this is allowed it is natural to define an RNP operator as an operator $T : X \longrightarrow Y$ that takes bounded sets in X to sets with the RNP. Such operators arise throughout the Grothendieck Memoir, and, in light of the Davis-Figiel-Johnson-Pelczynski factorization scheme, one might hope for an affirmative response to the next property.

(Q3) If $T : X \longrightarrow Y$ is an RNP operator, need T factor through a Banach space with the RNP ?

In case T is itself the odjoint of some other operator then S. HEINRICH [6], O. REINOV [19] and C. STEGALL [24] have independently given a positive response, using in fact variations on the theme of DAVIS and al. Whether or not the DAVIS and al scheme can be used in general is open to serious doubts due to some pathological examples of sets with the RNP discovered by Jean BOURGAIN.

Much of the work regarding the RNP and sets with the RNP has gone to establishing that these sets want to be weakly compact, but just do not quite make it. Their geometry is quite similar ; in fact a weakly closed set with the RNP is a Baire space in its weak topology (this was noticed by Y. BENYAMINI and G. EDGAR); weakly closed bounded sets with the RNP have lots of strongly exposing functionals (BOURGAIN again with HUFF and P. MORRIS providing background music) and RNP operators properly composed with integral operators give nuclear operators. What kinds of Banach spaces are RNP generated ? To be precise we say that the Banach space X is RNP generated (RNPG for short) if there is a set $K \subset X$ with the RNP such that K's linear span is dense. Little (practically nothing) is known regarding RNPG spaces. To date only one "theorem" exists regarding these spaces and that, as with most of the progress reported herein, is due to J. BOURGAIN.

THEOREM. - If X is a separable Banach space which contains no copy of ℓ_1, then X^* is RNPG if, and only if, X^* has the RNP.

(Q4) Is ℓ_∞ RNPG ?

By Bourgain's cited theorem, there are non RNPG spaces; indeed the dual JT^* of James tree space does the trick. Of course, (Q4) is aimed at obtaining a more natural example. In fact, one can ask a perhaps more specific question.

(Q5) Is every operator $T : \ell_\infty \longrightarrow X$, X RNPG, weakly compact ?

By way of examples, we might mention that weakly compactly generated spaces, spaces with the Radon-Nikodym property, arbitrary ℓ_p-sums ($1 \leq p < \infty$) of RNPG spaces and countable c_0-sums of RNPG spaces are RNPG. In particular, for any $1 \leq p < \infty$ and any measure μ and any RNPG space X, $L_p(\mu, X)$ is RNPG. Incidentally that $L_1(\mu, X)$ is RNPG whenever μ is finite and X is RNPG follows from the argument of TURETT and UHL [27] suitably localised ; this was noticed in a conversation with (who else ?) Jean BOURGAIN.

Finally as a terminal question regarding the RNP, we ask the following property.

(Q6) If X has the RNP and B_X denotes the closed unit ball of X, need every nonexpansive map $\varphi : B_X \longrightarrow B_X$ have a fixed point.

Of interest here is the work of Les KARLOVITZ [9] who has shown the answer to be yes if X is either ℓ_1 or JT. Of possible relevance here is the remarkable recent result of Charles STEGALL [25] which states (in particular) the following theorem.

THEOREM. - If X has the RNP, K is a closed bounded convex subset of X and $\varphi : K \longrightarrow Y$ (any Banach space) is continuous and bounded on K, then, given $\epsilon > 0$ (no matter how small), there is a bounded linear operator $T_\epsilon : X \longrightarrow Y$, dimension $T_\epsilon X \leq 1$, such that $\|T_\epsilon\| \leq \epsilon$ and $T_\epsilon + \varphi$ attains its maximum norm on K.

3. The Lebesgue-Bochner spaces.

In the past five years, the Lebesgue-Bochner spaces have found interesting and important applications in Banach space theory, probability and even harmonic analysis. Shunned for a considerable time, these spaces have only recently come to be recognized as mischieviously elusive objects of study in their own right. So, though much is known about them, most of it is still surface-level and a number of questions remain. Typical of many of these questions is the following repeater from [VM].

(Q1) If X is weakly sequentially complete, is $L_p(\mu, X)$ of the same type ?

Here $1 \leq p < \infty$.

Before entering a discussion of some recent progress related to (Q1) a few remarks are in order regarding other questions of the same genre as (Q1).

