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A SUMMARY OF IMPERFECT PREVENTIVE MAINTENANCE POLICIES WITH MINIMAL REPAIR (*)

by T. Nakagawa ([1])

Abstract. — *This paper considers three imperfect preventive maintenance (pm) models of a one-unit system with minimal repair at failures* : (i) *a unit after pm has the same failure rate as before pm with a certain probability.* (ii) *the age becomes younger by pm*; (iii) the age *and the failure rate are reduced in proportion to the pm cost. We obtain expected costs for each model and discuss optimum policies. An example is finally given.*

Résumé. — *On considère dans cet article trois modèles de maintenance préventive (mp) imparfaite d'un système à une unité avec réparation minimale aux défaillances* : (i) *une unité, après mp, a le même taux de défaillance qu'avant mp, avec une certaine probabilité*; (ii) *l'âge diminue par mp*; (iii) *l'âge et le taux de défaillance diminuent en fonction du coût de mp. Nous obtenons pour chaque modèle une espérance du coût et discutons les politiques optimales. Un exemple termine l'article.*

1. INTRODUCTION

Preventive maintenance (pm) problems have been discussed by many authors: Barlow and Hunter [1] have considered two types of pm policies. One policy is that an operating unit is maintained preventively at times $kT(k = 1, 2, \ldots)$ and a failed unit between periodic pm's undergoes a minimal repair. The policy is commonly used complex systems such as a computer and an airplane. Holland and McLean [4] have given a practical procedure for the policy to large motors and small electricl parts.

Earlier results of optimum pm policies have been summarızed in [5]. However, almost all models have assumed that a unit is as good as new after any pm. In practice this assumption is often not true: A unit after pm usually might be younger by pm, and occasionally, it might be worce than before pm because of faulty procedures. The models such that pm, inspection, test, and detection of failure are imperfect, have been treated in [3, 6, 7].

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This paper considers the following three imperfect pm models with minimal repair at failures:

(i) a unit after pm has the same failure rate as before pm or is as good as new with certain probabilities;

(ii) the age of a unit becomes x units of time younger by each pm;

(iii) the age and the failure rate of a unit are reduced to the original ones at the begining of all pm in proportion to the pm cost.

For each model, we obtain expected costs per unit of time and discuss optimum policies. An example is finally given when the failure time has a Weibull distribution.

2. IMPERFECT PREVENTIVE MAINTENANCE

Consider the periodic pm policy for a one-unit system which should operate for an infinite time horizon:

(i) the unit begins to operate at time 0. It has the failure time distribution $F(t)$ and the failure rate $r(t)$, i. e., $r(t) \equiv f(t)/[1-F(t)]$, where f is a density of F;

(ii) the failure rate $r(t)$ is monotonely increasing;

(iii) the unit is maintained preventively at times $kT (k = 1, 2, \ldots)$, where $T > 0$:

(iv) the unit undergoes only minimal repair at failures between pm's and the failure rate remains undisturbed by minimal repair;

(v) the repair and pm times are negligible;

(vi) a cost c_1 is suffered for each pm and c_2 is suffered for each minimal repair.

Then, we consider the three imperfect pm models with the above assumptions.

1. Model A

Suppose that the unit after pm has the same failure rate as it has been before pm with probability p $(0 \leqq p < 1)$ and is as good as new with probability $\bar{p}$ $(\equiv 1-p)$. Then, the expected total cost from $t = 0$ to the time that the unit is as good as new by perfect pm is

$$\sum_{j=1}^{\infty} \bar{p}p^{j-1}\left(jc_1 + c_2\int_0^{jT} r(t)\,dt\right), \tag{1}$$

and its mean time duration is

$$\sum_{j=1}^{\infty} \bar{p}p^{j-1}(jT). \tag{2}$$

Thus, dividing (1) by (2) and arranging them, we have the expected cost per unit of time

$$C_1(T;\, p) = \left[c_1 + c_2(\bar{p})^2 \sum_{j=1}^{\infty} p^{j-1} \int_0^{jT} r(t)\, dt\right] \Big/ T. \tag{3}$$

We seek an optimum pm time T^* which minimizes the expected cost $C_1(T;p)$. Differentiating $C_1(T;p)$ with respect T and setting it equal to zero imply

$$\sum_{j=1}^{\infty} p^{j-1} \int_0^{jT} t\, dr(t) = c_1/[c_2(\bar{p})^2]. \tag{4}$$

