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INFORMATION PROCESSING, HIERARCHICAL STRUCTURES AND MANAGEMENT PRODUCTION FUNCTIONS: A TENTATIVE STUDY (*)

by Félix MORA-CAMINO ([1])

Abstract. — *A model of the modern computerized firm is proposed with the aim of defining an efficient organization of its management. The duality theory of geometric Programming is applied to the analysis of the short planning period problem. Special results are derived in the case of a single output firm.*

Keywords: Theory of the firm, Organizations, Geometric Programming, Separable Programming.

Résumé. — *Un modèle théorique de l'entreprise munie d'un système informatique est proposé afin de définir une structure optimale pour son appareil administratif. La théorie de la dualité de la programmation géométrique est alors utilisée pour analyser le problème de la production à court terme. Des résultats spécifiques sont obtenus dans le cas où une seule production est considérée.*

Mots clés : Théorie de la firme, Théorie des organisations, Programmation Géométrique, Programmation séparable.

I. INTRODUCTION

In order to analyse the place of information processing and communication in the modern computerized firm, the classical Theory of the Firm is reappraised considering that the activities of the firm are of two kinds:

– material processing,

– data processing.

Here the focus is over the managerial organization of the firm which is responsible for information processing and communication.

In a recent study [1], M. Beckman developped a management production function to analyse the possible relationships between productivity of workers and size of the managerial organization of the firm as well as to determine its optimal structure. He treated the output of managers as an intermediate

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product of the firm, the "supervision" or "managerial control" applied to lower levels of management and characterized the structure of the organization with a "span of control" for each level.

In this study a different management production function is proposed, various managerial divisions are allowed at the same level in the organization and the multiproduct case is considered. A discrimination is made between capital for processing informations and communication, which is considered as a common ressource for the managerial divisions, and capital for processing materials, which is allocated to the material processing units.

The Duality Theory of Geometric Programming is used to analyse the short production planning period with the aim of defining in these conditions the optimal organization of the firm. Special results are derived in the case of a single output firm.

II. THE HIERARCHICAL PRODUCTION FUNCTION

The management of the firm is composed of R administrative levels, $r=1$ to R, each level is composed of $S(r)$ divisions, $s=1$ to $S(r)$, with at the top of the management $S(R)=1$. The existence of the management divisions is not discussed here, it is only supposed that each management division has definite and effective functions in the management of the firm.

Each division (r, s) produces a managerial control [1] y_{rs} over divisions of lower level under a supervision of divisions of higher level and with the use of informations provided by the divisions of the next lower level:

$$y_{rs}=f_{rs}(\underline{y}_{r+1}, I_{rs}, \underline{I}_{r-1}), \tag{1}$$

where:

$$I_{rs}=g_{rs}(x_{rs}, k_I), \tag{2}$$

is an intermediate input representing "the ability of division (r, s) in using informations" from other divisions.

x_{rs} is the amount of managerial labor in divisions of level r.

k_I is the amount of capital allocated to information processing in the firm, assumed a common ressource for all the management divisions.

$$\underline{y}'_r=(y_{r1}, y_{r2}, \ldots, y_{rS(r)}),$$

It will be assumed that possible conflicts beetween managerial controls of different higher levels are already taken into account in (1).

Equations (1) and (2) are associated with hierarchical structures of management control and of information flows such as those displayed in figure 1 and figure 2:

Specifying the functions f_{rs} and g_{rs} as Cobb-Douglas functions, (1) and (2) become:

$$y_{rs}=y_{rs}^{0}\cdot\prod_{t=1}^{S(r+1)} y_{r+1,t}^{a_{rs}^{t}}\cdot I_{rs}^{b_{rs}}\cdot\prod_{v=1}^{S(r-1)} I_{r-1,v}^{c_{rs}^{v}} \quad (3)$$

and:

$$I_{rs}=I_{rs}^{0}\cdot x_{rs}^{\alpha_{rs}^{I}}\cdot k_{I}^{\beta_{rs}^{I}}, \quad (4)$$

y_{rs}^{0} and I_{rs}^{0} being constant parameters.

The different elasticities in (3) and (4) are supposed non-negative and such that:

$a_{rs}^{t}=0$ when division (r, s) is not under direct supervision of division $(r+1, t)$.

$c_{rs}^{v}=0$ when information produced by division $(r-1, v)$ is of no direct use for division (r, s).

