Revue française d'automatique, informatique, recherche opérationnelle. Recherche opérationnelle

A. K. Govil

Priority effect on pointwise availability of the system

Revue française d'automatique, informatique, recherche opérationnelle. Recherche opérationnelle, tome 6, nº V1 (1972), p. 47-56

<http://www.numdam.org/item?id=RO_1972__6_1_47_0>

R.A.I.R.O.
(6e année, N° V-1, 1972, p. 47-56)

PRIORITY EFFECT ON POINTWISE AVAILABILITY OF THE SYSTEM

by A. K. GOVIL (1)

Abstract. — *In this paper, the effect of different repair priorities on the pointwise availability of the system has been investigated. In the end, a numerical example has been given in which pointwise availability is maximum in the case of Preemptive Resume Repair Discipline.*

INTRODUCTION

In the smooth running of an organisation, it is always desirable that the equipments or systems should be maintained properly. This will require to devise ways and means so as to increase the system's availability. One of the methods is to assign priorities depending upon the essentiality of the component in system's operation.

Keeping this in view, the behaviour of a system comprised of two types (denoted hereafter as type I and II), of components has been considered. Failure of type I, results in complete breakdown of the system whereas failure of type II, causes the system to work in reduced efficiency state. Three repair disciplines viz., Head-of-Line, Preemptive Resume, and Preemptive Repeat, have been imposed on the repair of these components. Further, it has been assumed that failures and repairs follow exponential and general distributions, respectively for both types of components. Use of Laplace Transforms and Supplementary variable technique have been made to arrive at the solution. In the end, a numerical example has been worked-out to illustrate the effect of above-mentioned repair disciplines on the pointwise availability of the system.

(1) Directorate of Scientific Evaluation, Research and Development Organisation, Ministry of Defence, New-Delhi-1.

NOTATIONS

λ & λ' = constant failure rates of the components of type I and II, respectively.

$S(x)$ = the general probability density for repair of the component of type I.

$S'(x)$ = the general probability density for repair of the component of type II.

$\eta(x)\,\Delta$ = the first order probability that the component of type I is being repaired in the time interval $(x, x + \Delta)$, conditioned that it was not repaired upto time x.

$\varphi(x)\,\Delta$ = the first order probability that the component of type II is being repaired in the time interval $(x, x + \Delta)$, conditioned that it was not repaired upto time x.

The stochastic models under different priority repair disciplines are discussed below :

(I.0) Head-of-Line Repair Discipline

The repair discipline is first come, first serve, which means that a failed component from either type i.e., type I or II, if taken-up for repair, will be repaired first and only after this, other failed components will be taken-up.

Define

$P_0(t)$ = the probability that at time t, the system is operating in normal efficiency.

$P_R(x, t)\,\Delta$ = the probability that at time t, the system is operating in reduced efficiency state due to the failure of component of type II and the elapsed repair time lies in the interval $(x, x + \Delta)$.

$P_f(x, t)\,\Delta$ = the probability that at time t, the system is in the failed state due to the failure of the component of type I, and the elapsed repair time lies in the interval $(x, x + \Delta)$.

$Q_f(x, t)\,\Delta$ = the probability that at time t, the system is in the failed state due to the failure of type I and the elapsed repair time of type II lies in the interval $(x, x + \Delta)$, whereas type I is awaiting repair.

From elementary probability considerations, we get the following forward equations governing the behaviour of the system :

$$P_0(t + \Delta) = P_0(t)[1 - \lambda\Delta][1 - \lambda'\Delta] + \int_0^\infty P_R(x, t)\varphi(x)\,dx + \int_0^\infty P_f(x, t)\eta(x)\,dx. \qquad \text{(I.1)}$$

$$P_R(x + \Delta, t + \Delta) = P_R(x, t)[1 - \lambda\Delta][1 - \varphi(x)\Delta] \tag{I.2}$$

$$P_f(x + \Delta, t + \Delta) = P_f(x, t)[1 - \eta(x)\Delta] \tag{I.3}$$

$$Q_f(x + \Delta, t + \Delta) = Q_f(x, t)[1 - \varphi(x)\Delta] + \lambda P_R(x, t)\Delta \tag{I.4}$$

