RENDICONTI
del
SEMINARIO MATEMATICO
della
UNIVERSITÀ DI PADOVA

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Rendiconti del Seminario Matematico della Università di Padova, tome 63 (1980), p. 51-60
<http://www.numdam.org/item?id=RSMUP_1980__63__51_0>

REND. SEM. MAT. UNIV. PADOVA, Vol. 63 (1980)

Notes on the Topologies and Uniformities of Hyperspaces.

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0. – Introduction.

Throughout the present paper X will denote a completely regular Hausdorff space, which we shall call simply « space »; μX will denote a uniform space and μ its uniformity: in the latter case X will indicate the (completely regular Hausdorff) space equipped with the topology induced by μ.

For axioms, terminology and notations about uniform spaces we refer to [I]. Furthermore we shall indicate with $\boldsymbol{H}(\mu X)$ the uniform space whose points are the closed sets of X and whose entourages are the sets of the type $\widetilde{\mathcal{U}} = \{(A, B)\colon St(A, \mathcal{U}) \supseteq B,\ St(B, \mathcal{U}) \supseteq A\}$ where $\mathcal{U}$ is a uniform covering belonging to μ: if $(A, B) \in \widetilde{\mathcal{U}}$ we say that A and B are near of order $\mathcal{U}$; dealing with topological properties we shall go on writing simply $\boldsymbol{H}(\mu X)$ meaning the induced topology, which we shall call τ_μ. Finally $\boldsymbol{K}(\mu X)$ denotes the subspace of $\boldsymbol{H}(\mu X)$ whose points are the compact sets of X.

It is well known that if μ and ν are admissible uniformities on X, τ_μ and τ_ν may differ, while their restrictions to $\boldsymbol{K}(\mu X)$ are identical. For this reason we shall denote by $\boldsymbol{K}(X)$ the topological space associated to $\boldsymbol{K}(\mu X)$. In this connection we may remark that $\boldsymbol{K}$ and $\boldsymbol{H}$ have functorial properties: given a function $f\colon X \to Y$ and a closed subset A of X, define $f^{\#}(A) = \mathrm{Cl}_Y f(A)$; if f is a continuous map, $f^{\#}\colon \boldsymbol{K}(X) \to \boldsymbol{K}(Y)$ turns out to be a continuous map; if $f\colon \mu X \to \nu Y$ is uniformly continuous, $f^{\#}\colon \boldsymbol{H}(\mu X) \to \boldsymbol{H}(\nu Y)$ is uniformly continuous, too.

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We devote the first section of this work to the stud y of completeness of hyperspaces and related questions. In the second one we deal with some topological properties: after observing that countably compactness is not preserved by $\boldsymbol{K}$ we give in theorem 2.1 a necessary and sufficient condition in order that $\boldsymbol{K}(X)$ is countably compact; moreover a proposition of E. Michael [Mi] on local compactness of $\boldsymbol{K}(X)$ is reviewed and some remarks on discrete uniformity and topology are made. In the third section we provide two counterexamples of an assertion formulated by J. R. Isbell in [I] and prove the related question in a particular case.

We wish to thank A. Le Donne and U. Marconi for discussing with us some problems connected with the matter of this paper.

1. – Following [I], we remind that a filter $\mathcal{F}$ in the space X is said to be semi-Cauchy if for every $\mathcal{U}$ belonging to μ there exists a finite number of elements of $\mathcal{U}$, say $U_1, U_2, \ldots, U_n$ such that $\bigcup_{i=1}^{n} U_i \in \mathcal{F}$. In [I] II.49 it is shown that if a semi-Cauchy filter is hyperconvergent (i.e. the closed sets of the filter form a convergent net in $\boldsymbol{H}(\mu X)$) then it is hyperconvergent to a precompact set.

If F_α is a net in $\boldsymbol{H}(\mu X)$ we put $\mathcal{F} = \left\{F \subseteq X : F \supseteq \bigcup_{\alpha \in A} F_\alpha \text{ for a residual set of indexes } A\right\}$ and we say that $\mathcal{F}$ is the filter associated to the net F_α.

