# Revue d'Histoire des Mathématiques

## Évariste Galois and the Social Time of Mathematics

Caroline Ehrhardt

Tome 17 Fascicule 2



# SOCIÉTÉ MATHÉMATIQUE DE FRANCE

Publiée avec le concours du Centre national de la recherche scientifique

### **REVUE D'HISTOIRE DES MATHÉMATIQUES**

#### RÉDACTION

**Rédacteur en chef :** Norbert Schappacher

**Rédacteur en chef adjoint :** Philippe Nabonnand

#### Membres du Comité de rédaction :

Tom Archibald Alain Bernard Frédéric Brechenmacher Marie-José Durand-Richard Étienne Ghys Hélène Gispert Jens Høyrup Agathe Keller Laurent Mazliak Karen Parshall Jeanne Peiffer Sophie Roux Joël Sakarovitch Dominique Tournès

**Directeur de la publication :** Bernard Helffer

#### COMITÉ DE LECTURE

Philippe Abgrall June Barrow-Greene , Liliane Beaulieu Umberto Bottazzini Jean Pierre Bourguignon Aldo Brigaglia Bernard Bru Jean-Luc Chabert Francois Charette Karine Chemla Pierre Crépel François De Gandt Moritz Epple Natalia Ermolaëva Christian Gilain **Catherine Goldstein** Jeremy Gray Tinne Hoff Kjeldsen Jesper Lützen Antoni Malet Irène Passeron Christine Proust David Rowe Ken Saito S. R. Sarma Erhard Scholz Reinhard Siegmund-Schultze Stephen Stigler Bernard Vitrac

#### Secrétariat :

Nathalie Christiaën Société Mathématique de France Institut Henri Poincaré 11, rue Pierre et Marie Curie, 75231 Paris Cedex 05 Tél.: (33) 01 44 27 67 99 / Fax : (33) 01 40 46 90 96 Mél : revues@smf.ens.fr / URL : http//smf.emath.fr/

Périodicité :	La Revue publie deux fascicules par an, de 150 pages chacun environ.	
Tarifs :	Prix public Europe : 67 €; prix public hors Europe : 76 €; prix au numéro : 38 €.	
	Des conditions spéciales	sont accordées aux membres de la SMF.
Diffusion :	SMF, Maison de la SMF, Case 916 - Luminy, 13288 Marseille Cedex 9 Hindustan Book Agency, O-131, The Shopping Mall, Arjun Marg, DLF Phase 1, Gurgaon 122002, Haryana, Inde	
	AMS, P.O. Box 6248, Providence, Rhode Island 02940 USA	
© SMF Nº ISS	SN: 1262-022X	Maquette couverture : Armelle Stosskopf

Revue d'histoire des mathématiques 17 (2011), p. 175–210

#### ÉVARISTE GALOIS AND THE SOCIAL TIME OF MATHEMATICS

#### CAROLINE EHRHARDT

ABSTRACT. — The thrust of this article is to offer a new approach to the study of Galois's *Mémoire sur les conditions de résolubilité des équations par radicaux*. Drawing on methodology developed by social and cultural historians, it contextualizes Galois's work by situating it in the parisian mathematical milieu of the 1820s and 1830s. By reconstructing the social process whereby a young man became an established mathematician at the time, this article shows that Galois's trajectory was far from unusual, and most importantly, that he was not treated differently from other aspiring mathematicians.

Second this article seeks to operate a shift from the writing of biographies of mathematicians to biographies of mathematical texts. Indeed, the meaning of a mathematical text is the product of a long social and scientific process, one that, in the case of Galois's text, took over one hundred years. During this long period, Galois's text was read, interpreted and recast by a large number of actors who did not agree as to its meaning and mostly construed it through local lenses. Only at the beginning of the 20th century, when Galois theory entered the realm of teaching in European countries, did it acquire a more unified meaning. By then, Galois, the aspiring mathematicians who had failed to convince the members of the *Académie des sciences*, was becoming a legend.

RÉSUMÉ (Évariste Galois et le temps social des mathématiques)

Cet article propose une nouvelle approche de l'étude du Mémoire sur les conditions de résolubilité des equations par radicaux de Galois. En se fondant sur la méthodologie de l'histoire sociale et culturelle, il contextualise le travail de Galois en le situant dans le milieu mathématique parisien des années 1820 et 1830. Tout d'abord, en reconstruisant le processus social par lequel un jeune homme peut devenir un mathématicien reconnu à cette époque, cet article

© SOCIÉTÉ MATHÉMATIQUE DE FRANCE, 2011

Texte reçu le 10 avril 2011, accepté le 17 juin 2011.