At the lower level of the firm (material processing), the material outputs are taken as the result of the combination of "effective labor of management" as defined by Beckman [1], of capital for material processing (k_i) and of material inputs. It is assumed in this study that there are no joint products. Using the same Coob-Douglas assumption, the final outputs are such that:

$$q_i=y_{0_i}^{A_i}\cdot k_i^{B_i}\cdot F_i(\underline{Q}), \qquad i=1 \text{ to } N, \quad (5)$$

with:

$$y_{0_i}=y_{0_i}^{0}\, x_{0i}^{\alpha i}\cdot\prod_{u=1}^{N} y_{1u}^{\alpha_i^u} \quad (6)$$

and:

$$A_i\geqq 0, \qquad B_i\geqq 0, \qquad \alpha_i\geqq 0, \qquad \alpha_i^u\geqq 0,$$

where $F_i(\underline{Q})$ is a function of the vector of material inputs $\underline{Q}$, and $y_{0_i}^{0}$ is a constant parameter.

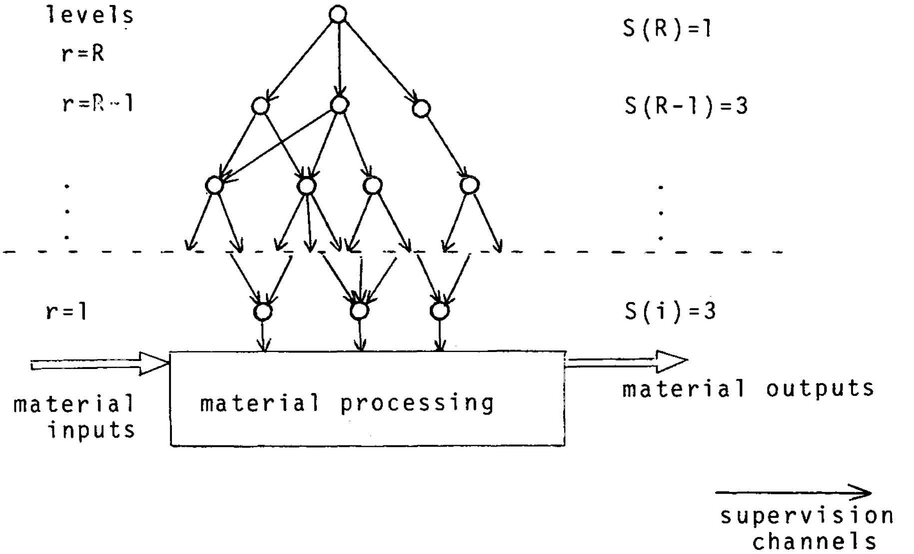

Figure 1. — Management control structure.

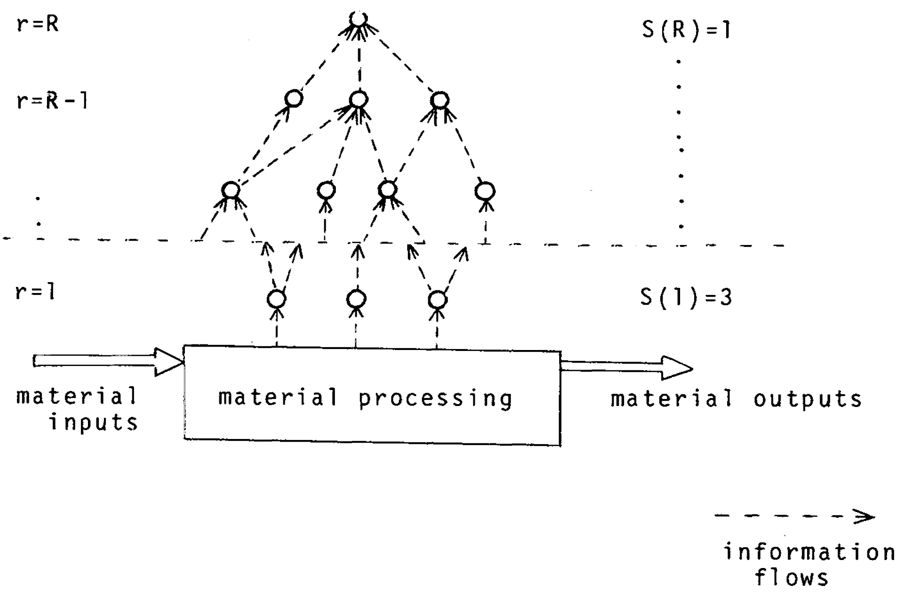

Figure 2. — Management information structure.