It is quite often the case that good properties shared by a given $L_p(\mu)$, and X are inherited by $L_p(\mu, X)$. For instance, we cite : separability, weakly compactly generated, reflexivity, uniform convexity ([3], [15]), uniform smoothness, the Banach-Saks property [23], strict convexity [27], local uniform convexity [27], uniformly nonsquare [27], smoothness [12], Fréchet differentiability of the norm [12] Beck convexity [21], type [16], cotype [16], the Radon-Nikodym property [27], being an Asplund space [27], noncontainment of c_0 [11] and noncontainment of ℓ_1 ([2], [18]). (It is to be remarked that many of the above are highly nontrivial though none are unexpected. In fact, this is probably the reason that $L_p(\mu, X)$ spaces have not been studied too much as yet-highly nontrivial results are often met with a shrug and a comment "of course !") Not every good property shared by a given $L_p(\mu)$ and X goes over to $L_p(\mu, X)$.

Undoubtedly the most interesting reversals in form are those exhibited by David ALDOUS [1]. He has shown that $L_p([0, 1], X)$ has an unconditional basis only when $L_p[0, 1]$ and X are superreflexive (i. e., $1 < p < \infty$ and X has an equivalent uniformly convex norm). In particular, $L_2([0, 1], c_0)$ does not have an unconditional basis. Further, ALDOUS has noted that $L_2([0,1], c_0)$ fails to have the weak Banach-Saks property though $L_2[0,1]$ and c_0 enjoy this property. All in all it appears clear from ALDOUS's work and other results to be discussed below that $L_p([0,1], c_0)$ is a space worth knowing.

The difficulty with (Q1) lies largely with the lack of understanding of the weak topology of $L_p(\mu, X)$. The classical duality of $L_p(\mu)$ and $L_{p^*}(\mu)$, where p and p^* are conjugate indices, goes only so far ; in the vector case it holds (i. e., $L_p(\mu, X)^* = L_{p^*}(\mu, X^*)$) if, and only if, X^* has the RNP with respect to (Ω, Σ, μ). Though the dual of $L_p(\mu, X)$ can be described in general, as yet no one has really cracked open the meaning of these alternative descriptions in terms of weak convergence. This gives gusto to several other repeaters from [VM].

(Q2) Give (simultaneous) necessary and sufficient conditions that a bounded (and, if $p = 1$, uniformly integrable) subset $\mathcal{K}$ of $L_p(\mu, X)$ be relatively weakly compact.

(Q3) What are the criteria that a bounded (and, if $p = 1$, uniformly integrable) set $\mathcal{K}$ in $L_p(\mu, X)$ be conditionally weakly compact ?

[A set B is conditionally weakly compact if each sequence in B has a weakly Cauchy subsequence.]

Some progress has been made regarding the above problems especially (Q3). In fact thanks to the beautiful Rosenthal ℓ_1-theorem [20], the work of Jean BOURGAIN [2] and Gilles PISIER [18] gives the following improvement of theorem from [VM].

THEOREM. - Suppose $X \not\supseteq \ell_1$ and $\mathcal{K} \subseteq L_p(\mu, X)$.

If $p = 1$, then $\mathcal{K}$ is conditionally weakly compact if, and only if, $\mathcal{K}$ is bounded and uniformly integrable.

If $p > 1$, then $\mathcal{K}$ is conditionally weakly compact if, and only if, $\mathcal{K}$ is bounded.

In trying to resolve (Q2), one is led to the following condition concerning a set $\mathcal{K}$ in $L_p(\mu, X)$:

(G_r) Given $\epsilon > 0$, there is a weakly compact set K_ϵ in X such that $\mu(\{\omega : f(\omega) \notin K_\epsilon\}) < \epsilon$, for each $f \in \mathcal{K}$.

Similarly, one can obtain the condition (G_c) by replacing the weakly compact set K_ϵ by a conditionally weakly compact set C_ϵ throughout. In [4], it is essentially shown that if $\mathcal{K}$ is bounded (and, if $p = 1$, uniformly integrable) and satisfies (G_r), then $\mathcal{K}$ is relatively weakly compact. More recently, BOURGAIN has observed that an analogous statement holds for conditional weak compactness : If $\mathcal{K}$ is bounded (and, if $p = 1$, uniformly integrable) and satisfies (G_c), then $\mathcal{K}$ is conditionally weakly compact.

Neither (G_r) nor (G_c) is necessary for relative or conditional weak compactness in general. In case of (G_r), if one looks at $X = c_0$, then given any bounded sequence (x_n) without a weakly convergent subsequence, the sequence $(r_n \otimes x_n)$, where r_n is the n-th Rademacher function, tends to zero weakly in $L_p([0, 1), c_0)$ but fails (G_r). The situation with (G_c) is a bit touchier, but BOURGAIN has constructed a counterexample in [2].

What at first appears to be a curiosity is the observation (not as trivial as it first appears !) that, if X is an $L_1(\lambda)$-space, then (G_r) is necessary for relative weak compactness in $L_p(\mu, X)$. Of course, this also occurs if X is reflexive. What we're leading upto is the keen observation of P. HIERMEYER [7] : If (G_r) is necessary for weak compactness in $L_p(\mu, X)$, then $L_p(\mu, X)$ is weakly sequentially complete. The preceding remarks made the next question obvious.

