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OPTIMAL TOOL PARTITIONING RULES FOR NUMERICALLY CONTROLLED PUNCH PRESS OPERATIONS (*)

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Abstract. – *The purpose of this paper is to formulate and solve an optimization problem arising in the operations of a numerical control machine such as a punch press. Holes of n different types must be punched by a press on a linear object such as a metallic bar. Each type of hole requires a different tool and tools are mounted on a fixed rotating carousel. Holes are punched in several passes on the bar, using each time a contiguous subset of tools. The problem is to determine a partition of the tools into subsets that will minimize expected completion time. The problem is first formulated as a shortest path problem on an acyclic graph. It is shown how this problem can be solved in* $O(n^2)$ *or in* $O(n^3)$ *time and that a closed form solution can sometimes be obtained in constant time.*

Keywords: Numerical control machines, punch presses, press drills, acyclic graphs, shortest path problem, flexible manufacturing.

Résumé. – *L'objet de cet article est de formuler et de résoudre un problème d'optimisation se posant dans la gestion des opérations de machines à commande numérique telles les presses-poinçons. On doit percer des trous de n types différents à l'aide d'une presse-poinçon, sur un objet lineaire comme une barre métallique. Chaque type de trou requiert un outil particulier et les outils sont installés sur un carrousel rotatif fixe. On perce les trous en faisant plusieurs passages sur la barre, en utilisant à chaque passage un sous-ensemble contigu d'outils. Le problème consiste à déterminer une partition des outils en sous-ensembles afin de minimiser le temps espéré de complétion. On formule d'abord le problème comme un problème de plus court chemin sur un graphe acyclique. On démontre comment résoudre ce problème en temps* $O(n^2)$ *ou* $O(n^3)$. *Dans certains cas, on peut même résoudre le problème de façon analytique en temps constant.*

Mots clés : Machines à commande numérique, presses-poinçons, presses-perforeuses, graphes acycliques, problème du chemin le plus court, ateliers flexibles.

1. INTRODUCTION

The purpose of this paper is to formulate and solve an optimization problem arising in the operations of numerical control machines, e.g., punch

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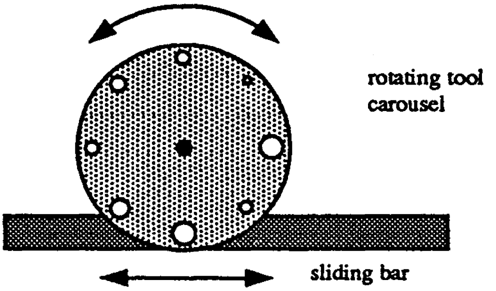
presses and press drills. The punch press problem can be described as follows. Holes of n different types must be punched by a press on a linear object such as a metallic bar. An equivalent situation is where holes are distributed along a predetermined path on a two-dimensional object such as a metallic sheet. Gaboune, Laporte and Soumis [4] describe optimal strip sequencing strategies for the determination of such paths. Each type of hole requires a particular tool and the n types of tools are equally spaced in a circular manner on a carousel. Tool change is carried out by rotating the carousel by the appropriate angle at constant speed. Examples of such machines are described in Ahmadi, Grotzinger and Johnson [1], in Groover [5] and in Winship [7]. Holes of any type are randomly distributed along a line segment according to a known distribution. Typically, the tool carousel is mounted on a fixed support and can rotate freely in either direction. The bar can slide back and forth under the carousel and stop whenever a hole is required. A schematic representation of this process is given in Figure 1.