Equations (I.1) through (I.4) are to be solved under the following boundary conditions :

$$P_f(0, t) = \lambda P_0(t) + \int_0^\infty Q_f(x, t)\varphi(x)\,dx \tag{I.5}$$

$$P_R(0, t) = \lambda' P_0(t) \tag{I.6}$$

$$Q_f(0, t) = 0 \tag{I.7}$$

Equations (I.1) through (I.4) become, when $\Delta \to 0$

$$\frac{\partial P_0(t)}{\partial t} + (\lambda + \lambda')P_0(t) = \int_0^\infty P_R(x, t)\varphi(x)\,dx + \int_0^\infty P_f(x, t)\eta(x)\,dx \tag{I.8}$$

$$\frac{\partial P_R(x, t)}{\partial t} + \frac{\partial P_R(x, t)}{\partial x} + [\lambda + \varphi(x)]P_R(x, t) = 0 \tag{I.9}$$

$$\frac{\partial P_f(x, t)}{\partial t} + \frac{\partial P_f(x, t)}{\partial x} + \eta(x)P_f(x, t) = 0 \tag{I.10}$$

$$\frac{\partial Q_f(x, t)}{\partial t} + \frac{\partial Q_f(x, t)}{\partial x} + \varphi(x)Q_f(x, t) = \lambda P_R(x, t) \tag{I.11}$$

Assume that initially the system is working in normal efficiency i.e., $P_0(0) = 1$, so that other state probabilities are zero.

Let $\bar{f}(s)$ be the Laplace Transform of the function $f(t)$, denoted by

$$\bar{f}(s) = \int_0^\infty \bar{e}^{st} f(t)\,dt \quad ; \quad R_e(s) > 0$$

Taking Laplace Transforms of equations (I.8) through (I.11) and (I.5) through (I.7) and using initial conditions, we get

$$[s + \lambda + \lambda']\bar{P}_0(s) = 1 + \int_0^\infty P_R(x, s)\varphi(x)\,dx + \int_0^\infty P_f(x, s)\eta(x)\,dx \tag{I.12}$$

$$\left[\frac{\partial}{\partial x} + s + \lambda + \varphi(x)\right]\bar{P}_R(x, s) = 0 \tag{I.13}$$

$$\left[\frac{\partial}{\partial x} + s + \eta(x)\right] \bar{P}_f(x, s) = 0 \tag{I.14}$$

$$\left[\frac{\partial}{\partial x} + s + \varphi(x)\right] \bar{Q}_f(x, s) = \lambda \bar{P}_R(x, s) \tag{I.15}$$

$$\bar{P}_f(0, s) = \lambda \bar{P}_0(s) + \int_0^\infty \bar{Q}_f(x, s)\varphi(x)\, \mathrm{d}x \tag{I.16}$$

$$\bar{P}_R(0, s) = \lambda' \bar{P}_0(s) \tag{I.17}$$

$$\bar{Q}_f(0, s) = 0 \tag{I.18}$$

Integrating (I.13) and (I.14) by parts and simplyfying, we obtain

$$\bar{P}_R(x, s) = \bar{P}_R(0, s) \exp\left[- (s + \lambda)x - \int_0^x \varphi(x)\, \mathrm{d}x\right] \tag{I.19}$$

$$\bar{P}_f(x, s) = \bar{P}_f(0, s) \exp\left[- sx - \int_0^x \eta(x)\, \mathrm{d}x\right] \tag{I.20}$$

Using relation (I.19) in (I.15), we get

$$\bar{Q}_f(x, s) = \lambda' \bar{P}_0(s)\left[1 - \bar{e}^{\lambda x}\right]\left[\bar{e}^{sx - \int_0^x \varphi(x)\, \mathrm{d}x}\right] \tag{I.21}$$

Again, making use of relation (I.21) in (I.16), we obtain

$$\bar{P}_f(0, s) = \bar{P}_0(s)[\lambda + \lambda' \{ \bar{S}'(s) - \bar{S}'(s + \lambda) \} \tag{I.22}$$

where $\bar{S}'(s)$ is the Laplace Transforms of $\bar{S}'(x)$ and $\bar{S}'(x)$ is connected by the relation

$$S'(x) = \varphi(x) \exp\left[- \int_0^x \varphi(x)\, \mathrm{d}x\right]$$

similarly,

$$S(x) = \eta(x) \exp\left[- \int_0^x \eta(x)\, \mathrm{d}x\right]$$

Using relations (I.17), (I.19), (I.20) and (I.22) in relation (I.12), we get

$$\bar{P}_0(s) = \frac{1}{[s + \lambda \{ 1 - \bar{S}(s) \} + \lambda' \{ 1 - \bar{S}'(s + \lambda) - \bar{S}(s)(\bar{S}'(s) - \bar{S}'(s + \lambda)) \}]} \tag{I.23}$$