The system of equations (3), (4), (5), (6) being recursive, we get the set of production functions:

$$q_i=\Phi_i(\underline{Q}).x_{0i}^{\alpha_i}.\left(\prod_{r=1}^{R}\prod_{t=1}^{S(r)}x_{rt}^{\beta_{rt}^i}\right).k_i^{b_i}.k_1^{c_i}, \qquad i=1 \text{ to } N, \tag{7}$$

where, as in [1], the presidential labor input is fixed to unity. It must be noted that relation (7) has an enough general form which can be obtained from different hypotheses considering, or not, concepts such as "effective labor of management" or "ability of using informations". It is also possible to allow communication (information exchange) between divisions of the same level without prejudice to the recursivity of the relations defining the managerial controls. Here where there are various divisions in the same level, the concept of span of control as defined in [1] loses its power to characterize the hierarchy.

III. EFFICIENT ORGANIZATION OF MANAGEMENT

In the short run analysis the variables considered are the level of operative labor and the level of managerial labor in each divisions. In a first step of the analysis, it is supposed that the level of operative labor in each material processing unit is fixed, so that the variable cost considered is reduced to the cost of management.

A program of production $(q_1^*, q_2^*, \ldots, q_N^*)$ of cost C will be said to be efficient if, for the same cost, it is not possible to rise the level of one output $i\,(i \in \{1, N\})$ without disminishing the level of at least another output $j\,(j \in \{1, N\}, j \neq i)$.

If $[x^*]$ is solution of the following problem (Problem I):

$$\min_{[x]} C([x]),$$

with:

$$C([x])=\sum_{r=1}^{R-1}\sum_{s=1}^{S(r)} w_{rs}.x_{rs} \tag{8}$$

and:

$$q_i([x]) \geqq q_i^*, \qquad i=1 \text{ to } N, \tag{9}$$

$$x_{rs}>0, \qquad s=1 \text{ to } S(r). \qquad r=1 \text{ to } R-1, \tag{10}$$

where w_{rs} is the average wage in division s of level r, and where the restrictions (10) insure that the corresponding divisions (r, s) are present in the management of the firm, and if $[x^*]$ is such that:

$$q_i([x^*])=q_i^*, \qquad i=1 \text{ to } N \tag{11}$$

and:

$$\delta C([x])/\delta q_i \,|\, [x]=[x^*] \geqq 0, \qquad i=1 \text{ to } N, \tag{12}$$

then it can be checked easily that the resulting program of production $(q_1^*, q_2^*, \ldots, q_N^*)$ of cost $C([x^*])$ is efficient in the above sense for the short run case where the levels of operative labor are fixed.

Problem I being convex [2], its solution is unique. To any efficient program of production $(q_1^*, q_2^*, \ldots, q_N^*)$ of feasible cost C^*, will correspond one organization of management (the solution of Problem I) in the case considered. This organization of management will be qualified as efficient.

If the function of the managerial divisions is to insure the existance of production (by taking decisions, fixing objectives, coordinating activities of lower divisions or processing units,...) without contributing directly to the material process of production. Problem I vanishes since the managerial costs are no more related to the level of the outputs. Then the structure of management will be determined using alternate considerations to the efficiency of the material production of the firm (*see* [3] for instance). This situation corresponds to the case in (7) where:

$$\text{for } i=1 \text{ to } N: \beta_{rt}^i=0, \qquad t=1 \text{ to } S(r), \qquad r=1 \text{ to } R-1, \tag{13}$$

which happens for instance when the elasticities α_i^u of material outputs with respect to direct managerial control are equal to zero.

If the firm is not concerned with a significant material processing activity, but instead is concerned with "production" of informations (ou rules, norms, controls,...) the set of production functions of the firm is such as (assuming that the different outputs are quantifiable and are not joint products):

$$\alpha_i=0 \qquad \text{and} \qquad b_i=0 \tag{14}$$

and (7) can be rewritten as:

$$I_i=\Phi_i(\underline{J})\left(\prod_{r=1}^{R-1}\prod_{s=1}^{S(r)} x_{rs}^{\beta_{rs}^i}\right).k_i^{c_i}, \qquad i=1 \text{ to } N, \tag{15}$$

where I_i is the amount of output i and $\underline{J}$ is the vector of the inputs (information inputs).

In this case the notion of efficiency as defined earlier can be useful to determine the organization of the management of the firm.

Finally, if the outputs of the firm are not quantifiable, the notion of efficiency is no more valid, other considerations must be used to analyse the organization of the firm.

REMARK: For the single output case, the concept of efficiency proposed here reduces to the classical concept of "cost of production".

IV. OPTIMAL ASSIGNMENT OF MANAGERIAL COSTS IN THE SHORT RUN-THE MULTI-PRODUCT CASE

In this section we return to the assumptions made in section II to analyse Problem I and to characterise the solution [x*] for the organization of management in the short run.