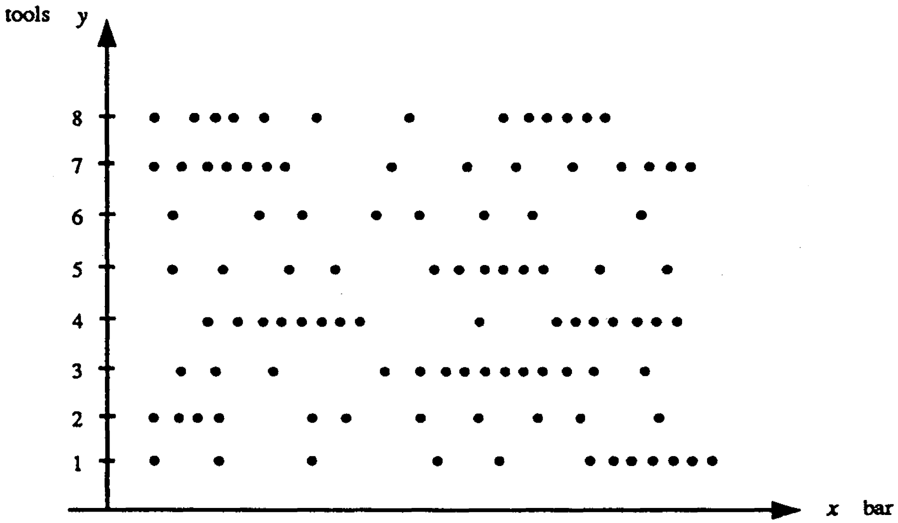
There are two extreme strategies for punching all holes. In one case, holes are punched sequentially, in a single pass of the bar, and the carousel rotates to provide the appropriate tools. This strategy minimizes bar movement but may require several carousel rotations. At the other extreme, the bar is shifted n times and a single tool is used during each pass. Depending on problem parameters, a strategy minimizing completion time would lie somewhere between these two extremes. A set of consecutive tools would then be selected and all corresponding holes would be punched sequentially in a single pass of the bar. Then, another set of consecutive tools would be used for the next pass of the bar, and so on until all holes are punched. Our aim is to determine a tool partitioning rule that will minimize the total expected completion time. Since the actual punching time is constant, we disregard it and minimize the total expected time spent between consecutive punching oprations. This amounts to determining a partition of an ordered set $N = \{1, 2, \ldots, n\}$ into K subsets $N_k = [j_{k-1}+1, j_k] = \{j_{k-1}+1, \ldots, j_k\}$, with $j_0 = 0$, in order to minimize

$$z = \sum_{k=1}^{K} f(N_k),$$

where K is an integer variable in $[1, n]$ representing the number of bar passes, and $f(N_k)$ is the expected time to execute a full pass of the bar with the tools of N_k.



a) Physical representation



b) Schematic two-dimensional representation with 8 tools, and hole requirements at various positions on the bar

Figure 1. – Two representations of a bar and tool carousel

2. THE MODEL

The problem can be formulated as a least cost path problem as follows. Let $G = (N \cup \{0\}, A)$ be a connected graph where $A = \{(i, j) : i, j \in N \cup \{0\}$ and $i < j\}$. Define costs $c_{ij} = f([i + 1, j])$. Then the problem consists of determining a least cost path from 0 to n on G. Assuming the c_{ij}'s are given and since G is acyclic, the problem can easily be solved in $O(n^2)$ time, as each arc needs processing only once (Ford and Fulkerson, [3]). This model is obviously valid if sets of *consecutive* tools are used.

Otherwise, a more involved model would in principle be required, but this is not in fact necessary as an interchange argument shows that it is never suboptimal to use consecutive tools. The model just described is of general applicability. In particular, our problem is similar to a classical equipment replacement problem described in some network flow textbooks – see, e.g. Jensen and Barnes ([6], p. 37). Another application of this structure to a line simplification problem arising in cartography is provided in Campbell and Cromley [2].

In what follows, we provide low complexity procedures for computing all c_{ij}'s so that the overall time complexity is $O(n^2)$ or in $O(n^3)$, depending on the system's operating rules. We also show how under some circumstances the problem can even be solved analytically, in constant time.

3. COST COMPUTATION FOR THE GENERAL CASE

In addition to the notation already introduced, define

p_i $(i = 1, \ldots, n)$: the probability of requiring a tool of type i (or the proportion of holes of type i);

d: the overall density of holes on the bar expressed in number of holes per unit length; note that the density of holes of type i is given by dp_i;

v_x: the bar velocity expressed in unit bar length per time unit;

v_y: the carousel velocity expressed in number of tools per time unit;

$X_{i+1,\,j}$: the time required to move the bar between two consecutive holes requiring tools in $[i + 1,\, j]$;

$Y_{i+1,\,j}$: the time associated with carousel movement between two consecutive holes requiring tools in $[i + 1,\, j]$.