C. EHRHARDT, IFÉ/ENS-Lyon, Service d'histoire de l'éducation, 45, rue d'Ulm, 75230 Paris Cedex 05 (France).

Courrier électronique : caroline.ehrhardt@ens-lyon.fr

<sup>2000</sup> Mathematics Subject Classification : 01A55, 01A60, 01A80; 11-03, 12-03.

montre que la trajectoire de Galois est loin d'être exceptionnelle et, surtout, qu'il n'a pas été traité différemment des autres aspirants-mathématiciens de sa géneration. Ensuite, cet article propose de passer de la biographie des mathématiciens à la biographie des textes mathématiques. De fait, le sens d'un texte mathématique est le produit d'un long processus social et scientifique, un processus qui, dans le cas de Galois, a pris plus de cent ans. Pendant cette longue période, le texte de Galois a été lu, interprété et reformulé, dans des contextes locaux, par un grand nombre d'acteurs qui ne s'accordaient pas nécessairement sur son sens. Ce n'est qu'au début du xx<sup>e</sup> siècle, alors que la théorie de Galois était en train de devenir une matière d'enseignement, qu'elle a acquis un sens plus uniforme. Mais à ce moment là Galois, l'aspirantmathématicien qui n'avait pas réussi à convaincre les membres de l'*Académie des sciences*, était déjà en train de devenir une légende.

#### 1. INTRODUCTION

Évariste Galois (1811–1832) has received extensive treatment by historians of mathematics, who have written dozens of biographies and monographs about his work, along with numerous studies of the development of 19th century algebra in which he is given an important role.<sup>1</sup> To these works one may add texts produced since the beginning of the 20th century for larger audiences, texts which have established a legend around Galois's personality.<sup>2</sup> This abundance of publications reflects the importance of Galois and his writings in mathematics and in the history of mathematics. It also invites us to explore in this paper questions of method. Some of these questions are immediately relevant to the social and cultural history of mathematics, but Galois's case actually leads straight to the more general question of how mathematical knowledge is constructed historically. This article is based on the empirical study I conducted for my doctoral dissertation, which focused on Galois's afterlife during the 19th century [Ehrhardt 2007]; detailed results of my work are about to be published; see [Ehrhardt 2011a] and [Ehrhardt 2011b].

The starting point of this study is a simple question: how did *Galois theory*, which is one of the fundamental theories in modern algebra, come

<sup>&</sup>lt;sup>1</sup> With no intention to cover one century of historiography, one may mention the biographies [Dupuy 1896], [Dalmas 1956], [Toti Rigatelli 1996], Taton's articles [Taton 1947], [Taton 1971], and [Taton 1983], the article [Infantozzi 1968], as well as the studies [Kiernan 1971], [Wussing 1984], [Hirano 1984], [Toti Rigatelli 1989], [Dahan-Dalmedico 1983], [Friedelmeyer 1991] and, more recently, [Galuzzi 2001].

<sup>&</sup>lt;sup>2</sup> Here one may cite biographical novels like [Bell 1937], [Infeld 1948], and most recently [Auffray 2004], as well as books like [Verdier 2003].

to be known as such? More precisely, why do we continue to attribute to Évariste Galois a theory which not only reaches far beyond the scope of his own writings, but which is entirely based on a mathematical machinerythe structural algebra of the 1920s and 1930s-which is completely alien to Galois. Indeed, looking at Évariste Galois's life and manuscripts, one is first struck by the fact that this mathematician, when he died at age twenty, did not leave behind more than some 60 sheets of manuscripts.<sup>3</sup> Among those, his most accomplished work, the Mémoire sur les conditions de résolubilité des équations par radicaux, is a short and cryptic text from which proofs are mostly missing and which was refused in 1831 by the Académie des sciences.<sup>4</sup> And it actually does not contain what one is used to recognize as "Galois theory." Furthermore, in Galois's case, the problem of the authorship of the theory generates another question, related to how one can properly write the history of mathematics; just as Galois theory is the product of a collective historical construction, is not Galois's personality equally the product of such a construction, due in no small part to historians and philosophers of mathematics?

The label "Galois theory" thus makes us wonder about the way in which mathematical knowledge is created and distilled over time, transforming a simple *mémoire* into a fully crystallized theory, and what this means for the alleged solidity and universality of the knowledge thus created. When we use the expression "Galois theory" both with respect to 1850, and to 1900, or 1950, we ought to be speaking of the "same" thing. But Galois's name in this expression already betrays a tension between historicity and transcendence of mathematical results—a tension which is germane to both mathematics and its historiography. Evariste Galois therefore offers an especially interesting occasion to analyze the way in which a text with a specific known origin in time and place, acquires the status of certified, transhistorical knowledge.