1.1 Lemma. If F_α is a Cauchy net in $\boldsymbol{K}(\mu X)$, the associated filter is semi-Cauchy.

Proof. For every $\mathcal{U} \in \mu$ let $\mathcal{V}$ be a star-refinement of the covering $\mathcal{U}$; there exists an index α such that for every $\beta > \alpha$ F_α and F_β are near of order $\mathcal{V}$ and then we have:

$$St(F_\alpha, \mathcal{V}) \subseteq St(V_1 \cup \ldots \cup V_n, \mathcal{V}) = \bigcup_{i=1}^{n} St(V_i, \mathcal{V}) \subseteq U_1 \cup \ldots \cup U_n$$

where $V_1, \ldots, V_n$ are elements of $\mathcal{V}$ which cover the compact F_α and $U_i \in \mathcal{U}$ contains $St(V_i, \mathcal{V})$: hence $\bigcup_{\beta > \alpha} F_\beta \subseteq U_1 \cup \ldots \cup U_n$. ■

This lemma easily provides a result due to K. Morita [Mo] which we have obtained in an indipendent way:

1.2 Theorem. μX is complete if and only if $\boldsymbol{K}(\mu X)$ is complete.

PROOF. The sufficiency is trivial since μX is a closed subspace of $\boldsymbol{K}(\mu X)$. The necessity is a consequence of lemma 1.1 and [I] II.49: in fact if we have a Cauchy net F_α in $\boldsymbol{K}(\mu X)$, its associated filter is semi-Cauchy and, since μX is complete, the filter is hyperconvergent to a compact set A. It is easy to verify that A is limit of the net F_α. ■

As a corollary one can easily show that the functor $\boldsymbol{K}$ commutes with completion, since $\boldsymbol{K}$ preserves (uniform) subspaces and density. It is known that this fact cannot be extended to the functor $\boldsymbol{H}$, since $\boldsymbol{H}$ does preserve uniform subspaces and density but it does not preserve completion. In any case, if we indicate by $(\mu X)\hat{}$ the completion of the uniform space μX, we can state the following proposition, whose proof is omitted:

1.3 PROPOSITION. $\boldsymbol{H}((\mu X)\hat{}\,) = (\boldsymbol{H}(\mu X))\hat{}$ if and only if $\boldsymbol{H}((\mu X)\hat{}\,)$ is complete. Hence the equality holds for metric and precompact uniform spaces. ■

REMARK. If X is a non compact space and T a compactification of X, then $\boldsymbol{K}(T)$ is a compactification of $\boldsymbol{K}(X)$ and the power of $\boldsymbol{K}(T)\setminus\boldsymbol{K}(X)$ is infinite: this enables us to say that $\boldsymbol{K}(X)$ always admits more than one precompact uniformity.

We collect now some observations about the subspace of $\boldsymbol{H}(\mu X)$ made of the precompact closed subsets of X: we denote it by $\boldsymbol{P}(\mu X)$; $\boldsymbol{F}(X)$ will indicate the subspace of finite subsets of X.

1.4 PROPOSITION.

i) $\mathrm{Cl}_{\boldsymbol{H}(\mu X)}(\boldsymbol{K}(X)) = \boldsymbol{P}(\mu X)$;

ii) $\boldsymbol{F}(X)$ is dense in $\boldsymbol{P}(\mu X)$;

iii) $\boldsymbol{F}(X)$ is dense in $\boldsymbol{H}(\mu X)$ if and only if μ is precompact.

PROOF. i): by lemma 1.1 and the considerations before it, every cluster point belongs to $\boldsymbol{P}(\mu X)$; on the other hand every precompact is limit of a net of finite subsets: let A be precompact; for every covering $\mathcal{U}$ belonging to μ, we can take a finite subset $A_{\mathcal{U}}$ of A such that $St(A_{\mathcal{U}}, \mathcal{U}) \supseteq A$. Clearly the net $A_{\mathcal{U}}$ converges to A.

ii) and iii) are proved by the argument above. ■

We remark that if μX is not a precompact space, then any precompact uniformity which induces the topology τ_μ on $\boldsymbol{H}(\mu X)$ cannot be of the form $\boldsymbol{H}(\nu X)$ for any ν: in fact ν needs to be precompact, but in this case $\boldsymbol{F}(X)$ would be dense in $\boldsymbol{H}(\nu X)$.

1.5 PROPOSITION. Let μX be a metric space. The following are equivalent:

i) μX is complete;

ii) $\boldsymbol{K}(X)$ is closed in $\boldsymbol{H}(\mu X)$.