Primal and dual geometric formulations

Problem I can be rewritten as (Problem I'):

$$\min_{[x]} \tilde{C}([x]),$$

with:

$$\tilde{C}([x]) = \sum_{r=1}^{R-1} \sum_{s=1}^{S(r)} \omega_{rs} . x_{rs} \tag{16}$$

and:

$$Z_i . \prod_{r=1}^{R-1} \prod_{s=1}^{S(r)} x_{rs}^{-\beta^i_{rs}} \leqq 1, \qquad i = 1 \text{ to } N, \tag{17}$$

$$x_{rs} > 0, \qquad s = 1 \text{ to } S(r), \qquad r = 1 \text{ to } R-1, \tag{18}$$

with:

$$\omega_{rs} = w_{rs} \Bigg/ \left(\sum_{k=1}^{R-1} \sum_{h=1}^{S(k)} w_{kh} \right) \tag{19}$$

and:

$$Z_i = q_i^* \left(\Phi_i(\underline{Q}) . x_{0i}^{a_i} . k_i^{b_i} . k_I^{c_i}\right)^{-1}, \qquad i = 1 \text{ to } N. \tag{20}$$

To this last formulation of Problem I corresponds the Dual Geometric Program [4] (Problem II):

$$\max_{[\xi],\, z} I([\xi], \underline{z}),$$

with:

$$I([\xi], \underline{z}) = \left(-\sum_{r=1}^{R-1} \sum_{s=1}^{S(r)} \xi_{rs} \operatorname{Log}(\xi_{rs}/\omega_{rs}) + \sum_{i=1}^{N} z_i \operatorname{Log}(Z_i) \right) \tag{21}$$

and:

$$\xi_{rs} \geqq 0, \qquad s = 1 \text{ to } S(r), \qquad r = 1 \text{ to } R-1, \tag{22}$$

$$z_i \geqq 0, \qquad i = 1 \text{ to } N, \tag{23}$$

$$\sum_{r=1}^{R-1} \sum_{s=1}^{S(r)} \xi_{rs} = 1, \tag{24}$$

$$A' . \left[\frac{\xi}{\underline{z}} \right] = \underline{0}, \tag{25}$$

where A is the matrix of the coefficients of the primal variables of Problem I. A is of dimension NXM, where M is the number of managerial divisions in the firm (less the presidencial division):

$$A = \left[\frac{I_{MM}}{[\beta]} \right] \qquad \text{with} \quad M = \sum_{r=1}^{R-1} S(r), \tag{26}$$

and:

$$[\beta] = \begin{bmatrix} \underline{\beta}_1 \\ \vdots \\ \underline{\beta}^N \end{bmatrix} \qquad \text{with} \quad \underline{\beta}^{i'} = (\beta_{11}^i, \ldots, \beta_{R-1\, S(R-1)}^i).$$

I_{MM} being the identity matrix of dimension MXM. The degree of difficulty [4] of Problem II is:

$$D = \left(\sum_{r=1}^{R-1} S(r) + N \right) - \left(\sum_{r=1}^{R-1} S(r) \right) - 1 = (M+N) - M - 1 = N - 1. \tag{27}$$

This index, characterising in some way the complexity of Problems I and II, is in this case independant of the number of managerial divisions in the firm. This index depends only of the dimension of the space of production of the firm.

Thus, if the firm considered has only one output, the degree of difficulty of this problem is zero, whatever the complexity of the managerial organization of the firm.

In this case, the solution of Problem II is trivial since the feasible set is, when it is not empty, restricted to one point: the solution of the system of equations (22) to (26).

Here also, it can be shown that Problem II is a convex mathematical programming problem which, when feasible, admits a unique solution. Next, the solution to this problem will be related to the solution of Problem I and its analysis will provide more insight about the properties of an efficient organization of management as defined in section III.

Duality relations and economic interpretation of Problem II

Since Problems I′ and II are a primal geometric programming problem and its respective dual, their solutions satisfy the following relations:

(1) $$\text{Log}(\tilde{C}([x^*]))=I([\xi^*], \underline{z}^*) \qquad (28)$$

or:

$$\text{Log}\,\tilde{C}^*=I^*; \qquad (28.1)$$

(2) $$\tilde{C}^*.\xi^*_{rs}=\omega_{rs}.x^*_{rs}, \qquad s=1 \text{ to } S(r), \qquad r=1 \text{ to } R-1; \qquad (29)$$

(3) $$(\delta\tilde{C}/\delta Z_i|_{[x]=[x^*]})/(C^*/Z_i)=z^*_i, \qquad i=1 \text{ to } N \qquad (30)$$

or:

$$e_{\tilde{C}^*z_i}=z^*_i, \qquad i=1 \text{ to } N. \qquad (30.1)$$

Relation (29) shows that the value ξ^*_{rs} is the optimal share of managerial cost allocated to the division (r, s) (*see* [5]).

