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C. MENINI

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ERRATA - CORRIGE

**« Linearly Compact Rings
and Strongly Quasi-Injective Modules ».**

C. MENINI (*)

Introduction.

I) Statement (d) of THE MAIN THEOREM of [M] must be changed in the following form:

« (d) Let ${}_R U$ be the minimal cogenerator of $\mathcal{C}_{\mathcal{F}}$, $T = \text{End}({}_R U)$. Then ${}_R U_T$ is faithfully balanced and the module U_T is s.q.i. »

Consequently the last assertion of the same theorem has to be erased.

The proof of (a) \Rightarrow (d) of this theorem is now the following:

« Let us remark, first of all, that in view of Lemma 5, R is linearly compact in its U -topology. Thus, since ${}_R U$ is a selfcogenerator, by Corollary 7.4 [2], $R = \text{End}(U_T)$ and hence ${}_R U_T$ is faithfully balanced. Moreover, since we already proved that (a) \Rightarrow (b), U_T is q.i.. Thus it is enough to prove that R separates points and submodules of U_T . Let $L \leq U_T$ and let $x \in U_T$. Assume that $\text{Ann}_R(x) \geq \text{Ann}_R(L)$. Since R is linearly compact in the U -topology, Rx is linearly compact discrete. Thus, since $Rx \leq_R U$, Rx is finitely embedded. Hence, by Lemma 8, there is a finite subset $\{y_1, \dots, y_n\} \subseteq L$ such that $\text{Ann}_R(x) \geq \bigcap_{j=1}^n \text{Ann}_R(y_j)$. Let $S_1, \dots, S_m \in \mathcal{S}_{\mathcal{F}}$ such that the R -submodule of ${}_R U$ spanned by x and y_j 's, $j = 1, \dots, n$, is contained in ${}_R M = \bigoplus_{i=1}^m t_{\mathcal{F}}(E(S_i)) \leq_R U$. Since

(*) Indirizzo dell'A.: Istituto di Matematica dell'Università di Ferrara, Via Machiavelli 35, 44100 Ferrara (Italy).

${}_R M$ is quasi-injective, by the proof of Lemma 7, there exists a morphism $f: {}_R M^n \rightarrow {}_R M$ such that $x = ((y_1, \dots, y_n))f$. Since ${}_R M$ is a direct summand of ${}_R U$, f extends to a morphism $g: {}_R U^n \rightarrow {}_R U$. Thus there exist $t_1, \dots, t_n \in T$ such that $x = \sum_{i=1}^n t_i y_i$ and hence $x \in L$. »

II) The last assertions of Theorem 10, concerning the explicit form of ${}_R K$ and K_A are false, while equivalence of statements (a), (b) and (c) is true and is also true that if (a), (b), (c) hold then A is linearly compact in its K -topology.

The proof of (c) \Rightarrow (b) runs as follows.

« Let $x \in K$. Rx is linearly compact discrete and hence $\text{Soc}(Rx)$ is a direct sum of a finite number of left simple R -modules S_1, \dots, S_n . By hypothesis, $\text{Soc}({}_R K)$ is essential in ${}_R K$. Hence $\text{Soc}(Rx)$ is essential in Rx . It follows that

$$(1) \quad Rx \leq \bigoplus_{i=1}^n t_{\mathcal{F}}(E(S_i)).$$

Let us prove that R separates points and submodules of K_A . Let $L < K_A$ and let $x \in K$. Assume that $\text{Ann}_R(x) \geq \text{Ann}_R(L)$. Note that, by (1), $R/\text{Ann}_R(x) \cong Rx$ is finitely embedded. Hence, by Lemma 8, there is a finite subset $F \subseteq L$ such that $\text{Ann}_R(x) \geq \bigcap_{l \in F} \text{Ann}_R(l)$. Thus, by Lemma 7, x belongs to the submodule of K_A spanned by F and hence $x \in L$.

III) Finally, the first part of the proof of Theorem 14 is modified as follows.

« PROOF. Let \mathcal{F} be the filter of open left ideals of τ and let U_R be the minimal injective cogenerator of $\mathcal{C}_{\mathcal{F}}: {}_R U = t_{\mathcal{F}}\left(E\left(\bigoplus_{s \in \mathcal{S}_{\mathcal{F}}} S\right)\right)$.

Set $A = \text{End}({}_R U)$. By theorem 10 and Lemma 6, ${}_R U_A$ is faithfully balanced and both the modules ${}_R U$ and U_A are s.q.i. »

At this point the remaining part of the proof works.

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REFERENCES

[M] C. MENINI, *Linearly compact rings and strongly quasi-injective modules*.