Looking more closely, we see that we are asking for the historical construction of what makes mathematical objects ideal. This process in general involves a great deal of everyday practices, graphic procedures (the use of curves, tables, diagrams...), symbolic manipulations (computations, combinations of objects in space), an institutional framework (teaching, places of learning and research, etc.), and also representations of research domains (what is interesting? what does a certain concept "mean"?). Such

<sup>&</sup>lt;sup>3</sup> Galois's works have recently been edited completely: [Galois 1962/1997].

<sup>4</sup> On this point, see [Ehrhardt 2010a].

a historical construction therefore takes place in social spaces of relevance for mathematical work that are themselves anything but universal.

The *Science Studies* approach however, with its focus on local studies and short term time horizons, fails to capture one of the most fundamental aspects of the constitution of mathematical knowledge: its claim to universality. Both the ways in which local mathematical practices are extended and elevated and the stakes involved in this process, remain a blind spot in the historiography. The ideotypical character that is easily granted to mathematical objects fits badly with the small scale at which they are actually produced.<sup>5</sup>

In order to retrace the combined establishment of the various mathematical theories that originated from Galois's work as well as the legend of the cursed mathematician which is today attached to his name, I offer the hypothesis that the historical character of mathematical knowledge actually lies in the way in which it is passed on. Far from being a neutral operation, this process of transmission is the very place where the historicity of mathematics lies, because it brings to the fore the successive, or intertwined, categories by which this knowledge is conceived, elaborated, and understood.<sup>6</sup> This means we need to analyze the composition and the reading of a mathematical text as resulting from the interaction between mental sets of tools, which depend on the apprenticeship of the authors and the social domains in which they move. Such an interaction takes place within a control system that is proper to the mathematical field in a given social space and time.

Galois's case further invites reflection on the multiple historical contexts into which a mathematical text is embedded and which serve to make sense of it: if Galois's *mémoire* was at first read in 1831 according to the criteria of the *Académie des sciences*, it subsequently remained a topical part of mathematical research all through the 19th century through several revivals. The passing of time did not transform this work into a "historical" mathematical text, for mathematicians renewed its intelligibility by continued usage.<sup>7</sup> In this sense today it still belongs to contemporary mathematics, as is revealed by the adjective use of the name Galois—or

<sup>&</sup>lt;sup>5</sup> Mathematics have been studied only very rarely along those lines of research. Recent works inspired by this approach—in spite of their great quality and the seminal way in which they open up new lines of research—are either locally focussed ([Warwick 2003]) or work in a short time range ([Rosental 2003]).

<sup>&</sup>lt;sup>6</sup> This point of view has been developed in [Cifoletti 1998].

 $<sup>^7</sup>$  The question of how the shelf live and the identity of mathematical objects are related to the their ongoing use is at the center of [Goldstein 1995].

the French adjective "galoisien", and similarly in other languages—to describe many notions and ideas. The *Mémoire sur les conditions de résolubilité des équations par radicaux* has thus accompanied the mathematical present for almost two centuries—but this mathematical present asks ever changing questions, from one generation to the next, because it takes different criteria and practices for granted, and because it takes place in cultural and social environments which evolve over time.

Finally, the posterity of Evariste Galois and his work is embedded into multiple social spaces and times. It cannot be understood unless one is prepared to frequently change scales in two different ways. Firstly, one needs to move between the level of the creation of mathematical knowledge at a certain instant and the long duration of its assimilation on the other, which requires that we think in terms of "social time" that is, of the time specific to each social group and space entering into the study, and their peculiar dynamics. Secondly, taking into account the local and global contexts in which the meaning of the texts is construed and passed on implies that we reconstruct as precisely as possible the links between mathematical proofs, the degree to which they overlap, and the way mathematicians use them. Also important is the social logic, whether institutional, local, international, personal, etc., which provide the frame of relevance for these practices.<sup>8</sup>

#### 2. EVARISTE GALOIS'S INTELLECTUAL BIOGRAPHY AS A HISTORIOGRAPHICAL TOOL

Galois is anything but an obscure and unknown mathematician, but paradoxically, the existing studies pose more questions than they answer. According to his biographers, Galois failed to convince his contemporaries, in particular the *Académie des sciences*, because his mathematics, which are thought to contain a glimpse of the structural viewpoint in algebra, were too far ahead of their time. In this narrative, his revolt against the unfair treatment given him by the *Académie* led him to get

