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A Note on Natural Tensor Products Containing Complemented Copies of c_0 .

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ABSTRACT - Let H be a Fréchet lattice containing a positive sequence equivalent to the unit basis of c_0 . We prove that the natural tensor product $H \widehat{\otimes}_{\mu} X$ contains a complemented subspace isomorphic to c_0 for every infinite dimensional Banach space X , which generalizes a previous result of Cembranos and Freniche.

0. - Introduction.

Cembranos [4] shows in 1984 that $\mathcal{C}(K, X)$ contains a complemented subspace isomorphic to c_0 if K is an infinite compact Hausdorff space and X an infinite dimensional Banach space. The theorem was extended in 1986 by E. Saab and P. Saab [14] who proved that the injective tensor product $X \widehat{\otimes}_{\xi} Y$ of two infinite dimensional Banach spaces X and Y contains a complemented copy of c_0 if X or Y contains c_0 , using a proof inspired by the Cembranos's one. However both results have been obtained also in 1984, indeed in a little more general version, in a paper of Freniche [8]. On the other hand, Emmanuele [7] showed in 1988 that if $(\Omega, \mathcal{A}, \nu)$ is a not purely atomic measure space and X is a Banach space containing c_0 , the space $L^p(\mu, X)$, $1 \leq p < \infty$, of Bochner integrable X -valued functions, contains a complemented copy of c_0 .

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The spaces used by Freniche, Cembranos and Emmanuele are particular cases of the Levin natural tensor product $H \widehat{\otimes}_{\mu} X$ of a Banach lattice H and a Banach space H (see the original paper [11] and also [5]). *The main purpose of this paper is to extend the above results to this more general setting.* Our method will be useful also to deal with the same problem in the case of the Saphar tensor product $X \widehat{\otimes}_{d_{\infty}} Y$ (see [15]).

The notation and terminology is standard (see [10] for general theory of locally convex spaces and [1] for Banach lattices). If H is a Fréchet space (resp. a Fréchet lattice), $(\|\cdot\|_s)_{s=1}^{\infty}$ will be an increasing fundamental system of continuous seminorms (resp. continuous lattice seminorms) in H and $U_s = \{x \in H \mid \|x\|_s \leq 1\}$. The usual Schauder basis of ℓ^1 and c_0 will be denoted by $(e_n)_{n=1}^{\infty}$. A sequence (h_i) in the Fréchet space H is said to be weakly absolutely summable if, for every $n \in \mathbb{N}$, the inequality

$$\varepsilon_n((h_i)) = \sup_{h' \in U_n^0} \sum_{i=1}^{\infty} |\langle h_i, h' \rangle| < \infty$$

holds. We denote by $\ell^1(H)$ the set of all weakly absolutely summable sequences in H and by $\ell_0^1(H)$ the set of all $(h_i) \in \ell^1(H)$ such that, for every $n \in \mathbb{N}$, we have

$$\lim_{k \rightarrow \infty} \varepsilon_n((0, 0, \dots, 0, h_k, h_{k+1}, \dots)) = 0.$$

1. - Levin natural tensor products.

Let H be a Fréchet lattice and F a Fréchet space. The Levin (or natural) tensor product $H \widehat{\otimes}_{\mu} F$ is the completion of $H \otimes_{\mu} F$ where μ is the topology defined by the family of seminorms $\{\mu_s \mid s \in \mathbb{N}\}$ and

$$\mu_s(z) = \inf \left\{ \left\| \sum_{i=1}^n \|x_i\|_s h_i \right\|_s \mid z = \sum_{i=1}^n h_i \otimes x_i \in H \otimes X, h_i \geq 0 \right\}.$$

This tensor product was introduced by Levin [11] in the case of a Banach lattice H and a Banach space G , although with a different equivalent definition, (see [9] for a proof of the equivalence) and by Chaney [5] and Lotz [13] with the present definition. The μ -topology verifies that $\varepsilon \leq \mu \leq \pi$ (ε and π are the injective and projective topologies of Grothendieck respectively) but μ is not a tensor norm since it does not verify in general the metric mapping property.

If $H = \mathcal{C}(K)$ or $H = L^\infty(\nu)$ then μ coincides with ε . If H is an $L^1(\mu)$ space, $\mu = \pi$ (see [5]).