Since time units have not been specified, v_y can be set equal to 1 without loss of generality and v_x can therefore be replaced by the ratio $v = v_x/v_y$. Also, the bar length can be taken as one unit. When tools in $[i + 1,\, j]$ are used, the time between two punching operations is expressed as $\rho\,(X_{i+1,\,j},\, Y_{i+1,\,j})$, where ρ is a metric defined according to the operating system. For example, if the bar and the carousel move sequentially, then $\rho\,(X_{i+1,\,j},\, Y_{i+1,\,j}) = X_{i+1,\,j} + Y_{i+1,\,j}$, *i.e.*, ρ is the l_1 or Manhattan metric; if the bar and the carousel move simultaneously, $\rho\,(X_{i+1,\,j},\, Y_{i+1}) = \max\,(X_{i+1,\,j},\, Y_{i+1,\,j})$, *i.e.*, ρ is the l_∞ or Chebychev metric. The expected time to execute a complete pass of the bar using the tools of $[i + 1,\, j]$ is obtained for each metric by multiplying

$E(\rho(X_{i+1,j}, Y_{i+1,j}))$ by the expected number of holes requiring those tools, *i.e.*,

$$f([i+1, j]) = E(\rho(X_{i+1,j}, Y_{i+1,j}))(d P_{i+1,j}), \tag{1}$$

where

$$P_{ij} = \sum_{k=i}^{j} p_k. \tag{2}$$

For a Manhattan metric

$$E(\rho(X_{i+1,j}, Y_{i+1,j})) = E(X_{i+1,j}) + E(Y_{i+1,j}). \tag{3}$$

The first term of (3) can be expressed as

$$E(X_{i+1,j}) = (vd\, P_{i+1,j})^{-1}, \tag{4}$$

while

$$E(Y_{i+1,j}) = \sum_{y=0}^{j-i-1} y\, P(Y_{i+1,j} = y),$$

where y represents the absolute difference between any two tool indices r and s in $[i+1, j]$. In other words,

$$E(Y_{i+1,j}) = \sum_{r=i+1}^{j} \sum_{s=i+1}^{j} |s-r| p_r\, p_s / P_{i+1,j}^2 \tag{5}$$

$$= 2 \sum_{r=i+1}^{j-1} \sum_{s=r+1}^{j} (s-r)\, p_r\, p_s / P_{i+1,j}^2. \tag{6}$$

The value of $E(X_{i+1,j})$ in (4) was obtained by dividing $(d P_{i+1,j})^{-1}$, the expected distance between two consecutive holes requiring tools in $[i+1, j]$, by the bar velocity v. In (5), the numerator sums all interpoint distances $|s-r|$, weighted by their joint probability $p_r\, p_s / P_{i+1,j}^2$; to avoid computing symmetric cases twice, the double summation in (6) extends over $s > r$ only and is multiplied by 2. Dividing by $P_{i+1,j}^2$ is necessary as we seek to compute a *conditional* expectation relative to tools $i+1$ to j; in other words, when computing the expectation, each of the two probabilities p_r and p_s

must be divided by the probability $P_{i+1,\,j}$ that a tool in the interval $[i+1,\, j]$ will be required. Finally note that if $j = i + 1$, the value of $E\,(Y_{i+1,\,j})$ is then equal to 0 since in this case only tool $i + 1$ is used during a pass of the bar, which requires no carousel movement.

From (1), it follows that for a Manhattan metric

$$f\,([i + 1,\, j]) = v^{-1} + 2d\, S_{i+1,\,j}/P_{i+1,\,j} \qquad (0 \le i \le j \le n), \qquad (7)$$

where

$$S_{ij} = \sum_{r=i}^{j-1} \sum_{s=r+1}^{j} (s - r)\, p_r\, p_s \qquad (1 \le i \le j \le n). \qquad (8)$$

In the case of a Manhattan metric, the computation of (3) uses the additivity property of expectations, but this does not apply to the Chebychev metric. The computation of $E\,(\max\,(X_{i+1,\,j},\, Y_{i+1,\,j}))$ is more involved and now depends on the distribution of holes along the bar. We will compute this expression for the case where the holes are distributed according a Poisson process. To avoid the boundary effect, the expected time between two consecutive holes is computed assuming the bar length is infinite. It is then multiplied by $dP_{i+1,\,j}$ to obtain the expected execution time $f\,([i + 1,\, j])$ for a unit bar length. Under these assumptions, $X_{i+1,\,j}$ is an exponential random variable of parameter $\lambda_{i+1,\,j} = vd\, P_{i+1,\,j}$. The interpoint expected time is therefore