<sup>&</sup>lt;sup>8</sup> Even if the subject matters are very different, our approach turns out to be quite close to the type of sociocultural history developed by historians like Daniel Roche ([Roche 1988]), Roger Chartier ([Chartier 1989]) or Bernard Lepetit ([Lepetit 1999], in particuliar pp. 88–119 and pp. 142–168). The heuristic power of changing scales is emphasized in the well-known book [Revel 1996]. We have elaborated on this approach with a view to the history of mathematics in [Ehrhardt 2010b].

involved in politics, and eventually to his tragic death in a duel.<sup>9</sup> But how are we to reconcile then, this revolutionary image with Galois's interest in the resolution of algebraic equations, one of the most classical subjects of the 19th century? Indeed, if mathematicians had long been able to algebraically solve the equations up to the fourth degree, the general problem of algebraic solution had been attacked head on by Lagrange at the end of the 18th century, without, however, reaching the desired definite answer. And in 1830, the question of the equations of the fifth degree had just been solved while the general case still remained to be settled.

The legend of Galois also has it that the recognition of his mathematical genius only came with the publication of his works by Joseph Liouville (1809–1882), fourteen years after the author's death, see [Galois 1846]. However, this does not explain why Liouville undertook this publication. He certainly was an influential mathematician at the time, but nothing destined him to take interest in the solution of algebraic equations. Nor does the legend explain either why mathematicians of the 1850s who, just as Galois, had no notion of algebraic structures, finally turned to exploiting his works.

Studying the posterity of Galois and his work therefore cannot be done without first deconstructing the representations of the person and of his works that have been woven during the 20th century. In the absence of new archival evidence providing fresh clues,<sup>10</sup> the only possibility is to approach the subject from a new vantage point, trying to make out the actual meaning that the events of Galois's academic life could possibly have in the thick Parisian context of the 1830s. Now, this context is not a coherent whole within which Galois's trajectory could be neatly traced in all its complexity. In the eyes of his contemporaries, Galois was not a mathematician; he was a student, an *écolier*—this category has its roots in the 18th century, and at the beginning of the 19th century it included the students in the *Quartier Latin*—who *aspired to become* a mathematician. At the same time, he was seen as a member of the Republican movement, as is reflected in the media of the time, which are marked by the tension between

<sup>&</sup>lt;sup>9</sup> The biographical infatuation for Galois and its aberrations have been analysed in [Taton 1993] and [Rothman 1989]. See also the last contribution to the present special issue, by A. Albrecht et A.- G. Weber.

<sup>&</sup>lt;sup>10</sup> An exception is Galois's 1829 mathematics test for the admission competition to the *École préparatoire*, which was first published and commented in [Ehrhardt 2008].

the events of 1789 and those of 1830.<sup>11</sup> Galois's trajectory thus took place in a micro-historical space whose logic and the mechanism of its structures are embedded in longterm history.

The methodology of intellectual biography turns out to be useful here to reconstruct Galois's social identity.<sup>12</sup> This approach strives at resituating a historical character in the various social spaces of his evolution—the *Académie des sciences*, mathematical programs in High schools, and the Republican *milieu*—these spaces functioning fairly autonomously according to their specific rules. It is thus Galois's *œuvre*, and not the story of his life, which provides the guiding thread: reading it as a historiographical object which is the concrete product of a work of abstraction, it is possible to account for the multiple interactions between scientific and institutional factors, without having to rank them.<sup>13</sup>

Considered first of all as a result of scholarly work, this *œuvre* immediately points to the knowledge and mathematical know-how that undergirded the theory of equations at the time. In this perspective, Galois appears as the heir to a mathematical tradition that was passed on to those who were trained in the *classes préparatoires*, an institution created after the French Revolution. In spite of their apparent novelty, Galois's writings did not constitute an abrupt break with the ways of thinking of the early 19th century. His theorems and arguments did not arise *ex nihilo*; they relied on previous results owed to mathematicians like Gauss, Lagrange or Poinsot who were well-known to scholars at the time. They were based on techniques of proof, on methods of definition, as well as on graphic and symbolic ways of manipulating objects which were also found in the work of the contemporary mathematicians and in certain textbooks dating from the end of 18th or beginning of the 19th century like Bézout's

<sup>&</sup>lt;sup>11</sup> Galois became a member of the *Société des amis du peuple*, which counted Raspail and Auguste Blanqui in its ranks, after the Revolution of July 1830. His Republican activism brought him into jail twice; the first time in May 1831, for having offered a toast to kill the King at a banquet (he was acquitted), and the second for having dressed up in the uniform of the *garde républicaine* on Bastille day, 14 July 1831. He was then imprisoned at Sainte-Pélagie, and subsequently retained at the pension of Doctor Faultrier until May 1832. These various episodes have found a broad echo in the press of the time, whether favourable or hostile to Republican ideas. Also during the first days after his death, Galois was most often described as a young Republican.