The importance of the Levin tensor product lies in the fact that many usual function spaces can be represented by means of such a product. For instance, let $(\Omega, \mathcal{A}, \mu)$ be a complete σ -finite measure space and $\mathfrak{M}(\Omega, \mathcal{A})$ be the set of classes, modulo equality almost everywhere, of measurable real functions on (Ω, \mathcal{A}) . A Köthe function space (or a Banach ideal function space) on $(\Omega, \mathcal{A}, \mu)$ will be a Banach space H which is an ideal in $\mathfrak{M}(\Omega, \mathcal{A})$. Given a Banach space X , we define $H(X)$ as the set of strongly measurable functions $f: \Omega \rightarrow X$ such that $\|f(\cdot)\| \in H$ endowed with the norm $\|f\| = \|\|f(\cdot)\|_X\|_H$. It can be proved (see [11], [2], [3]) that, when H has an order continuous norm, we have $H(X) = H \widehat{\otimes}_\mu X$. In particular, the familiar Lebesgue-Bochner spaces $L^p(\mu, X)$, $1 \leq p < \infty$, are isometric to $L_p(\Omega) \widehat{\otimes}_\mu X$.

We shall need the following result about the representation of the elements of $H \widehat{\otimes}_\mu X$.

LEMMA 1. *Let H be a Fréchet lattice and F a Fréchet space. Every $z \in H \widehat{\otimes}_\mu F$ has a representation $z = \sum_{i=1}^\infty h_i \otimes x_i$ with $h_i \geq 0$, where $(x_i)_{i=1}^\infty$ is a bounded sequence in F and $(h_i)_{i=1}^\infty \in \ell_0^1(H)$. Moreover, if $n \in \mathbb{N}$, $\mu_n(z) \leq \left(\sup_{i \in \mathbb{N}} \|x_i\|_n\right) \varepsilon_n((h_i))$ for every such representation of z and $\mu_n(z) = \inf \left\{ \left\| \sum_{i=1}^\infty \|x_i\|_n h_i \right\|_n \right\}$, where the infimum is taken over all representations of z as above.*

PROOF. If $z \in H \widehat{\otimes}_\mu F$, there exists a sequence $(z_n)_{n=0}^\infty \subset H \otimes F$ such that $\lim_{n \rightarrow \infty} \mu_k(z - z_n) = 0$ for each $k \in \mathbb{N}$. Choosing a subsequence if it is needed, we can suppose that $z = z_0 + \sum_{n=1}^\infty (z_n - z_{n-1})$ with

$$\mu_n(z_n - z_{n-1}) < 2^{-2n}, \quad n \in \mathbb{N}.$$

For every $n \in \mathbb{N}$ there exists a representation $z_n - z_{n-1} = \sum_{i=1}^{j(n)} h_{ni} \otimes x_{ni}$, such that $h_{ni} \geq 0$ and $\left\| \sum_{i=1}^{j(n)} \|x_{ni}\|_n h_{ni} \right\|_n < 2^{-2n}$. By the monotony of the lattice seminorms, z can be represented by a convergent series in $H \widehat{\otimes}_\mu F$, $z = \sum_{k=1}^\infty h_k \otimes x_k$ with every $h_k \geq 0$, $x_k \neq 0$ and $\sum_{k=1}^\infty \|x_k\|_n h_k$ converges in H for every $n \in \mathbb{N}$. We define a sequence $(\alpha_k)_{k=1}^\infty$ in \mathbb{K} as follows: given $k \in \mathbb{N}$, there are $n, i \in \mathbb{N}$, $1 \leq i \leq j(n)$, such that $x_k = x_{ni}$.

We put $\alpha_k = \|x_{ni}\|_n$ if $\|x_{ni}\|_n \neq 0$ and $\alpha_k = 2^{-2n} \left(\left\| \sum_{i \in J_n} h_{ni} \right\|_n \right)^{-1}$, where $J_n = \{i \in \mathbb{N} \mid 1 \leq i \leq j(n) \text{ and } \|x_{ni}\|_n = 0\}$, if $\left\| \sum_{i \in J_n} h_{ni} \right\|_n \neq 0$; if $\|x_{ni}\|_n = 0$ and $\left\| \sum_{i \in J_n} h_{ni} \right\|_n = 0$, we put $\alpha_k = 1$. Then $z = \sum_{k=1}^{\infty} (\alpha_k h_k) \otimes (x_k / \alpha_k)$, is a representation of the announced type. In fact, it is clear that $(x_k / \alpha_k)_{k=1}^{\infty}$ is a bounded sequence in E . On the other hand, H' is a lattice with its canonical order. Denoting $V_n = \{h \in H \mid \|h\|_n \leq 1\}$, we have that V_n^0 is a solid set in H' . Then, if $k_n = 1 + \sum_{i=1}^{n-1} j(i)$, we have