$$E\,(\max\,(X_{i+1,\,j},\, Y_{i+1,\,j})) = \sum_{y=0}^{j-i-1} \int_0^{\infty} \max\,(x,\, y)\, \lambda_{i+1,\,j}\, e^{-\lambda_{i+1,\,j} x}\, P\,(Y_{i+1,\,j} = y)\, dx,$$

assuming independence of $X_{i+1,\,j}$ and $Y_{i+1,\,j}$. This expression can be rewritten as

$$\sum_{y=0}^{j-i-1} \left[y \int_0^y \lambda_{i+1,\,j}\, e^{-\lambda_{i+1,\,j} x}\, dx \right.$$
$$\left. + \int_y^{\infty} \lambda_{i+1,\,j}\, x e^{-\lambda_{i+1,\,j} x}\, dx \right] P\,(Y_{i+1,\,j} = y)$$
$$= \sum_{y=0}^{j-i-1} [y\,(1 - e^{-\lambda_{i+1,\,j} y}) + (y + \lambda_{i+1,\,j}^{-1})\, e^{-\lambda_{i+1,\,j} y}]\, P\,(Y_{i+1,\,j} = y)$$
$$= \lambda_{i+1,\,j}^{-1} \sum_{y=0}^{j-i-1} e^{-\lambda_{i+1,\,j} y}\, P\,(Y_{i+1,\,j} = y) + E\,(Y_{i+1,\,j})$$
$$= \lambda_{i+1,\,j}^{-1} \sum_{r=i+1}^{j} \sum_{s=i+1}^{j} e^{-\lambda_{i+1,\,j} |s-r|}\, p_r p_s / P_{i+1,\,j}^2 + E\,(Y_{i+1,\,j})$$
$$= \left(2\,\lambda_{i+1,\,j}^{-1} \sum_{r=i+1}^{j-1} \sum_{s=i+1}^{j} e^{-\lambda_{i+1,\,j}(s-r)}\, p_r p_s \right.$$
$$\left. + \lambda_{i+1,\,j}^{-1} \sum_{k=i+1}^{j} p_k^2 \right) \Big/ \; P_{i+1,\,j}^2 + E\,(Y_{i+1,\,j}), \tag{9}$$

where $E\,(Y_{i+1,\,j})$ is defined as in (6). Note that it is now necessary to consider cases where $s = r$ as the corresponding terms do not vanish. This explains the presence of the term $\lambda_{i+1,\,j}^{-1} \sum_{k=i+1}^{j} p_k^2$ in (9). Defining

$$T_{ij} = \sum_{r=i}^{j-1} \sum_{s=r+1}^{j} e^{-\lambda_{ij}\,(s-r)}\, p_r\, p_s \qquad (1 \le i < j \le n) \tag{10}$$

and combining (1) and (9), $f\,([i+1,\, j])$ can be rewritten as

$$f\,([i+1,\, j]) = \left(2\, T_{i+1,\,j} + \sum_{k=i+1}^{j} p_k^2 \right) \Big/ \; v\, P_{i+1,\,j}^2 + 2 d S_{i+1,\,j} / P_{i+1,\,j}$$
$$(0 \le i < j \le n). \tag{11}$$

If all $O\,(n^2)$ values of $f\,([i+1,\, j])$ defined by (7) or (11) are computed independently, and assuming the P_{ij} values are precomputed, the total computation will require $O(n^4)$ time because of the double summations in S_{ij} and T_{ij}. However, a better time complexity is possible if some additional preliminary computations are carried out. First consider the Manhattan metric. For $1 \leq i \leq j \leq n$, compute in $O(n^2)$ time the coefficients

$$Q_{ij} = \sum_{k=i}^{j} k p_k. \tag{12}$$

Then the computation of S_{ij} in (8) can be decomposed as

$$\begin{aligned} S_{ij} &= S_{i,\, j-1} + \sum_{r=i}^{j-1} (j - r)\, p_r p_j \\ &= S_{i,\, j-1,} + j p_j\, P_{i,\, j-1} - p_j\, Q_{i,\, j-1} \qquad (1 \leq i < j \leq n). \end{aligned} \tag{13}$$

The value of $f([i+1,\, j])$ is easily computed for each i, starting with $S_{ij} = 0$ for $j = i$ and then increasing j. If the value of $S_{i,\, j-1}$ is stored, then $f([i+1,\, j])$ is obtained in constant time from $f([i+1,\, j-1])$ once the P_{ij} and Q_{ij} coefficients are known. Therefore, the overall time complexity clearly reduces to $O(n^2)$ for the Manhattan metric.