It is easily seen that the left hand side of (4) is monotonely increasing from the assumption (ii). Thus, if $\int_0^{\infty} t\, dr(t) > c_1/[c_2(\bar{p})^2]$ then there exists a finite and unique T^* which satisfies (4), and the resulting expected cost is

$$C_1(T^*;\, p) = c_2(\bar{p})^2 \sum_{j=1}^{\infty} p^{j-1} jr(jT). \tag{5}$$

2. Model B

Suppose that the age of the unit becomes x units of time younger by each pm, where x $(0 \leqq x \leqq T)$ is constant and previously specified. When $x = T$, the unit after pm is as good as new by perfect pm, and when $x = 0$, it has the same age as before pm. Further, suppose that the unit is replaced if it operates for the time interval NT, where N is a positive integer, and a cost c_3 is suffered for the planned replacement of the unit at time NT and is greater than the pm cost c_1. Then, the expected cost per unit of time is

$$C_2(T,\, N;\, x) = \left[(N-1)c_1 + c_2 \sum_{j=0}^{N-1} \int_{j(T-x)}^{T+j(T-x)} r(t)\, dt + c_3\right] \Big/ (NT), \tag{6}$$

which is decreasing in x from the assumption (ii). Thus, we have the inequalities

$$C_2(T,\, N;\, 0) \geqq C_2(T,\, N;\, x) \geqq C_2(T,\, N;\, T). \tag{7}$$

First, suppose that N is constant and T is a variable on $(0, \infty)$. A necessary condition that a finite T^* minimizes $C_2(T, N; x)$ is that it satisfies

$$\sum_{j=0}^{N-1} \int_{j(T-x)}^{T+j(T-x)} t\, dr(t) = [(N-1)c_1 + c_3]/c_2. \tag{8}$$

Next, suppose that T is constant. Further, $C_2(T, 0; x) = \infty$ formally for simplicity of analysis. Then, a necessary condition that there exists a finite and unique N^* minimizing $C_2(T, N; x)$ is that N^* satisfies

$$C_2(T, N+1; x) \geqq C_2(T, N; x)$$

and

$$C_2(T, N; x) < C_2(T, N-1; x) \qquad (N = 1, 2, \ldots).$$

Thus, from these inequalities, we have, respectively,

$$\begin{gathered} L(N) \geqq (c_3 - c_1)/c_2 \qquad \text{and} \qquad L(N-1) < (c_3 - c_1)/c_2 \\ (N = 1, 2, \ldots), \end{gathered} \tag{9}$$

where

$$L(N) \equiv \begin{cases} N \displaystyle\int_{N(T-x)}^{T+N(T-x)} r(t)\, dt - \sum_{j=0}^{N-1} \int_{j(T-x)}^{T+j(T-x)} r(t)\, dt \\ \qquad (N = 1, 2, \ldots), \\ 0 \qquad (N = 0). \end{cases} \tag{10}$$

Further, we have

$$L(N+1) - L(N) = (N+1) \left[\int_{(N+1)(T-x)}^{T+(N+1)(T-x)} r(t)\, dt - \int_{N(T-x)}^{T+N(T-x)} r(t)\, dt \right] \geqq 0. \tag{11}$$

Thus, an optimum number N^* of pm cycles is given by the minimum value such that $L(N) \geqq (c_3 - c_1)/c_2$ if $L(\infty) \geqq (c_3 - c_1)/c_2$, and otherwise, we make no replacement.

3. Model C

Model B has assumed that the age x reduced by pm is independent of the pm cost c_1. In this model, we suppose that the age and the failure rate of the unit after pm are reduced in proportion to the pm cost c_1.

First, suppose that the age of the unit after pm reduces to $[1-(c_1/c_0)](y+T)$ by each pm when it was $y+T$ immediately before pm, where c_0 $(c_0 \geqq c_1)$ is the initial cost of the unit. If the operation of the unit enters into the steady-state then we have the equation

$$[1-(c_1/c_0)](y+T)=y,$$

i. e.,

$$y=[(c_0/c_1)-1]T. \tag{12}$$

Thus, the expected cost per unit of time is

$$C_3(T)=\left[c_1+c_2\int_0^T r(t+y)\,dt\right]\Big/T=\left[c_1+c_2\int_{[(c_0/c_1)-1]T}^{(c_0/c_1)T} r(t)\,dt\right]\Big/T. \tag{13}$$