<sup>&</sup>lt;sup>12</sup> This concept is at the heart of [Ehrhardt 2011a], where the empirical results sketched here are developed in greater detail. It was defined by [Perrot 1992] and applied to Condorcet's case in [Brian 1994], and to that of François de Neufchâteau in [Margairaz 2005].

<sup>&</sup>lt;sup>13</sup> On the "concrete history of abstraction", see [Perrot 1998] and [Brian 1996].

or Lacroix's. Innovative though it may have been, Galois's work remained firmly grounded on the mental toolkit of his time.

Next, trying to restore the original meaning of Galois's works, one learns to read them as one would read the research papers of a student whose training was rather similar to that of many young men of his generation-Galois was steeped in Lacroix's teaching; he prepared for the system of the competitive exams (concours), and he tried to stand out in a supposedly conservative Parisian mathematical *milieu*. In 1830, this mathematical milieu, both at the Académie and in the educational system at large, was still dominated by the generation of scholars who had been educated just before or during the French Revolution. In this context, young people found it quite difficult to make a name for themselves, and Galois was hardly the only one deploring this state of affairs-the Gazette des écoles denounced the aberrations of the school system, Raspail and Saigey lamented over the academic system in the Annales des sciences d'observations, and Dupin complained in his correspondence with Lacroix about the lack of attention his works received at the Académie.<sup>14</sup> Galois's trajectory is thus inseparable from the "libération de la parole" which emerged at that time-a quest for freedom of speech that crystallized at the beginning of the July Monarchy in a series of legal proceedings against the Republican movement as well as against other groups trying to curb this freedom.<sup>15</sup>

If one compares Galois's case to that of other young mathematicians, like Charles Sturm (1803–1855) and Joseph Liouville, it becomes fairly clear that Galois was not the victim of any specific injustice. Like Galois, Liouville and Sturm's first submissions were also ignored or rejected, and they too chose a flanking strategy by publishing in scientific journals instead. Moreover, taking a close look at the *Procès-verbaux des séances de l'Académie des sciences*, we can gain purchase on the way submissions were assessed in the 1820s and 1830s and see that in Galois's case, the academicians evinced an interest in the work of a student they knew, and did not firmly reject him. In fact, at the time even a negative response—as long as there was a report—was a positive sign. Usually, many manuscripts sent to the academy were buried without a reply. This impression is partly confirmed by certain indices, like a letter of Sophie Germain's where Galois was introduced as a "student showing propitious dispositions" (*un* 

<sup>&</sup>lt;sup>14</sup> See [Raspail 1830], [Saigey 1829]; Dupin's letter to Lacroix: Bibliothèque de l'Institut, Ms 2396.

<sup>&</sup>lt;sup>15</sup> On the public coverage of the sessions of the *Académie des sciences* and its effects, see [Belhoste 2006].

élève annonçant des dispositions heureuses), and an article in the Gazette des écoles mentionning Poisson's interest in Galois's work.<sup>16</sup>

It bears emphasizing then, that when Galois's writings were examined by his contemporaries, they were assessed and interpreted according to the mathematical practices and ways of thinking that were current in the early 19th century. They were works in algebra written in a period of conceptual latency between the analytic art of the 18th and the structural viewpoint, which would slowly gain dominance starting in the 1850s.<sup>17</sup>

However, if the algebraic tradition of the early 19th century constituted the framework through which Galois developed his ideas, and if the reception of his works followed the academic norms of the day, Galois's research remained unrepresentative of the mathematical orthodoxy of the time. As a matter of fact, Galois's trajectory was not the typical trajectory of an aspiring mathematician and he did not share the academicians' habitus of analysis. The memoir he submitted to the Académie did not follow the traditional method, which was to first decompose the initial problem, then show how one could *effectively* execute the theory for a class of examples, and finally apply it numerically to special cases. These themes were touched upon in the memoir—which shows that he was not unaware of the expectations of his contemporaries-but a close inspection of his manuscripts suggests that he only decided to make them more explicit after having received the report of the academicians.<sup>18</sup> Moreover, what he tried to promote as "analysis of the analysis" actually shifted the equilibrium between objects of the theory and its applications. Galois advocated lifting mathematical research to a higher level of "abstraction" so that objects under study-for instance elliptic functions, which before would have been considered as belonging to the domain of classical analysis in an extended sense accepted at the time—could now figure among its applications. If the academicians Siméon-Denis Poisson (1781–1840) and Sylvestre-Francois Lacroix (1765– 1843), who were in charge of evaluating Galois's work, criticized his lack of clarity, this was because this work did not easily fit with the interpretive

<sup>&</sup>lt;sup>16</sup> [Henry 1879], pp. 631-632; Gazette des écoles of 20 January 1831.