$$\begin{aligned} \sup_{h' \in V_n^0} \sum_{k=k_n}^{\infty} \alpha_k |\langle h_k, h' \rangle| &\leq 2 \left(\sup_{h' \in V_n^0, h' \geq 0} \sum_{k=k_n}^{\infty} \alpha_k \langle h_k, h' \rangle \right) \leq \\ &\leq 2 \left\| \sum_{k=k_n}^{\infty} \alpha_k h_k \right\|_n \leq 2 \sum_{k=n}^{\infty} \left\| \sum_{i=1}^{j(k)} \|x_{ki}\|_k h_{ki} \right\|_k < 4 \sum_{k=n}^{\infty} 2^{-2k} \leq 2^{-2n-3}, \end{aligned}$$

and thus, $(\alpha_k h_k)_{k=1}^{\infty}$ is in $\ell_0^1(H)$.

Finally, we fix $n \in \mathbb{N}$. It is clear that $\mu_n(z) \leq \left\| \sum_{k=1}^{\infty} \|x_k\|_n h_k \right\|_n$ for every representation of z of the above type. Given one of them, for every $\varepsilon > 0$, there is a k_0 such for every $k \geq k_0$, $\left\| \sum_{j=1}^{\infty} \|x_j\|_n h_j \right\|_n < \varepsilon/3$. Putting $w_k = \sum_{j=1}^{k-1} h_j \otimes x_j$, we have $\mu_n(z - w_k) < \varepsilon/3$. Now it is possible to take a new representation of w_k , say $w_k = \sum_{j=1}^t \bar{h}_j \otimes \bar{x}_j$ with $\bar{h}_j \geq 0$, such that

$$\begin{aligned} \left\| \sum_{j=1}^t \|\bar{x}_j\|_n \bar{h}_j \right\|_n &\leq \\ &\leq \mu_n(w_k) + \varepsilon/3 \leq \mu_n(z - w_k) + \mu_n(z) + \varepsilon/3 \leq \mu_n(z) + 2\varepsilon/3. \end{aligned}$$

Then $z = \sum_{j=1}^t \bar{h}_j \otimes \bar{x}_j + \sum_{j=k}^{\infty} h_j \otimes x_j$, so we get another representation $z = \sum_{i=1}^{\infty} h_i \otimes x_i$ of the above mentioned type, satisfying $\left\| \sum_{i=1}^{\infty} \|x_i\|_n h_i \right\|_n \leq \mu_n(z) + \varepsilon$. The remaining inequality follows easily. ■

2. Complemented copies of c_0 in Levin tensor products.

Next result generalizes the theorem of Cembranos and Freniche quoted in the introduction from the lattice point of view.

THEOREM 2. *Let H be a Fréchet lattice which contains a positive sequence equivalent to the unit basis of c_0 . Then for every infinite dimensional Banach space E , $H \widehat{\otimes}_\mu E$ contains a complemented subspace isomorphic to c_0 .*

PROOF. Let $(b_n)_{n=1}^\infty$ be a sequence in H^+ equivalent to the standard basis $(e_n)_{n=1}^\infty$ of c_0 by an isomorphism $\Phi: c_0 \rightarrow H$ such that $\Phi(e_n) = b_n$. As c_0 is isomorphic to a subspace of H , ℓ^1 is isomorphic to a quotient of H'_β . The sequence $(e_n)_{n=1}^\infty$ is also bounded in ℓ^1 and verifies that $\langle e_m, e_n \rangle = \delta_{nm}$. As the quotients of the DF spaces lift bounded sets [10, 12.4.8], there exist a bounded sequence $(b'_n)_{n=1}^\infty$ in H'_β such that $\langle b'_m, b_n \rangle = \delta_{nm}$. Since $(b'_n)_{n=1}^\infty$ is equicontinuous, there is $k \in \mathbb{N}$ such that, $(b'_n)_{n=1}^\infty \subset U_k^0$ holds. By the theorem of Josefson-Nissenzweig, there is a $\sigma(E', E)$ null sequence $(a'_n)_{n=1}^\infty$ in E' such that $\|a'_n\| = 1, \forall n \in \mathbb{N}$. An application of the principle of local reflexivity gives us a sequence $(a_n)_{n=1}^\infty$ in E such that $\langle a'_n, a_n \rangle = 1$ and $\|a_n\| \leq 2$, for every $n \in \mathbb{N}$.