An $O(n^3)$ time complexity can be obtained for the Chebychev metric. For this, first define for $1 \leq i < j \leq n$ and for $1 \leq y \leq j-1$ the coefficients

$$R_{ijy} = \sum_{k=i}^{j-y} p_k\, p_{k+y}. \tag{14}$$

In the square defined by $[1,\, n]^2$, R_{ijy} sums the products of row and column probabilities in the y^{th} diagonal above the main diagonal, starting in row i and ending in column j. These values are easily computed starting with $R_{i,\, i+y,\, y} = p_i\, p_{i+y}$ and using the recursion $R_{ijy} = R_{i,\, j-1,\, y} + p_{j-y}\, p_j \; (i + y + 1 \leq j \leq n)$, so that all R_{ijy} values can be obtained in $O(n^3)$ time. Then T_{ij} can be rewritten as a summation over all diagonals having the same value of $y = s - r$ for $s > r$, *i.e.*,

$$T_{ij} = \sum_{y=1}^{j-i} R_{ijy}\, e^{-vdy\, P_{ij}}. \tag{15}$$

In other words, each T_{ij} can be computed in linear time once the P_{ij} and the R_{ijy} coefficients are known. It follows that all T_{ij} and $f([i+1,\, j])$

computations can be carried out in $O(n^3)$ time. Note that memory requirements can be limited to $O(n^2)$ by computing the T_{ij}'s and the R_{ijy}'s in increasing order of j, using at each step a two-dimensional array of R_{ijy} coefficients. Whenever j is incremented, the $R_{i,\,j+1,\,y}$ array can easily be derived in $O(n^2)$ time from the R_{ijy} array.

4. NUMERICAL EXAMPLE

We now illustrate the computations for the Manhattan metric. Let $n=4$, $p_1=0.4$, $p_2=0.1$, $p_3=0.3$, $p_4=0.2$, and $d=10$. We first concentrate on the computation of the S_{ij} values in (8). The relevant numerical values of P_{ij} and Q_{ij} for this example are given by TABLE I.

TABLE I
Numerical values of P_{ij} and Q_{ij}

P_{ij}

$i \backslash j$	1	2	3	4
1	0.4	0.5	0.8	1.0
2	–	0.1	0.4	0.6
3	–	–	0.3	0.5

Q_{ij}

$i \backslash j$	1	2	3
1	0.4	0.6	1.5
2	–	0.2	1.1
3	–	–	0.9

Then, using (13) and $S_{ii} = 0$, we obtain

$$\begin{aligned}
S_{1,1} &= 0,\\
S_{1,2} &= S_{1,1} + 2p_2\, P_{1,1} - p_2\, Q_{1,1} = 0.04,\\
S_{1,3} &= S_{1,2} + 3p_3\, P_{1,2} - p_3\, Q_{1,2} = 0.31,\\
S_{1,4} &= S_{1,3} + 4p_4\, P_{1,3} - p_4\, Q_{1,3} = 0.65,\\
S_{2,2} &= 0,\\
S_{2,3} &= S_{2,2} + 3p_3\, P_{2,2} - p_3\, Q_{2,2} = 0.03,\\
S_{2,4} &= S_{2,3} + 4p4\, P_{2,3} - p_4\, Q_{2,3} = 0.13,\\
S_{3,3} &= 0,\\
S_{3,4} &= S_{3,3} + 4p_4\, P_{3,3} - p_4\, Q_{3,3} = 0.06,\\
S_{4,4} &= 0.
\end{aligned}$$

The values of $f([i+1,\, j])$ are then easily obtained by using (7). These values are reported in TABLE II for $v = 1/3$, 2 and 1/10.

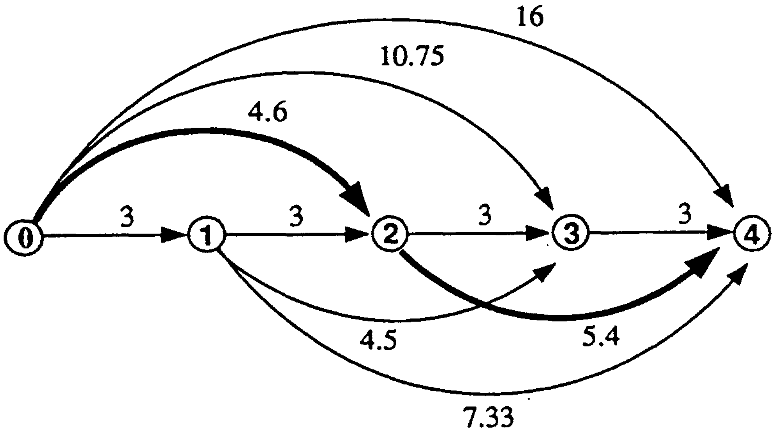


Figure 2. – Acyclic graph for $v = 1/3$. The optimal path (0, 2, 4) is indicated by the bold arcs