Since,

$$\bar{P}_R(s) = \int_0^\infty \bar{P}_R(x, s)\, dx \tag{I.24}$$

Employing relations (I.19) and (I.23) in (I.24), we obtain

$$\bar{P}_R(s) = \frac{\lambda'[1 - \bar{S}'(s+\lambda)]}{[s+\lambda][s + \lambda\{1 - \bar{S}(s)\} + \lambda'\{1 - \bar{S}'(s+\lambda) - \bar{S}(s)(\bar{S}'(s) - \bar{S}'(s+\lambda))\}]} \tag{I.25}$$

The Laplace Transform of the probability $\bar{P}_{down}(s)$, that at time t, the system is in the failed state, is given by

$$\bar{P}_{down}(s) = \int_0^\infty \bar{P}_f(x, s)\, dx + \int_0^\infty Q_f(x, s)\, dx. \tag{I.26}$$

Using relations (I.20) through (I.23), we get

$$\bar{P}_{down}(s) = \left[\frac{\lambda\{1 - \bar{S}(s)\} + \lambda'\{1 - \bar{S}'(s+\lambda) - \bar{S}(s)(\bar{S}'(s) - \bar{S}'(s+\lambda))\}]}{s[s + \lambda\{1 - \bar{S}(s)\} + \lambda'\{1 - \bar{S}'(s+\lambda) - \bar{S}(s)(\bar{S}'(s) - \bar{S}'(s+\lambda))\}]} - \frac{\lambda'[1 - \bar{S}'(s+\lambda)]}{[s+\lambda][s + \lambda\{1 - \bar{S}(s)\} + \lambda'\{1 - \bar{S}'(s+\lambda) - \bar{S}(s)(\bar{S}'(s) - \bar{S}'(s+\lambda))\}]}\right] \tag{I.27}$$

It may be noted here that

$$\bar{P}_0(s) + \bar{P}_R(s) + \bar{P}_{down}(s) = \frac{1}{s} \tag{I.28}$$

(II.0) Preemptive Resume Repair Discipline

In this discipline, on failure the component of type I is taken under repair immediately. In this way, the component of type I preempts repair facility even when this facility was engaged in the repair of type II. However, when type II is taken back in the repair facility, the repair starts from the point it was left earlier. With this in view, in addition to the definitions given in (I.0), the following definitions are given to develope the mathematical model under this repair discipline :

$Q_F(x, y, t)\, \Delta =$ the probability that at time t, the system is in the failed state due to the failure of the component of type I and the elapsed repair time lies in the interval $(x, x + \Delta)$ and at the instant when it preempted in the repair facility, the elapsed repair time of type II was in the interval $(y, y + \Delta)$.

and $P_R(y, t)$ and $\varphi(y)$ have the same meaning as in (I.0) except that the elapsed repair time is y instead of x.

From continuity arguments, we get the following differential equations under this discipline, as

$$\left[\frac{\partial}{\partial t} + \lambda + \lambda'\right] P_0(t) = \int_0^\infty P_R(y, t)\varphi(x)\,dx + \int_0^\infty P_f(x, t)\eta(x)\,dx \quad \text{(II.1)}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \lambda + \varphi(y)\right] P_R(y, t) = \int_0^\infty Q_F(x, y, t)\eta(x)\,dx \quad \text{(II.2)}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \eta(x)\right] P_f(x, t) = 0 \quad \text{(II.3)}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \eta(x)\right] Q_F(x, y, t) = 0 \quad \text{(II.4)}$$

The above equations are subjected to the following boundary conditions :

$$P_f(0, t) = \lambda P_0(t) \quad \text{(II.5)}$$

$$P_R(0, t) = \lambda' P_0(t) \quad \text{(II.6)}$$

$$Q_F(0, y, t) = \lambda P_R(y, t) \quad \text{(II.7)}$$

Taking Laplace Transforms of equations (II.1) through (II.7), using initial conditions and solving, we get

$$\bar{P}_0(s) = \frac{1}{[s + \lambda\{1 - \bar{S}(s)\} + \lambda'\{1 - \bar{S}'(s + \lambda - \lambda\bar{S}(s))\}]} \quad \text{(II.8)}$$

$$P_R(s) = \frac{\lambda'[1 - \bar{S}'\{s + \lambda - \lambda\bar{S}(s)\}]}{[s + \lambda\{1 - \bar{S}(s)\}][s + \lambda\{1 - \bar{S}(s)\} + \lambda'\{1 - \bar{S}'(s + \lambda - \lambda\bar{S}(s))\}]} \quad \text{(II.9)}$$

and

$$\bar{P}_{down}(s) = \int_0^\infty \bar{P}_f(x, s)\,dy + \int_0^\infty \int_0^\infty \bar{Q}_F(x, y, s)\,dy\,dx$$

$$= \frac{\lambda[1 - \bar{S}(s)]}{s[s + \lambda\{1 - \bar{S}(s)\}]} \quad \text{(II.10)}$$