We define $Q: c_0 \rightarrow H \widehat{\otimes}_\mu E$ such that $Q((\xi_i)) = \sum_{i=1}^\infty \xi_i b_i \otimes a_i$, for every $(\xi_i) \in c_0$. The map Q is well defined: $(b_i)_{i=1}^\infty$ being equivalent to $(e_i)_{i=1}^\infty$, we have that $(\xi_i b_i) \in \ell^1_0(H)$. Moreover, Q is linear and continuous: in fact, given $s \in \mathbb{N}$

$$\begin{aligned} \mu_s(Q((\xi_i))) &\leq \sup_{x' \in U_s^0} \sum_{i=1}^\infty |\langle \xi_i \Phi(e_i), x' \rangle| \|a_i\| \leq \\ &\leq 2 \sup_{i \in \mathbb{N}} |\xi_i| \sup_{x' \in U_s^0} \sum_{i=1}^\infty |\langle e_i, \Phi'(x') \rangle| \leq 2 \sup_{i \in \mathbb{N}} |\xi_i| \sup_{x' \in U_s^0} \|\Phi'(x')\|_{\ell^1}. \end{aligned}$$

Then we define $T: H \widehat{\otimes}_\mu E \rightarrow c_0$ in the following way. By Lemma 1, every $H \widehat{\otimes}_\mu E$ can be represented as $z = \sum_{i=1}^\infty u_i \otimes v_i$ where, for each $n \in \mathbb{N}$

$$(1) \quad \lim_{m \rightarrow \infty} \sup_{h' \in U_n^0} \sum_{i=m}^\infty |\langle h', u_i \rangle| = 0.$$

and $\sup_{i \in \mathbb{N}} \|v_i\| \leq M$ for some $M > 0$. Then we put

$$T(z) = (\langle z, b'_n \otimes a'_n \rangle)_{n=1}^\infty = \left(\sum_{i=1}^\infty \langle u_i, b'_n \rangle \langle v_i, a'_n \rangle \right)_{n=1}^\infty.$$

The map T is well defined: it is easy to see that $b'_n \otimes a'_n \in \left(H \widehat{\otimes}_\mu E \right)'$; on the other hand, since $(u_i)_{i=1}^\infty$ is bounded in H , we have $M_k := \sup_{i \in \mathbb{N}} \|u_i\|_k < \infty$; as $b'_n \in U_k^0$ for all $n \in \mathbb{N}$, from (1), given $\varepsilon > 0$, there is $h \in \mathbb{N}$ such that, for all $n \in \mathbb{N}$

$$\left| \sum_{i=1}^\infty \langle u_i, b'_n \rangle \langle v_i, a'_n \rangle \right| \leq \sum_{i=1}^h |\langle u_i, b'_n \rangle \langle v_i, a'_n \rangle| + \\ + M \sum_{i=h+1}^\infty |\langle u_i, b'_n \rangle| \leq M_k \sum_{i=1}^h |\langle v_i, a'_n \rangle| + \frac{\varepsilon}{2}$$

and since $(a'_n)_{n=1}^\infty$ is $\sigma(E', E)$ null, there is n_0 such that for every $n \geq n_0$

$$\left| \sum_{i=1}^\infty \langle u_i, b'_n \rangle \langle v_i, a'_n \rangle \right| \leq M_k \sum_{i=1}^h |\langle v_i, a'_n \rangle| + \frac{\varepsilon}{2} \leq \varepsilon,$$

and hence $T(z) \in c_0$.

Moreover T is continuous. In fact, by Lemma 1, for every $z \in H \widehat{\otimes}_\mu E$, given $\varepsilon > 0$, there exist a representation $z = \sum_{i=1}^\infty h_i \otimes x_i$ with $h_i \geq 0$, such that

$$\mu_k(z) + \frac{\varepsilon}{2} > \sup_{h' \in U_k^0} \left| \sum_{i=1}^\infty \|x_i\| \langle h', h_i \rangle \right|.$$

As U_k^0 is a solid set we have $b'_n = (b'_n)^+ - (b'_n)^-$ with $(b'_n)^+, (b'_n)^- \in U_k^0$ for each $n \in \mathbb{N}$. Then we obtain

$$\|T(z)\| = \sup_{n \in \mathbb{N}} \left| \sum_{i=1}^\infty \langle h_i, b'_n \rangle \langle x_i, a'_n \rangle \right| \leq \\ \leq 2 \sup_{n \in \mathbb{N}} \sup_{h' \in U_k^0} \left| \sum_{i=1}^\infty \langle h_i, h' \rangle \|x_i\| \right| \leq 2\mu_k(z) + \varepsilon,$$

and hence $\|T(z)\| \leq 2\mu_k(z)$.