TABLE II
Numerical values of $f\ ([i+1,\ j])$

$v = 1/3$

$i \backslash j$	1	2	3	4
0	3	4.6	10.75	16
1	–	3	4.5	7.33
2	–	–	3	5.4
3	–	–	–	3

$v = 2$

$i \backslash j$	1	2	3	4
0	0.5	2.1	8.25	13.5
1	–	0.5	2	4.83
2	–	–	0.5	2.9
3	–	–	–	0.5

$v = 1/10$

$i \backslash j$	1	2	3	4
0	10	11.6	17.75	23
1	–	10	11.5	14.33
2	–	–	10	12.4
3	–	–	–	10

The acyclic graph corresponding to $v = 1/3$ is depicted in Figure 2. It can readily be verified that the optimal path is (0, 2, 4) and has a cost of $4.6 + 5.4 = 10$. In other words, tools 1 and 2 should be used in a first pass of the bar, and tools 3 and 4 in a second pass. Note that this optimal policy is directly dependent on the value of v. For example, if $v = 2$, then the optimal path becomes (0, 1, 2, 3, 4) and has a cost of 2. This corresponds to using a single tool in each of four passes of the bar. At the other extreme, if $v = 1/10$, the optimal path is simply (0,4) and has a cost of 23. In this case, all holes should be punched in a single pass of the bar.

5. COST COMPUTATION FOR EQUAL PROBABILITIES

In the case of the Manhattan metric, and assuming $p_i = 1/n$ for all i, the optimum can be determined in constant time. Indeed, the computation of $f\ ([i+1,\ j])$ given by (7) reduces to

$$f([i+1, j]) = v^{-1} + [2d/n(j-i)] \sum_{r=i+1}^{j-1} \sum_{s=r+1}^{j} (s-r)$$
$$= v^{-1} + [2d/n(j-i)] [(j-i)^3 - (j-i)]/6$$

This function can be rewritten in terms of $t = j - i \geq 1$, as

$$f(t) = v^{-1} + (d/3n)(t^2 - 1),$$

in other words, f depends only on t, the number of tools, and not on the particular tool sequence that is used. If integrality requirements are disregarded, an optimal least cost path solution of cost $z(t)$ will contain n/t arcs. Therefore,

$$z(t) = (n/t) f(t) = nv^{-1}/t + (d/3)(t - 1/t),$$

a function attaining a minimum at

$$t^* = \left\{ \begin{array}{ll} \min(n, [(3n/dv) - 1]^{1/2}) & \text{if } 3n > dv \\ 1 & \text{otherwise.} \end{array} \right\} \qquad (16)$$

The optimum provided by (16) must be interpreted in an asymptotic sense since t^* is in general non-integer and does not necessarily divide n. For small values of n, it is usually necessary to explore the neighbourhood of t^* to identify an optimal value. To illustrate the computation of t^* in the case of equal probabilities, in a 12 tool problem with $v = 1/10$ and $d = 25$, the value of t^* in (16) is equal to 3.6606. Since $z(3) = 62.22$ and $z(4) = 61.25$, four consecutive tools should be used during each pass.

Finally, it is worth noting that a similar analysis can be conducted in the case of a Chebychev metric. However, the computations are more involved and although the total cost $z(t)$ can still be expressed as a function of t, there is no closed form expression for the minimum and a numerical optimization method is required. As a result, this case presents less interest and is not presented here.

6. CONCLUSION

The objective of this study was to formulate and solve a partitioning problem arising in a flexible manufacturing context. We have shown that the corresponding optimization problem can be modelled as a least cost path problem on an acyclic graph. By appropriately exploiting data structures, the problem can be solved in $O(n^2)$ or $O(n^3)$ time, depending on the metric considered. When all tool probabilities are equal, a closed form solution can sometimes be obtained in constant time.

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