PROOF. i) $\Rightarrow$ ii): by theorem 1.2 $\boldsymbol{K}(\mu X)$ is complete, hence closed. ii) $\Rightarrow$ i): every Cauchy sequence $(x_n)_{n\in\mathbb{N}}$ is precompact: then it is compact and convergent. ∎

Clearly the proposition holds for any space μX in which every Cauchy net is precompact, and examples in which $\boldsymbol{K}(\mu X)$ is closed in $\boldsymbol{H}(\mu X)$ and not complete can be given.

EXAMPLE. Let X be a totally ordered set whose power is uncountable. We equip X with the uniformity μ given as follows: for any $x \in X$, put $\mathcal{U}_x = \{\{y\}, \{z\colon z > x\};\ y \leqslant x\}$: the coverings $\mathcal{U}_x$ form a basis for a uniformity μ. We have $\boldsymbol{P}(\mu X) = \boldsymbol{K}(X) = \boldsymbol{F}(X)$ and clearly the space is not complete.

2. – In this section we are going to deal with some topological properties of hyperspaces.

It is known that the functor $\boldsymbol{K}$ preserves several properties of the space X such as metrizability, connectedness, zero-dimensionality, first and second countability. Our purpose is to investigate if $\boldsymbol{K}$ preserves countably compactness or pseudocompactness: both questions have negative answers but we have the following result:

2.1 THEOREM. The following are equivalent:

i) $\boldsymbol{K}(X)$ is countably compact;

ii) $\boldsymbol{K}(X)$ is strongly countably compact (i.e. the closure of every countable set is compact; see [K2]);

iii) in $\boldsymbol{K}(X)$ every countable union of compact subspaces is contained in a compact;

iv) in X every countable union of compact subspaces is contained in a compact.

PROOF. iii) $\Rightarrow$ ii) $\Rightarrow$ i): trivial.

i) $\Rightarrow$ iv): let $(K_n)_{n\in\mathbf{N}}$ be a sequence of compact subspaces of X which we can suppose to be increasing, and take $A = \bigcup_{n\in\mathbf{N}} K_n$. By the assumption there exists $B \in \boldsymbol{K}(X)$ which is an accumulation point for $\{K_n : n \in \mathbf{N}\}$, hence given $\mathcal{U} \in \mu$ there is n_0 such that for any $n \geqslant n_0$ we have: $St(K_n, \mathcal{U}) \supseteq B$, $St(B, \mathcal{U}) \supseteq K_n$ and so $St(B, \mathcal{U}^*) \supseteq \supseteq \mathrm{Cl}_X A$, $St(\mathrm{Cl}_X A, \mathcal{U}) \supseteq B$, that is to say $\mathrm{Cl}_X A$ and B are near of order $\mathcal{U}^*$ for every $\mathcal{U}$, therefore Cl_X A is B.

iv) $\Rightarrow$ iii): let $(\mathcal{C}_n)_{n\in\mathbf{N}}$ be a sequence of compact subspaces of $\boldsymbol{K}(X)$ and $\mathcal{C} = \bigcup_{n\in\mathbf{N}} \mathcal{C}_n$. A routine argument (see [Mi]) shows that $C_n = \bigcup_{A\in\mathcal{C}^u} A$ is a compact subspace of X and by the hypothesis $\bigcup_{n\in\mathbf{N}} C_n$ is contained in a compact C; finally $\mathcal{C} \subseteq \boldsymbol{K}(C)$. ∎

It has been proved now that if X is countably compact, $\boldsymbol{K}(X)$ needs not to be countably compact; moreover, following the example given in [Kl] p. 765, one can exhibit an example of a countable compact space X such that $\boldsymbol{K}(X)$ is not even pseudocompact. This fact has an interesting consequence for fine uniform spaces: if α denotes the fine uniformity and X the above space, $\boldsymbol{K}(\alpha X) \neq \alpha\boldsymbol{K}(X)$ since $\boldsymbol{K}(\alpha X)$ is precompact and $\boldsymbol{K}(X)$ has a continuous unbounded real-valued function.

We go on with the following lemma whose (routine) proof we omit:

2.2 LEMMA. Let K be a compact of the space X. If U is a neighbourhood of K, the covering $\{U, X\setminus K\}$ is uniform for every admissible uniformity. ∎

2.3 THEOREM. For a space X the following are equivalent:

i) X is locally compact;

ii) $\boldsymbol{K}(X)$ is open in $\boldsymbol{H}(\mu X)$ for every admissible uniformity μ.