Differentiating $C_3(T)$ with respect to T and setting it equal to zero, we have

$$\int_{[(c_0/c_1)-1]T}^{(c_0/c_1)T} t\,dr(t)=c_1/c_2. \tag{14}$$

Next, suppose that the failure rate of the unit after pm reduces to $[1-(c_1/c_0)]\,r(y+T)$ by each pm when it was $y+T$ before pm. In the steady-state, we have

$$[1-(c_1/c_0)]\,r(y+T)=r(y). \tag{15}$$

The expected cost per unit of time is

$$C_3(T;\,y)=\left[c_1+c_2\int_0^T r(t+y)\,dt\right]\Big/T. \tag{16}$$

Thus, the age y of the unit after pm is computed from (15), and hence, an optimum pm time T^* is obtained by substituting y into (16) and changing T to minimize it.

3. CONCLUSIONS AND EXAMPLE

We have considered the three imperfect pm models and have obtained the expected costs per unit of time for each model. It is noted that all models are identical and agree with ([2], p. 97) when $p=0$ in Model A, $N=1$ and $x=T$ in Model B, and $c_0=c_1$ in Model C.

Further, we consider a modified model of Model B such that the age of the unit after pm reduces to αt $(0 \leqq \alpha \leqq 1)$ when it was t before pm, i. e., the age becomes $t(1-\alpha)$ units of time younger by pm. In this case, we could make similar discussions to ones of Model B.

It is very difficult to make discussions about optimum pm times of Models B and C. We finally consider an example when the failure time has a Weibull distribution and show how to get optimum pm times.

Suppose that the failure time distribution is a Weibull with a shape parameter α, i. e., $F(t) = 1-\exp(-\lambda t^{\alpha})$ ($\lambda > 0$, $\alpha > 1$). Then, the failure rate is $r(t) = \lambda\alpha t^{\alpha-1}$, which is monotonely increasing, taking the values from 0 to ∞. Thus, we have the following results for each model.

1. Model A

The expected cost per unit of time is, from (3),

$$C_1(T;\, p) = [c_1 + c_2 \bar{p}\lambda T^{\alpha} g(\alpha)]/T, \tag{17}$$

where $g(\alpha) \equiv \bar{p}\sum_{j=1}^{\infty} p^{j-1} j^{\alpha}$ which represents the α-th moment of the geometric distribution with parameter p. The optimum pm time is, from (4),

$$T^* = \{c_1/[c_2\bar{p}\lambda(\alpha-1)g(\alpha)]\}^{1/\alpha}. \tag{18}$$

2. Model B

The expected cost per unit of time is, from (6),

$$C_2(T, N;\, x) = \left((N-1)c_1 + c_3 + c_2\lambda\sum_{j=0}^{N-1} \{[T+j(T-x)]^{\alpha} - [j(T-x)]^{\alpha}\}\right)\Big/(NT). \tag{19}$$

From (8),

$$\sum_{j=0}^{N-1} \{[T+j(T-x)]^{\alpha} - [j(T-x)]^{\alpha}\} = [(N-1)c_1 + c_3]/[c_2\lambda(\alpha-1)], \tag{20}$$

which is monotonely increasing in T, taking the values from 0 to ∞. Thus, the optimum pm time T^* exists uniquely, which satisfies (20). Further, the left hand side of (20) is decreasing in x for a fixed T, and hence, the optimum pm time T^* is an increasing function of x. Thus, putting $x = 0$ and $x = T$ in (20), we have the lower and upper limits:

$$\frac{1}{N}\left[\frac{(N-1)c_1 + c_3}{c_2\lambda(\alpha-1)}\right]^{1/\alpha} \leqq T^* \leqq \left[\frac{1}{N}\,\frac{(N-1)c_1 + c_3}{c_2\lambda(\alpha-1)}\right]^{1/\alpha} \tag{21}$$

3. Model C

Suppose that the age of the unit is reduced by pm. Then, the expected cost per unit of time is, from (13),

$$C_3(T) = (c_1 + c_2 \lambda \{ (c_0/c_1)^\alpha - [(c_0/c_1) - 1]^\alpha \} T^\alpha)/T, \tag{22}$$

and the optimum pm time is, from (14),

$$T^* = (c_1 / \{ c_2 \lambda (\alpha - 1) [(c_0/c_1)^\alpha - ((c_0/c_1) - 1)^\alpha] \})^{1/\alpha} \tag{23}$$

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