<sup>17</sup> Our undertaking would have been impossible were it not for the recent results of the historiography of the Parisian scientific institutions during the 19th century, both methodologically and factually. Among the many references that have enriched our investigations, we mention [Belhoste 2003], [Crosland 1992], [Dhombres & Dhombres 1989].

<sup>&</sup>lt;sup>18</sup> See [Lacroix & Poisson 1828–1831] for this report. The exact dating of Galois's writings is often problematic; we follow here the chronology established by Bourgne and Azra in [Galois 1962/1997].

framework applied by these readers. They did not find in Galois's investigation the mathematics that they themselves practiced and which constituted the model of legitimate work.<sup>19</sup>

At this point the history of mathematics can follow the methodogy used by historians of literature and of reading. The work of Catherine Goldstein ([Goldstein 1995]) has mapped out this common ground. As she argues, the distance that inevitably separates the writing of a text from its reception is an object of historical study in its own right.<sup>20</sup> The ideas of a mathematician are never transparent and they have no abstract or absolute existence. A mathematical result can only be validated by the reading, which is made of it through a specific interpretative framework. This framework is intimately linked to the mathematical toolkit that the reader has at his disposal, and to what the reader considers to be at stake in the text he is assessing.

However, contextualising Galois's work in this way only takes us so far. It does not explain the sudden posthumous success which his writings met with when Joseph Liouville published them in the Journal de mathématiques pures et appliquées almost 15 years after the death of their author [Galois 1846]. Nor does it give us a clue about how his works were integrated into the mathematical activity after 1846. Liouville's discovery of Galois actually took place on two distinct scales; first, it emerged from the coincidental combination of a micro-historical phenomenon and what one may call a structural transformation of the mathematical field. In fact, the reason for Liouville's interest in Galois's works between 1843 and 1846 was related to his personal trajectory: his research on differential equations and his controversy with Guillaume Libri.<sup>21</sup> But in order to draw on these works, he had to rehabilitate them first. The publication of Galois's works was thus part of a personal strategy by Joseph Liouville, and the endorsement that the published works received in the process was based on the scientific authority of the editor of the Journal de mathématiques pures et appliquées who elected them for his strategy.

Second, Liouville's undertaking was part of a gradual redifinition of the boundaries, the objects, and the methods of the discipline of algebra. Indeed, there was a major evolution in the importance and the status of algebra in French mathematics between 1830 and 1850. The boundaries of the discipline were expanded, both in teaching and in research, and new

<sup>&</sup>lt;sup>19</sup> These points are spelled out in [Ehrhardt 2010a].

<sup>&</sup>lt;sup>20</sup> See for example [Chartier 2003].

 $<sup>^{21}</sup>$  On the rivalry between the two mathematicians see [Ehrhardt 2010c] as well as [Belhoste & Lützen 1984].

objects found their place in algebra, quite beyond equations and things comparable to what Galois used, such as substitutions. The theory of equations itself, which had somewhat drifted out of fashion in 1830, also came back to the fore. This happened on one hand through the mathematics taught in the *classes préparatoires*, which were reflected in the *Nouvelles annales de mathématiques* where Abel and Wantzel were now published.<sup>22</sup> On the other hand, the theory of algebraic equations was seen as a potential source of inspiration for other sorts of equations; both Libri and Liouville tried in 1836–1837 to extend the principles of algebraic solvability to differential analysis. In this setting, what seemed heterodoxy in the 1830s was about to turn into a new algebraic *doxa* in the 1840s and 1850s, with the result that the problems that Galois had tackled became interesting. If the rehabilitation of Galois's works was possible at the beginning of the 1850s, it was because they had become compatible with the intepretative lenses of their potential readers.<sup>23</sup>

Furthermore, the *Journal de mathématiques pures et appliquées* put the reception of Galois's writings in very different context from that at the *Académie des sciences* in 1830: these works now addressed a far greater mathematical audience since the *Journal* was directed towards all professional mathematicians, teachers as well as researchers, and it was well distributed all over Europe.

#### 3. RE-READING GALOIS: REFLECTIONS OF LOCAL MATHEMATICAL PRACTICE

While the unfavourable reaction of the *Académie* and the premature death of its author in 1832 appeared to doom Galois's research to oblivion, a new appreciation made possible by the publication in the *Journal de Liouville* allowed these works to meet with a broad recognition across Europe between 1850 and 1870. This process was initiated by a series of interpretations of his *Mémoire sur les conditions de résolubilité des équations par radicaux* worked out by mathematicians from various personal, national and professional backgrounds : Enrico Betti (1823–1892), Arthur Cayley (1821–1895), Camille Jordan (1838–1922), Joseph-Alfred Serret (1819–1885), Richard Dedekind (1831–1916), and also Leopold Kronecker (1823–1891), to mention only the most famous among them. While the

<sup>&</sup>lt;sup>22</sup> See [Wantzel 1845], [Abel 1845].