PROOF. i) $\Rightarrow$ ii): let K belong to $\boldsymbol{K}(X)$; since X is locally compact, there is a compact neighbourhood U of K in X. In view of Lemma 2.2 $\{U, X\setminus K\} = \mathcal{U} \in \mu$ and K' is near of order $\mathcal{U}$ to K if and only if $K' \subseteq U$ hence $\boldsymbol{K}(U)$ is a (compact) neighbourhood of K in $\boldsymbol{H}(\mu X)$ which is contained in $\boldsymbol{K}(X)$.

ii) $\Rightarrow$ i): suppose X not locally compact: then there exists $x \in X$ such that no neighbourhood of x is compact. Given $\mathcal{U} \in \mu$, take $\mathcal{V}$ such that $\mathcal{V}^{**} < \mathcal{U}$: then $x \in \mathrm{Cl}_X\, St(x, \mathcal{V}) \subseteq St(x, \mathcal{U})$ and so the neighbourhood of $\{x\}$ made of the closed sets that are near of order $\mathcal{U}$ to $\{x\}$ contains $\mathrm{Cl}_X\, St(x, \mathcal{V})$ which is not compact. ∎

REMARK. In Proposition 4.4.2 [Mi] E. Michael shows that if X is locally compact, then $\boldsymbol{K}(X)$ is open in the space of closed sets equipped with the finite topology: since this last condition is equivalent to ii) of the previous theorem, then Proposition 4.4.2 can be reversed.

Lemma 2.2 and an easy argument are used to prove:

2.4 PROPOSITION. The following are equivalent for the space X:

i) X is discrete;

ii) $\{\{x\}: x \in X\}$ is clopen in $\boldsymbol{K}(X)$;

iii) $\{\{x\}: x \in X\}$ is clopen in $\boldsymbol{H}(\mu X)$ for every admissible uniformity μ of X.

PROOF. i) $\Rightarrow$ iii): by Lemma 2.2 $\mathcal{U}_x = \{x, X \setminus \{x\}\}$ is uniform for every admissible μ, hence the singleton $\{x\}$ is an isolated point in $\boldsymbol{H}(\mu X)$ and $\{\{x\}: x \in X\}$ is clopen.

iii) $\Rightarrow$ ii): trivial.

ii) $\Rightarrow$ i): if x were not an isolated point, then for every uniform covering $\mathcal{U}$ belonging to an admissible uniformity of X, there would be an element $U \in \mathcal{U}$ such that $U \ni x$, $U \setminus \{x\} \neq \emptyset$; let y belong to $U \setminus \{x\}$: the set $\{x, y\}$ is near of order $\mathcal{U}$ to $\{x\}$. ∎

To conclude this section, we give the following proposition which characterizes the isolated points of $\boldsymbol{H}(\mu X)$.

2.5 PROPOSITION. $A \in \boldsymbol{H}(\mu X)$ is an isolated point if and only if the covering $\{X \setminus A, \{a\}: a \in A\}$ belongs to μ.

PROOF. If A is an isolated point there exists an open covering $\mathcal{U} \in \mu$ such that if B is near of order $\mathcal{U}$ to A then B is equal to A. First $St(A, \mathcal{U}) = A$; furthermore if $U \subseteq A$, $U \in \mathcal{U}$ and U is not a singleton, for $x \in U$ we have that $A \setminus U \cup \{x\}$ is a closed set near of order $\mathcal{U}$ to A; then $\mathcal{U}$ is a refinement of the covering described in the statement. The converse is trivial. ■

2.6 COROLLARY. $\boldsymbol{H}(\mu X)$ has isolated points if and only if X has isolated points. ■

2.7 COROLLARY. If X is discrete and μ is precompact, then the set of isolated points of $\boldsymbol{H}(\mu X)$ coincides with the subspace of finite subsets of X. ■

2.8 COROLLARY. The following are equivalent:

i) τ_μ is discrete;

ii) X is an isolated point of $\boldsymbol{H}(\mu X)$;

iii) μ is the discrete uniformity. ■

3. The last corollary of the previous section points out that the discrete topology on the space of closed subsets arises only from a single uniformity (the discrete one). This question can be generalized: do different uniformities generate different topologies? In [I] p. 35, Exercise 17 there is a positive answer to this problem and a hint for its solution which seems to be not correct; namely the family of the « arbitrarily large sets » does not seem to determine the uniformity, against the assertion at the foot of the same page.