The levels of operative labor, x_{0i}, $i=1$ to N, are held fixed in this analysis, so that relation (20) implies:

$$(\delta\tilde{C}/\delta Z_i\,|_{[x]=[x^*]})/(\tilde{C}^*/Z_i)$$

$$=(\delta\tilde{C}/\delta q_i\,|_{[x]=[x^*]})/(\tilde{C}^*/q_i^*),\qquad i=1\text{ to }N\quad(31)$$

or:

$$e_{C^*Z_i}=e_{C\ q_i},\qquad i=1\text{ to }N.\qquad(31.1)$$

Then, relation (29) shows that z_i^* represents the elasticity $e_{C^*q_i}$ of the optimal management cost with respect to the level q_i^* of the i-th output. Relations (23), (30) and (31) insures now that the relation of efficiency (12) is satisfied at the solution of Problem I.

Writing the managerial cost as:

$$C=C(q_1,\, q_2,\, \ldots,\, q_N),\qquad(32)$$

a marginal variation of managerial cost resulting of small variations of managerial labor can be written as:

$$dC=\sum_{i=1}^{N}(\delta C/\delta q_i).dq_i,\qquad(33)$$

with:

$$dq_i=\sum_{r=1}^{R-1}\sum_{s=1}^{S(r)}(\delta q_i/\delta x_{rs}).dx_{rs}\qquad(34)$$

or since the elasticities β_{rs}^i are constant:

$$dq_i=\sum_{r=1}^{R-1}\sum_{s=1}^{S(r)}(q_i/x_{rs}).\beta_{rs}^i.dx_{rs}.\qquad(35)$$

Taking into account (34) and (35), the marginal variation of managerial cost can be rewritten as:

$$dC=\sum_{r=1}^{R-1}\sum_{s=1}^{S(r)}\left(\sum_{i=1}^{N}((\delta C/\delta q_i).(q_i/x_{rs}).\beta_{rs}^i).dx_{rs}\right)\qquad(36)$$

and:

$$dC/C=\sum_{r=1}^{R-1}\sum_{s=1}^{S(r)}\left(\sum_{i=1}^{N}((\delta C/\delta q_i).(q_i/C).\beta_{rs}^i).(dx_{rs}/x_{rs})\right). \tag{37}$$

Since:

$$C=W.\tilde{C}$$

with:

$$W=\sum_{r=1}^{R-1}\sum_{s=1}^{S(r)} w_{rs}, \tag{38}$$

we have:

$$d\tilde{C}/\tilde{C}=dC/C \tag{39}$$

and:

$$(\delta\tilde{C}/\delta q_i).(q_i/\tilde{C}=(\delta C/\delta q_i).(q_i/C), \qquad i=1 \text{ to } N. \tag{40}$$

Then, if the marginal variations of managerial labor are taken around the solution of Problem I, considering relations (30) and (31), it is possible to write:

$$dC/C^*=\sum_{r=1}^{R-1}\sum_{s=1}^{S(r)}\left(\sum_{i=1}^{N} z_i^*.\beta_{rs}^i\right).(dx_{rs}/x_{rs}^*), \tag{41}$$

so that, the term $\left(\sum_{i=1}^{N} z_i^*.\beta_{rs}^i\right)$ represents the elasticity of managerial cost with respect to the level of the managerial labor x_{rs} of division (r, s) when the organization of management is efficient.

Relation (25) can be written more explicitly as:

$$\xi_{rs}^*=\sum_{i=1}^{N} z_i^*.\beta_{rs}^i. \tag{42}$$

Finally, considering relations (41) and (42), it is possible to conclude that, the managerial costs associated with an efficient program of production $(q_1^*, q_2^*, \ldots, q_N^*)$ of cost C^* are such that the share of the cost of each division is equal to the elasticity of cost with respect to the level of managerial labor of the division:

$$w_{rs} \,.\, x_{rs}^{*}/C^{*} = e_{C x_{rs}\,[x] = [x^{*}]_{r=1 \text{ to } R}^{s=1 \text{ to } S(r)}} \tag{43}$$