(Q4) If X is weakly sequentially complete, need weakly compact sets in $L_p(\mu, X)$ satisfy (G_r) ?

This question has several natural test cases. Indeed, two of the most interesting examples of Banach spaces produced in the past five years are weakly sequentially complete. They are the dual of JH (James Hagler spaces) and the Bourgain example cited above. Each in fact has the Schur property.

Somewhat noteworthy here is that $L_1([0, 1), JH^*)$ is not sequentially complete in the weak topology generated by $L_\infty([0, 1), JH^{**})$. However, since in this case $L_\infty([0, 1), JH^{**})$ is a small part of $L_1([0, 1), JH^*)^*$, this observation (due to L. EGGHE) is far from solving (Q1).

Another unnerving indication of the present lack of understanding of weak compactness in the Lebesgue-Bochner spaces is reflected in the following property.

(Q5) When does a weakly Cauchy sequence in $L_p(\mu, c_0)$ converge weakly ?

[Keep in mind that the conditionally weakly compact sets in $L_p(\mu, c_0)$ are known.]

The answer to (Q5) may well lie in building good nontrivial operators on $L_p(\mu, X)$ spaces. In fact, the following was observed in conversations with HIERMEYER and Jurgen BATT : If X is a separable Banach space, then a necessary and sufficient condition that $\mathcal{K} \subseteq L_p(\mu, X)$ be relatively weakly compact is that, for each bounded linear operator $T : L_p(\mu, X) \longrightarrow c_0$, $T\mathcal{K}$ is relatively weakly compact. The difficulty in using this criterion at present lies in the fact that nontrivial operators on $L_p(\mu, X)$ are elusive. In fact, constructing subsets of $L_p(\mu, X)$ that don't come about coordinate-by-coordinate is a stumbling block. For instance, I don't know the answer to the following problem.

(Q6) If $\mathcal{K} \subseteq L_p(\mu, X)$ is weakly compact, is there a weakly compactly generated $X_0 \subseteq X$ such that $\mathcal{K} \subseteq L_p(\mu, X_0)$?

Enough about weak compactness.

There's another line of questions about the Lebesgue-Bochner spaces that seem intriguing. An indicative sampling from this line :

(Q7) For which Banach spaces X are $\ell_2(X)$ and $L_2([0, 1], X)$ isomorphic ?

If X is Hilbert space or Pelczynski's universal space [LT], then $\ell_2(X)$ and $L_2([0, 1], X)$ are isomorphic. They are not isomorphic for every X ; for instance if $X = c_0$ (or ℓ_1), then $\ell_2(X)$ has an unconditional basis, but $L_2([0, 1], X)$ hasn't thanks to the Aldous results cited above.

(Q8) For what separable X 's does $\ell_2(X)$ imbed isomorphically in $L_1([0, 1], X)$?

Sometimes this occurs. Again if X is a Hilbert space and (r_n) denotes the Rademacher sequence, then the map $R : \ell_2(X) \longrightarrow L_1([0, 1], X)$ defined by $R(x_n) = \sum_n r_n \otimes x_n$ defines an imbedding of $\ell_2(X)$ into $L_1([0, 1], X)$; on the other hand, it is a beautiful result of S. KWAPIEN [10] that only in case of a Hilbertian X does R perform such an imbedding. Another example is given by $X = C[0, 1]$; this time the embedding exists thanks to Banach and Mazur. Still another example is given by the Pelczynski universal space. On the other hand, $\ell_2(\ell_1)$ does not imbed in $L_1([0, 1], \ell_1)$.

4. Miscellaneous.

We close with a couple of questions that are open-ended. Their asking is motivated by the beautiful and effective Riesz representation theory for operators on spaces $C(K)$ of continuous real (or complex) valued functions defined on a compact Hausdorff space K . This theory is outlined for example in chapter VI of [VM]. The questions (and these are obvious variations on these themes) :

(Q1) Develop a representation theory for operators on the disk algebra A .

(Q2) Develop a representation theory for operators on the spaces $C^k(I^n)$ of k-times continuously differentiable scalar-valued functions defined on the n-cube $(n > 1)$.

In the case of (Q1) there is a body of results already known ; the monograph of PELCZYNSKI [17] is an excellent source for such information. In case of (Q2), I don't even know of a description of $C^1(I^2)^*$. In any case, the overriding consideration in the representation theories that are developed should be the effectivity of the theory. Here a quote of one J. Jerry UHL, Jr. is worth repeating : "A representation theorem is good for a publication ; an application of that theorem is good for mathematics".

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