3.1 EXAMPLES.

a) Let $X = \{x_\lambda : \lambda \in \Lambda\}$ an infinite discrete space. Let μ be the coarsest precompact (admissible) uniformity: a basis of μ is given by the partitions of the form $\mathcal{U}_F = \{\{x_{i_1}\}, \ldots, \{x_{i_n}\}, X \setminus F\}$ where $F = \{x_{i_1}, \ldots, x_{i_n}\}$ denotes any finite subset of X. Let ν be the finest precompact uniformity: a basis of ν consists of all the finite partitions of X. Plainly τ_ν is finer than τ_μ: furthermore given a partition $\mathcal{V} = \{V_1, \ldots, V_n\} \in \nu$ call F a finite subset of X such that $F \cap V_i \neq \emptyset$ for every $i = 1, \ldots, n$: if a closed is near of order $\mathcal{U}_F$ to X, then it is near of order $\mathcal{V}$ to X, too. This implies that τ_μ and τ_ν coincide at X.

b) (A. Le Donne) Let $X = \mathbf{N} \times \mathbf{N}$ equipped with the discrete topology. Denote by $\mathcal{U}_n$ the partition whose elements are the singletons $\{(x, y)\}$ if $x \leqslant n$ or $y \leqslant n$, otherwise the horizontal lines $U_n^y = \{(x, y) : x > n\}$. $\mathcal{V}_n$ is obtained changing the horizontal lines U_n^y with the vertical ones $V_n^x = \{(x, y) : y > n\}$ for $x > n$. Since $\mathcal{U}_n <^* \mathcal{U}_m$

if $n > m$, the family of the coverings $\mathcal{U}_n$ forms a basis for a metric uniformity which we call μ; similarly the coverings $\mathcal{V}_n$ form a basis for a metric uniformity ν. Both μ and ν are compatible with the discrete topology on X and they are not comparable with each other. However the filters of neighbourhoods of the closed X in τ_μ and τ_ν coincide since if a closed A is near of order $\mathcal{U}_n$ to X, then it is near of order $\mathcal{V}_m$ to X for $m < n$.

We point out that the spaces above provide two conuterexamples of the hint and comment of the cited exercise, but not of its statement. In fact if in 3.1 *a*) one takes a subset A of X such that A and $X \setminus A$ are both infinite and in 3.1 *b*) $A = \{(2n, 2n): n \in \mathbb{N}\}$ one can easily show that τ_μ and τ_ν differ at A. In the solution of the problem the trouble is that the closed at which the topologies are different has to be choosen looking at the structure of the uniformities. The next theorem partially solves this question. Following [I] we denote by $p\mu$ the precompact reflection of the uniformity μ.

3.2 THEOREM. Let X be a space, μ, ν two admissible uniformities. Suppose that one of the following cases occurs:

i) μ is not finer than $p\nu$;
ii) μ is not finer than ν and μ has a linearly ordered basis.

Then τ_μ is not finer than τ_ν.

PROOF. Case i): since $p\mu$ has a sub-basis of two-element uniform open coverings (see [I] Lemma 36, p. 25) there exists such a covering $\mathcal{U} = \{U_1, U_2\}$ belonging to $p\nu$ and not to μ. For every uniform covering $\mathcal{V} \in \mu$ there is $V \in \mathcal{V}$ such that V is contained neither in U_1 nor in U_2: then take $x_{\mathcal{V}} \in V \setminus U_2$, $y_{\mathcal{V}} \in V \setminus U_1$.

Let $A = \mathrm{Cl}_X \{x_{\mathcal{V}}: \mathcal{V} \in \mu\}$, $B_{\mathcal{W}} = A \cup \{y_{\mathcal{W}}\}$ for $\mathcal{W} \in \mu$.

An easy argument shows that $B_{\mathcal{W}}$ and A are near of order $\mathcal{W}$ and are not near of order $\mathcal{U}$, that is the neighbourhood of A made of the closed near of order $\mathcal{U}$ does not contain any neighbourhood of A in τ_ν.