(III.0) Preemptive Repeat Repair Discipline

This discipline differs from the Preemptive Resume Repair Discipline, in so much that the component of type II which have been removed from the repair facility, due to priority given to the repair of the component of type I, will be taken-up again for repair without giving any consideration to the repair already carried-out on this component. In other words, the repair of the component of type II, when restarts is considered as the repair on a fresh failed component, for example consider the repair of engine crank-shafts by rematallizing process. To avoid scale formation and to make it homogeneous, this process has to be carried-out continuously. If once left, the whole process is repeated and the previous repair goes waste.

In addition to the previous definitions given in (I.0), the following definition is given to develope the mathematical model :

$R_f(x, t)\,\Delta =$ the probability that at time t, the system is in the failed state due to the failure of the component of type I and the elapsed repair time lies in the interval $(x, x + \Delta)$, at the instant when it preempted in the repair facility and type II which had been in service, is awaiting repair.

By connecting the various state probabilities of the system from continuity arguments, the following differential equations may be obtained as :

$$\left[\frac{\partial}{\partial t} + \lambda + \lambda'\right] P_0(t) = \int_0^\infty P_R(x, t)\varphi(x)\,dx + \int_0^\infty P_f(x, t)\eta(x)\,dx \quad \text{(III.1)}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda + \varphi(x)\right] P_R(x, t) = 0 \quad \text{(III.2)}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \eta(x)\right] P_f(x, t) = 0 \quad \text{(III.3)}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \eta(x)\right] R_f(x, t) = 0 \quad \text{(III.4)}$$

The above equations are solved under the following boundary conditions :

$$P_f(0, t) = \lambda P_0(t) \quad \text{(III.5)}$$

$$R_f(0, t) = \lambda \int_0^\infty P_R(x, t)\,dx \quad \text{(III.6)}$$

$$P_R(0, t) = \lambda' P_0(t) + \int_0^\infty R_f(x, t)\eta(x)\,dx. \quad \text{(III.7)}$$

Again, taking Laplace Transforms of equations (III.1) through (III.7), using initial conditions and solving them, we get

$$\bar{P}_0(s) = \frac{[s + \lambda \{ 1 - \bar{S}(s) \} + \lambda \bar{S}(s) \bar{S}'(s + \lambda)]}{[s + \lambda \{ 1 - \bar{S}(s) \}] [s + \lambda - \lambda \bar{S}(s) \{ 1 - \bar{S}'(s + \lambda) \} + \lambda' \{ 1 - \bar{S}'(s + \lambda) \}]} \tag{III.8}$$

$$\bar{P}_R(s) = \frac{\lambda'[1 - \bar{S}'(s + \lambda)]}{[s + \lambda \{ 1 - \bar{S}(s) \}] [s + \lambda - \lambda \bar{S}(s) \{ 1 - \bar{S}'(s + \lambda) \} + \lambda' \{ 1 - \bar{S}'(s + \lambda) \}]} \tag{III.9}$$

and

$$\bar{P}_{down}(s) = \int_0^\infty \bar{P}_f(x, s)\, dx + \int_0^\infty \bar{R}_f(x, s)\, dx$$

$$= \frac{\lambda[1 - \bar{S}(s)]}{s[s + \lambda \{ 1 - \bar{S}(s) \}]} \tag{III.10}$$

PARTICULAR CASES

Repairs follow exponential distribution with rates μ and μ'.

Setting $\bar{S}(s) = \dfrac{\mu}{s + \mu}$ and $\bar{S}'(s) = \dfrac{\mu'}{s + \mu'}$ in relations (I.23), (II.8) and (III.8), we get

$$\bar{P}_0(s) = \frac{(s + \lambda + \mu')}{s\left[(s + \lambda + \mu' + \lambda')\left(1 + \dfrac{\lambda}{s + \mu}\right) + \dfrac{\lambda\lambda'\mu}{(s + \mu)(s + \mu')}\right]}$$

(Head-of-Line)

(III.11)

$$\bar{P}_0(S) = \frac{\left[s\left\{1 + \dfrac{\lambda}{\mu}\right\} + \mu'\right]}{s\left[1 + \dfrac{\lambda}{s + \mu}\right]\left[s\left\{1 + \dfrac{\lambda}{s + \mu}\right\} + \lambda' + \mu'\right]}$$