 $<sup>^{23}</sup>$  For more details on this new context for the reception of Galois, see [Ehrhardt 2010c].

first re-readers of Galois all endeavoured to fill in the holes in Galois's proofs, they did not stop there—they also undertook genuine reconstructions and recastings; they endowed Galois's work with a new meaning, a mathematical "added value." But these interpretations were far from being compatible with one another.

The Italien Enrico Betti, for instance, decided in 1851-1852 to take Galois's work as a starting point to complete the theory of the solution of algebraic equations.<sup>24</sup> He turned groups of substitutions into mathematical objects in their own right-even if this notion was not yet considered interesting independently of the theory of equations. Orienting his research in that direction, Betti swept another fundamental aspect of Galois's memoir under the rug: the notion of adjunction. Moving in both registers at the same time-or, using Thomas Kuhn's terminology, in both disciplinary matrices-without being able to establish a boundary between them, Betti's re-reading established a transitory state in the development of algebra, one which is often neglected by the type of historiography which insists more on the first origins and the final outcome of mathematical theories. Alhough he was well acquainted with the most recent developments of algebra, Betti did remain a somewhat isolated beginner in mathematics in the 1850s and his research would not meet with the recognition he had hoped for. His place in the afterlife of Evariste Galois would remain marginal.

In the British Isles, the reception of Galois's works took place within a specific algebraic tradition that was independent of the theory of equations.<sup>25</sup> Arthur Cayley offered a re-reading of Galois, which one might almost call axiomatic, and which reminds us of the Cambridge algebraic school of the early 19th century. Here, the characteristic properties of the general group concept dominated the scene, quite independently of its applications.<sup>26</sup> The notion of the group actually aroused interest precisely because it was generic, that is, it allowed to identify the similarities between widely differing mathematical situations such as algebraic equations, quaternions, functions, etc. Studying this notion followed a logic of trying to establish an inventory of all groups meeting a given particular condition, like having a given number of elements.

But if Cayley's works in the 1850s aimed at developing a theory of groups, he was not the only one in Great Britain to make such an attempt

<sup>&</sup>lt;sup>24</sup> See [Betti 1851] and [Betti 1852].

<sup>&</sup>lt;sup>25</sup> See [Nový 1968], [Durand-Richard 1996].

<sup>&</sup>lt;sup>26</sup> See [Cayley 1854a], [Cayley 1854b], [Cayley 1859].

and to refer to Galois. The group concept triggered various approaches. Cayley's, which is the most famous one today, was based on symbolic algebra. The other one owed more to the combinatorial tradition, which had not given rise to a theory but belonged to the domain of "tactics", with ramifications in recreational mathematics. This approach was developed in the works of Thomas Kirkman (1806–1895), who, interestingly, largely corresponded with James Joseph Sylvester on this topic.<sup>27</sup> The dialogue between these three mathematicians—as one can see it in [Crilly 2006], [Hunger-Parshall 1998], and [Hunger-Parshall 2006]—provides evidence of the coexistence of different practices at the time when they tried to theoretically construct a new object. But these differences did not hamper scholarly exchange: Kirkman and Cayley partly worked together on similar objects. They apparently conversed in private, but hardly quoted each other publicly, in their articles.

Meanwhile, other mathematicians in Great-Britan in the 1850s and 1860s referred to Galois in their work on the solution of algebraic equations, for instance James Cockle (1819–1895).<sup>28</sup> They typically imported Galois's works together with other recent papers on the theory of equations, such as Wantzel's, for example, without marking them as decisive contributions to the problem.

In the end, the only thing that all these re-readings had in common was probably that none of them actually used the contents and substance of Galois's work. However, they did contribute to the weaving of a local memory of Galois, as the name was spread in association with certain problems and objects. The reception of Galois's work in Great-Britain is evidence for the fact that the coherence which a community may acquire through education, through specialised journals, or simply through friendships, is not sufficient to unify the ways in which a common mathematical object is being worked out and developed by the members of this community. Not all re-readings and usages of Galois's papers in the UK complied with what one would typically expect of British algebraic practice at that time.