Case ii): The uniformity μ has a well ordered basis $(\mathcal{V}_\alpha)_{\alpha<\xi}$ indexed in a regular cardinal ξ ($\beta > \alpha$ implies $\mathcal{V}_\beta$ finer than $\mathcal{V}_\alpha$). Take an open covering $\mathcal{U}$ belonging to ν and not to μ. For every $\alpha < \xi$ there exist $x_\alpha, y_\alpha \in X$ which are near of order $\mathcal{V}_\alpha$ and are not near of order $\mathcal{U}$. Take $\mathcal{U}'$ a star-refinement of $\mathcal{U}$; for every $U \in \mathcal{U}'$ put $\Phi_U = \{\alpha: x_\alpha \in U\}$, $\Psi_U = \{\alpha: y_\alpha \in U\}$. Two cases may present:

a) there exists $U \in \mathcal{U}'$ such that Φ_U or Ψ_U is cofinal in ξ, say Φ_U; put $A = \mathrm{Cl}_X\{x_\alpha: \alpha \in \Phi_U\}$, $B_\beta = A \cup \{y_\beta\}$ and argue as in i) replacing $\mathcal{U}$ with $\mathcal{U}'$.

b) let $\mathcal{U}''$ be a star-refinement of $\mathcal{U}'$. Define: $\bar{x}_1 = x_1$, $\bar{y}_1 = y_1$ then by transfinite induction, for any $\lambda < \xi$: $\bar{x}_\lambda = x_\beta$ where

$$\beta = \min\{\alpha: x_\alpha \notin St(\bar{x}_\gamma, \mathcal{U}'') \cup St(\bar{y}_\gamma, \mathcal{U}''), \forall \gamma < \lambda\}, \qquad \bar{y}_\lambda = y_\beta$$

where

$$\beta = \min\{\alpha: x_\alpha \notin St(\bar{x}_\gamma, \mathcal{U}'') \cup St(\bar{y}_\gamma, \mathcal{U}'')\ \forall \gamma < \lambda, x_\alpha \notin St(\bar{x}_\lambda, \mathcal{U}'')\}.$$

The regularity of the cardinal ξ ensures the existence of $\bar{x}_\lambda, \bar{y}_\lambda$ just defined: it is enough to observe that, since $St(z, \mathcal{U}'') \subseteq U$ for some $U \in \mathcal{U}'$, the power of the set $\{\alpha: x_\alpha \in St(z, \mathcal{U}'')\}$ is strictly less than ξ for any $z \in X$. Put $A = \mathrm{Cl}_X\{\bar{x}_\lambda: \lambda < \xi\}$, $B_\lambda = A \cup \{\bar{y}_\lambda\}$ and argue as before replacing $\mathcal{U}$ with $\mathcal{U}''$ to conclude that τ_μ is not finer than τ_ν at the point A. ■

Looking at the uniformities which verify the assert of Theorem 3.2, τ is a lattice-homomorphism on its image, in the following sense: $\tau_{\mu\wedge\nu} = \tau_\mu \wedge \tau_\nu$ where the infimum in the second member is taken among the topologies of the hyperspace originated by a uniformity of X: if the infimum is taken in the lattice of the topologies of the hyperspace, clearly the assertion is true no longer: in example 3.1 *b*) $\mu\wedge\nu$ is the discrete uniformity on X, hence the closed X is isolated in $\tau_{\mu\wedge\nu}$ while we have proved that the filters of neighbourhoods of X in τ_μ and τ_ν are the same non trivial filter.

We conclude with some open questions.

It is easy to show that the precompact reflection p does not commute with the functor $\boldsymbol{K}$: take a uniformly discrete space μX and observe that the covering $\mathcal{U} = \{U_1, U_2\}$ where U_1 is the subset of singletons and $U_2 = \boldsymbol{K}(X)\setminus U_1$ is a covering of $p\boldsymbol{K}(\mu X)$ and not of $\boldsymbol{K}(p\mu X)$; this implies that $\boldsymbol{K}$ does not commute with the Stone-Čech compact reflection β too. Putting $\boldsymbol{F}_n(X) = \{A \in \boldsymbol{F}(X): A$ has at most n elements$\}$, it is known (see [*K*2]) that $\beta(\boldsymbol{F}_n(X)) = \boldsymbol{F}_n(\beta X)$ if and only if X^n is pseudocompact, so one may ask: is there any hypothesis which ensures that $\beta\boldsymbol{K}(X) = \boldsymbol{K}(\beta X)$?

It would be of certain interest to know whether the following properties are preserved by $\boldsymbol{K}$: Lindelöf property; paracompactness; normality.

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Manoscritto pervenuto in redazione il 10 luglio 1979.