Analysis of the dual objective function

In relation (21), the dual objective function is split in two terms:

$$I = H + G$$

with:

$$H = -\sum_{r=1}^{R-1} \sum_{s=1}^{S(r)} \xi_{rs} \operatorname{Log}(\xi_{rs}/\omega_{rs}) \tag{44}$$

and:

$$G = \sum_{i=1}^{N} z_i \,.\, \operatorname{Log} Z_i \tag{45}$$

and the relation of duality (28) can be rewritten as:

$$C^{*} = W \,.\, e^{H([\xi^{*}])} \,.\, \prod_{i=1}^{N} Z_i^{z_i} \tag{46}$$

or, taking into account relation (20).

$$C^{*} = \mathscr{R} \,.\, (\mathrm{W} \,.\, e^{H([\xi^{*}])}) \,.\, \left(\prod_{i=1}^{N} q_i^{*z_i^{*}} \right), \tag{47}$$

where:

$$\mathscr{R} = \mathscr{R}(\underline{x}_0, \underline{K}, K_I, \underline{z}^{*}) \tag{48}$$

This last expression displays the multiplicative effect of wages and levels of production on the cost of production.

The program of production ($q_1^{*}, q_2^{*}, \ldots, q_N^{*}$) of cost C^{*} being supposed efficient, relation (11) holds, and we have:

$$Z_i \,.\, \prod_{r=1}^{R-1} \prod_{s=1}^{S(r)} x_{rs}^{*\,-\beta_{rs}^{*}} = 1, \qquad i = 1 \text{ to } N \tag{49}$$

so that:

$$e^{G} = \prod_{r=1}^{R-1} \prod_{s=1}^{S(r)} x_{rs}^{*\left(\sum_{i=1}^{N} \beta_{rs}^{i} r_i^*\right)} \tag{50}$$

or:

$$e^{G^*} = \prod_{r=1}^{R-1} \prod_{s=1}^{S(r)} x_{rs}^{*\beta_{rs}}. \tag{51}$$

Defining the "average wage of management" (different from the mean wage) as:

$$w = W/M, \tag{52}$$

relation (24) implies that:

$$w = \prod_{r=1}^{R-1} \prod^{S(r)} w^{\xi_{rs}^*}. \tag{53}$$

Now, the cost C^* can be written as:

$$C^* = M . \tilde{C}^* . e^{H(\xi^*)} \tag{54}$$

with:

$$\tilde{C}^* = \prod_{r=1}^{R-1} \prod_{s=1}^{S(r)} (w . x_{rs}^*)^{\xi_{rs}^*}. \tag{55}$$

These last relations lead to the conclusion that the cost associated with an efficient program of production can be considered as the product of three terms:

— M the number of managerial divisions (less the presidential division);

— $\tilde{C}^*$ a geometric mean of the salaries paid in the managerial divisions with a wage equal to the "average wage of management";

— $e^{H([\xi^*])}$ where $H([\xi^*])$ is a conditional entropy of managerial costs with respect to the repartition of wages between the managerial divisions:

From relation (19) we have:

$$\omega_{rs} \geqq 0, \qquad s = 1 \text{ to } S(r), \qquad r = 1 \text{ to } R-1 \tag{56}$$

and:

$$\sum_{r=1}^{R-1} \sum_{s=1}^{S(r)} \omega_{rs} = 1, \tag{57}$$

ω_{rs} can be considered as the *a priori* probability to spend one unit of the money allocated to the managerial costs, in the (r, s) division (it is supposed that all the managerial divisions are effectively involved in the process of production, that the wages of the managerial divisions are known and that nothing is known about the levels of production of the different outputs). ξ_{rs} can be considered as the probability of the same event, when the program of production is known.

V. OPTIMAL ASSIGNMENT OF MANAGERIAL COSTS IN THE SHORT RUN-THE SINGLE OUTPUT CASE

General considerations

This case is also analysed using the Duality Theory of Geometric Programming, although it could be treated *via* the Lagrange Duality Theory, since this tool allows to deal directly with the management costs of the firm and not with levels of managerial labor which are a consequence of them in some way. Thus, here:

$$N=1 \quad \text{and} \quad D=0. \tag{58}$$

The restrictions (22) and (24) of Problem II are unchanged while (23) and (25) become (dropping index i):

$$z^* \geqq 0 \tag{59}$$

and:

$$\xi_{rs}^* = \beta_{rs} . z^*, \qquad s=1 \text{ to } S(r), \qquad r=1 \text{ to } R-1. \tag{60}$$

Solving the system of equations (24)-(25), we get easily:

$$z^* = 1/\beta \tag{61}$$

and:

$$\xi_{rs}^* = \beta_{rs}/\beta, \qquad s=1 \text{ to } S(r), \qquad r=1 \text{ to } R-1, \tag{62}$$

with:

$$\beta = \sum_{r=1}^{R-1} \sum_{s=1}^{S(r)} \beta_{rs}. \tag{63}$$

The elasticity of the cost of management with respect to the level of output is a constant, as well as the shares of managerial cost allocated to each division: They depend neither of the level q of the output, nor of the wages w_{rs} of the managerial divisions.