It is easy to see that $TQ((\xi_i)) = (\xi_i)$ for every $(\xi_i) \in c_0$, and then $Q(c_0) = QT\left(H \widehat{\otimes}_\mu E\right)$ is a complemented of $H \widehat{\otimes}_\mu E$ isomorphic to c_0 . ■

REMARK 3. If K is an infinite compact Hausdorff space, $\mathcal{C}(K)$ contains a subspace isomorphic to c_0 by a positive isometry, and this fact is implicit in the Cembranos's proof and more explicit in the Freniche's one. To see that, take a sequence $(G_n)_{n=1}^\infty$ of open non empty pairwise disjoint sets in K , and a sequence $(t_n)_{n=1}^\infty$, $t_n \in G_n$, $\forall n \in \mathbb{N}$. The Urysohn lemma gives us a sequence $(b_n)_{n=1}^\infty \subset \mathcal{C}(K)^+$ such that $b_n: K \rightarrow [0, 1]$ with $b_n(t_n) = 1$ and $b_n(t) = 0$ if $t \in K \setminus G_n$. Let F be the linear span of $\{b_n | n \in \mathbb{N}\}$. The map $Q: F \rightarrow c_0$ such that $Q(b_n) = e_n$ is an isometry since if $f = \sum_{i=1}^s \lambda_i b_{n_i} \in C(K)$, then $\|f\| = \sup_{i \in \mathbb{N}} |\lambda_i| = \|Q(f)\|$, and it is clearly positive. Moreover, if we take $b'_n = \delta_{t_n}$ (the Dirac measure at t_n), choosing (a_n) as in Theorem 2, we get an easy representation of the projection P from $\mathcal{C}(K, X) = \mathcal{C}(K) \widehat{\otimes}_\varepsilon X$ onto the closure of the linear span of $\{b_n(\cdot) a_n | n \in \mathbb{N}\}$.

EXAMPLE 4. If $M(t)$ is an Orlicz function which does not satisfies the Δ_2 condition at 0, the Orlicz sequence space h_M has a sublattice order isomorphic to c_0 (see [3] and [12]) and ℓ^∞ is not a subspace of h_M . By Theorem 2, for every infinite dimensional Banach space X , c_0 is a complemented subspace of $h_M \widehat{\otimes}_\mu X$.

COROLLARY 5. Let $(\Omega, \mathcal{A}, \mu)$ be a complete σ -finite measure space such that $L^\infty(\mu)$ is infinite dimensional. Let X be a Banach space which does not contain c_0 . Then $L^\infty(\mu) \widehat{\otimes}_\mu X$ is not complemented in $L^\infty(\mu, X)$.

PROOF. By Theorem 2, c_0 is complemented in $L^\infty(\mu) \widehat{\otimes}_\mu X$. By [6], if c_0 were complemented in $L^\infty(\mu, X)$, c_0 would be a subspace of X . Then the result follows. ■

We note also, for the sake of completeness, the alternative result:

THEOREM 6. If H is an order continuous norm Köthe function space, $H(X) = H \widehat{\otimes}_\mu X$ contains a complemented copy of c_0 if X has a copy of c_0 .

PROOF. It is essentially the same one of Emmanuele for $L^p(\mu, X)$ in [7]. ■

Our method can be easily applied also in the context of the d_∞ -tensor product of Saphar (see [15] for definitions and details):

THEOREM 7. *Let F be a Fréchet space containing a subspace isomorphic to c_0 . Then for every infinite dimensional Banach space X , $F \widehat{\otimes}_{d_\infty} X$ contains a complemented subspace isomorphic to c_0 .*

PROOF. It is similar to the proof of Theorem 2, since the topology of $F \widehat{\otimes}_{d_\infty} X$ is determined by the seminorms given, for each $z \in F \widehat{\otimes}_{d_\infty} X$, by

$$d_{\infty s}(z) = \inf \left\{ \left(\sup_{i \in \mathbb{N}} \|x_i\| \right) \varepsilon_s((y_i)) \right\} \quad s \in \mathbb{N},$$

where the infimum is taken over all representations $z = \sum_{i=1}^{\infty} y_i \otimes x_i$, such that $\sup_{i \in \mathbb{N}} \|x_i\| < \infty$ and $(y_i) \in \ell_0^1(F)$. ■

A Grothendieck space is a Banach space X such that $\sigma(X', X)$ and $\sigma(X', X'')$ null sequences in X' are the same. In consequence a Grothendieck space can not contain a complemented copy of c_0 . Then we have:

COROLLARY 8. *Let H, G be Banach lattices such that H contains a positive sequence equivalent to the standard basis of c_0 and G is a Köthe function space with order continuous norm. Let X and Y be infinite dimensional Banach spaces such that Y contains a subspace isomorphic to c_0 . Then neither $G(Y)$, $H \widehat{\otimes}_{\mu} X$ nor $Y \widehat{\otimes}_{d_\infty} X$ are Grothendieck spaces.*

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