(Preemptive Resume)

(III.12)

$$\bar{P}_0(s) = \frac{\left[s(s+\lambda+\mu')\left(1+\frac{\lambda}{s+\mu}\right)+\frac{\lambda\mu\mu'}{(s+\mu)}\right]}{s\left[1+\frac{\lambda}{(s+\mu)}\right]\left[s(s+\lambda+\mu')\left(1+\frac{\lambda}{s+\mu}\right)+\frac{\lambda\mu\mu'}{(s+\mu)}+\lambda'(s+\lambda)\right]}$$

(Preemptive Repeat)

(III.13)

On inverting relations (III.11) through (III.13) and putting $\lambda = 0.04$, $\lambda' = 0.06$, $\mu = 0.4$ and $\mu' = 0.6$, we get the following results which have been tabulated and graphed in table I and fig. I, respectively.

TABLE I. — *Pointwise availability for different priority repair disciplines*

TIME	HEAD-OF-LINE	PREEMPTIVE RESUME	PREEMPTIVE REPEAT
0	1	1	1
0.5	0.97	0.99	0.95
1	0.94	0.98	0.92
2	0.89	0.94	0.88
4	0.86	0.92	0.84
5	0.85	0.90	0.83
8	0.83	0.89	0.80

A comparison of pointwise availabilities for different repair disciplines [Table I and fig. I], indicates that it is maximum for preemptive Resume Repair Discipline. Therefore, it may be inferred that in some practical situations (repair of components in a service workshop), when repair of essential components is required, preemptive resume repair discipline may be adopted to carry out such repairs.

ACKNOWLEDGEMENTS

The author is grateful to D[r] S. S. Srivastava, Director, Scientific Evaluation and to D[r] R. C. Garg, Senior Scientific Officer, Grade-I, for their keen interest in the preparation of this paper.

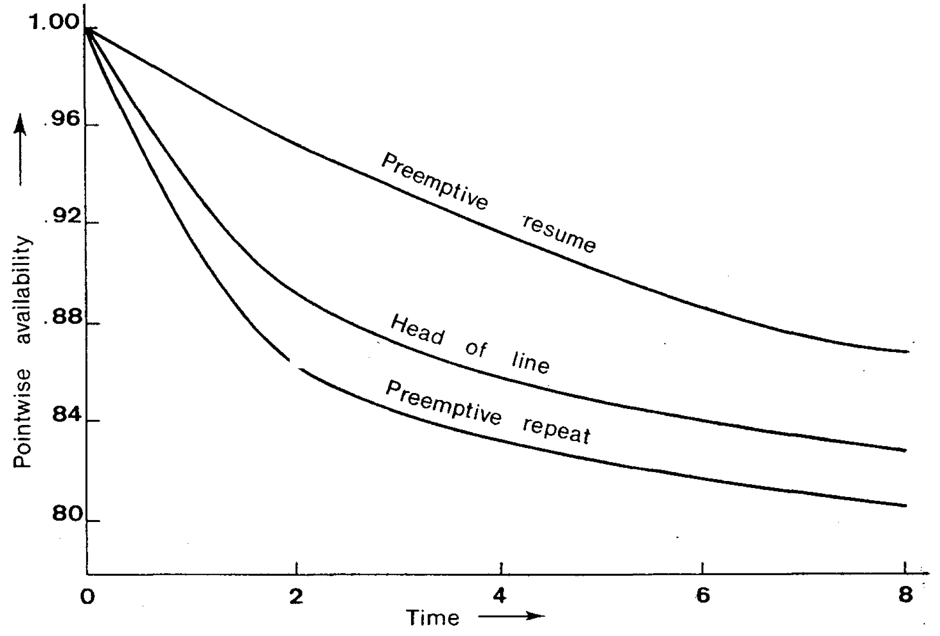

Figure 1.

REFERENCES

[1] EDWARD L. Pugh, « The best estimate of Reliability in the exponential case », *Opns. Res.*, vol. 31, n° 1, 57-61.

[2] GARG R. C., « Dependability of a complex system having two types of components », *IEEE Transactions on Reliability*, (1963).

[3] GOVIL A. K., *Dependability of a complex system with general repair distribution under preemptive priority resume*, Zamm (Germany), vol. 50, n° 8, (1970).

[4] THIRUVENGADAM K., Queuing with breakdown, *Opns. Res.* vol. 11, n° 1, 62-71 (1965).