The cost of management becomes:

$$C^* = R \, . \, W \, . \, e^H \, . \, \mathrm{q}^{1/\beta}, \tag{64}$$

where the conditional entropy H is now a constant characterising the organization of the management of the firm by providing a measure of the distorsion existing between the share of managerial cost and the relative wage of the managerial divisions:

$$H = - \sum_{r=1}^{R-1} \sum_{s=1}^{S(r)} (\beta_{rs}/\beta) \operatorname{Log}((\beta_{rs}/\beta)/\omega_{rs}). \tag{65}$$

Special case

It is interesting to consider what happens when the wages are chosen such that:

$$\omega_{rs} = \beta_{rs}/\beta, \qquad s = 1 \text{ to } S(r), \qquad r = 1 \text{ to } R-1. \tag{66}$$

From relation (65):

$$H([\xi^*]) = 0 \tag{67}$$

and:

$$x_{rs}^* = Z^{1/\beta} \tag{68}$$

so that:

$$C^* = W \, . \, Z^{1/\beta} \tag{69}$$

or:

$$C^* = \mathscr{R} \, . \, W \, . \, q^{1/\beta}. \tag{70}$$

Thus, when the relative wages of the managerial divisions are equal to their relative elasticity of production, the number of managers in each division is constant.

If the level of operative labor in the short run is such that:

$$q=\Phi(\underline{Q}).x_0^a.k^b.k_I^c, \tag{71}$$

then:

$$x_{rs}^*=1, \qquad s=1 \text{ to } S(r), \qquad r=1 \text{ to } R-1 \tag{72}$$

and:

$$C^*=W. \tag{73}$$

Then, the cost of managerial labor (as well as the cost of capital for processing informations) can be considered as a fixed cost (independant of the level of output).

This is exactly what happens in the classical theory of the firm where managerial labor is transparent in the process of production. Nevertheless, here, the level of operative labor will not be taken in accordance with relation (71) but will be determined simultaneously with the levels of managerial labor as a solution of a problem of minimization of the total cost of production in the short run (*see* section III). This problem will be investigated in section VI.

With respect to relation (66), it may be worth to note that, since the values β_{rs} come from multiplicative as well as additive effects in the organization (*see* figures 1 and 2), the ω_{rs} are not necessarily decreasing with r. So that, the intuitive feeling that the relative wages are increasing with the corresponding level in the organization, is not damaged by this assumption. Since other means of payment are available for the managers (such as participation to the benefits), even if the wages are chosen according to relation (66) in a decreasing way, the income supplied by the firm to the managers may be an increasing function of their level in the hierarchy.

VI. SUBSTITUTION EFFECTS BETWEEN MANAGERIAL AND OPERATIVE LABOR

In this section managerial costs and operative costs are taken into account simultaneously. The problem of finding an efficient program of production in the short run can be formulated as (Problem III):

$$\min_{[x],\, x_0} C([x], \underline{x_0})$$

with:

$$C([x],\ \underline{x_0})=\sum_{r=1}^{R-1}\ \sum_{s=1}^{S(r)} w_{rs}x_{rs}+\sum_{i=1}^{M} v_i x_{0i} \tag{74}$$

and:

$$q_i([x],\ x_{0i})\geqq q_i^*, \qquad i=1 \text{ to } M, \tag{75}$$

$$x_{rs}>0, \qquad s=1 \text{ to } S(r), \qquad r=1 \text{ to } R-1, \tag{76}$$

$$x_{0i}>0, \qquad i=1 \text{ to } M, \tag{77}$$

where v_i is the average wage in the i-th processing unit.

Although the determination of the levels of operative and managerial labor through the direct use of geometric programming like in Section III leads to a $M-1$ degree primal geometric program, in this section we make use of a different strategy of resolution, with the objective of preserving the previous results relative to the internal aspects of the managerial organization. This strategy is based on the observation that Problem III is a separable problem since constraints (75) can be rewritten as:

$$-\alpha_i.\operatorname{Log}(x_{0i})-\left(\sum_{r=1}^{R-1}\ \sum_{s=1}^{S(r)} \beta_{rs} \operatorname{Log} x_{rs}\right) \leqq \operatorname{Log}(q_i^*/\tilde{Z}_i), \qquad i=1 \text{ to } M, \tag{78}$$

where:

$$\tilde{Z}_i=Z_i.x_{0i}^{\alpha i}, \tag{79}$$

so that the property shown in Annex I can be used to analyse the single output case.

Here Problem III is thus replaced by:

$$\min_{x_0} v.x_0+e^H.w.(q/\tilde{Z})^{1/\beta}.x_0^{-\alpha/\beta}, \tag{80}$$

with $x_0>0$.

Defining C_M:

$$C_M=e^H.W.(q/\tilde{Z})^{1/\beta}.x_0^{-\alpha/\beta}, \tag{81}$$

as the cost of management for levels of output q and of operative labor x_0.

Problem (80) is convex and its first order conditions of optimality are:

$$v - C_M . (\alpha/\beta)/x_0^* = 0 \quad (82)$$

or:

$$\frac{v . x_0^*}{C_M} = \alpha/\beta. \quad (83)$$

So that, at solution, the proportion of operative costs over managerial costs is equal to the proportion of their production elasticities.

If no substitution is permitted and if the wage of the workers is modified, to maintain optimality (a minimum cost of production), a simple rule can be to choose x_0 such that:

$$d(v . x_0) = 0. \quad (84)$$

If substitutions are allowed, x_0 must be chosen such that:

$$d(v . x_0/C_M(x_0)) = 0. \quad (85)$$

Relation (84) implies that x_0 must be modified of a quantity:

$$dx_0^1 = -(x_0/v) . dv. \quad (86)$$

While relation (85) implies that x_0 must be modified of a quantity:

$$dx_0^2 = -(x_0/(v(1+\alpha/\beta)) . dv. \quad (87)$$

Since, $|dx_0^2| < |dx_0^1|$, the substitution of managerial labor to operative labor leads to a greater stability of the level of the later. Considering relation (80), it can be seen that if the wages of the managerial divisions are modified uniformly by a factor k, since H is unchanged, the level of the managerial labor will be modified by a factor $k^{(\beta/(\alpha+\beta))}$.

VII. CONCLUSION

The classical theory of the firm has been developped long before the "Computer Revolution" and although the information processing activities of the managerial organization have always been present, the facilities offered by Electronic Data Processing [7] have been such an important factor of reorganization of production in the medium and large firms, that a theory of the firm considering explicitly its information processing activities was needed.

The purpose of this study has been to make an attempt to provide such a theory: A simplistic model connecting the information and material processing activities of the firm has been proposed using Cobb-Douglas functions where the capital allocated to information processing has been taken as a distinct ressource. In this paper the short planning period has been analysed, in [9] this approach is extented to consider the medium planning period problem.

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ANNEX I

A RESULT IN SEPARABLE PROGRAMMING

Consider the following problem:

$$\min_{\underline{x}\,\underline{y}} D(\underline{x}) + F(\underline{y}) \qquad (A.1)$$

with:

$$(\underline{x}, \underline{y}) \in S,$$

where D and F are definite convex functions over $\mathbb{R}^p$ and $\mathbb{R}^q$ respectively and where S is the feasible set.

This problem can be rewritten as [6]:

$$\min_{\underline{x}} \{ D(\underline{x}) + \min_{\underline{y}} F(\underline{y}) \} \tag{A.2}$$

with:

$$\underline{x} \in E, \qquad \underline{y} \in S_x,$$

where:

$$S_x = \{ \underline{x} \mid (\underline{x}, \underline{y}) \in S \} \tag{A.3}$$

and:

$$E = \{ \underline{x} \mid (\underline{x}, \underline{y}(x)) \in S \text{ and } \underline{y}(\underline{x}) \text{ is solution of the inner problem in } \underline{y} \}. \tag{A.4}$$

If the inner problem in $\underline{y}$ is a primal geometric program whose degree of difficulty is zero, the performance of the inner problem at solution is an analytical function $C(\underline{x})$ given by the expression of the criterion of the geometric dual of the inner primal problem. In this case the outer problem becomes:

$$\min_{\underline{x}} D(\underline{x}) + C(\underline{x}) \tag{A.5}$$

with:

$$\underline{x} \in E.$$

Thus, the separation and subsequent geometric dualization of the inner primal program leads to the resolution of a non linear outer problem. No iteration between the inner and outer problems is needed, contrarily to what happens with general separable problems.