In France, the re-readings of Galois's memoir partly depended on a contingent event: in 1846, Liouville had announced that he was preparing something on the subject. Priority was thus reserved for him—voluntarily or under pressure—until the mid 1860s.<sup>29</sup> Furthermore, the question of substitutions and the solution of algebraic equations was linked with the

<sup>&</sup>lt;sup>27</sup> See for instance [Kirkman 1860].

<sup>&</sup>lt;sup>28</sup> See [Cockle 1859].

<sup>&</sup>lt;sup>29</sup> According to [Bertrand 1899], pages dealing with Galois's work in the second edition of Serret's *Cours d'algèbre supérieure* had to be left out because Liouville protested.

number of values a function takes on when one permutes the letters which it depends on; this taps into a tradition that went back to Lagrange and which had been revived by Cauchy, and afterwards by Joseph-Alfred Serret and Joseph Bertrand.<sup>30</sup>

There were two mathematicians fighting over Galois's mathematical legacy, each one reading Galois in a specific institutional context. Serret occupied the domain of higher education and offered in the successive editions of his well-known *Cours d'algèbre supérieure* an interpretation of Galois's works, which rendered them accessible and usable to the beginners. In other words, he interpreted them so that they might be completely compatible with in the classical algebraic tradition. In particular, he developed for certain parts of Galois's work a practical dimension that they were originally lacked.<sup>31</sup> This recasting, thanks to longterm history and to the specificities of textbook circulation, remained influential until the end of the 19th century.

Camille Jordan on the other hand launched a development for which the solution of algebraic equations is secondary. Undergirding his reading of Galois was the new theory of substitutions, which was elaborated in the 1840s in connection with the problem of the number of values that a function takes on when one permutes the letters it contains.<sup>32</sup> Jordan's interpretation of Galois's works was not a commentary on them: it was not linear; results were reformulated and reshuffled, and Jordan moved out of the framework of the resolution of equations. In fact, if the contents of the Traité des substitutions et des équations algébriques was to a great extent inspired by Galois's work, it also went substantially beyond Galois, towards a general theory of groups of substitutions. Jordan's reappropriation of Galois concerned the results themselves as well as basic "ideas" or "methods" which could be extracted in particular from Galois's line of thought in the Mémoire sur les conditions de résolubilité des équations par radicaux. Thus, in outlining for the first time what could be a proper, adequate, and seminal way of reading Galois, Jordan's Traité added a new layer to the scholarly memory of Galois; henceforth it would include representations of the sense one ought to attribute to his works. The treatise also remained a standard reference, in spite of the criticism voiced or the limitations pointed out by contemporaries. Serret's and Jordan's recastings initiated Galois's

<sup>&</sup>lt;sup>30</sup> See [Lagrange 1808], [Cauchy 1815], and [Cauchy 1844], [Serret 1850], [Bertrand 1845].

<sup>&</sup>lt;sup>31</sup> See [Serret 1849], [Serret 1854], and [Serret 1866].

<sup>&</sup>lt;sup>32</sup> See [Jordan 1970].

posterity on a scale which not only went beyond the French context of their conception, but also beyond the circle of specialists of algebra.

Finally, the example of the mathematicians Leopold Kronecker and Richard Dedekind shows that the transfer of Galois's works from France to Germany in the 1850s was not simply a matter of content, concepts, and theorems.<sup>33</sup> It went along with a modification of the system of representations of mathematics, which affected the practices and types of problems associated with Galois's memoir, as well as the organisation of concepts and results. Dedekind and Kronecker took off in the same direction, insisting on the notion of adjunction and the group concept, i.e., on the most abstract aspects of Galois's work. From this point of view, their readings can be said to reflect the organization of the German university system in which mathematics tended to be taught with an emphasis on questions of foundations and method rather than on applications, and they probably also reflected the prominence given to questions about numbers.<sup>34</sup> Their readings also allowed a glimpse at the peculiarities of Berlin and Göttingen, the centres of mathematical production in which they originated, but they also reflected the professional trajectories and philosophical orientations of the two mathematicians.<sup>35</sup> As a matter of fact, both seemed to have been influenced by Lejeune-Dirichlet, who had himself left Berlin for Göttingen.

Furthermore, these two interpretations illustrate that one has to take into account different temporalities when trying to outline what may have been the "influence" of a mathematical text: Kronecker played an important role through the 1880s, during the golden age of Berlin mathematics, before his work began to be criticized. Dedekind, on the other hand, taught his recasting of Galois only to a small group of students attending his seminar, and it was not before the 20th century, when Emmy Noether (1882–1935) turned him into one of her intellectual heroes, that his works came into the limelight. The dynamics of the opposition between these two recastings of Galois were thus completely inverted within one or two generations.

Because it allows us to see the mental toolkits and lenses through which a number of mathematicians conducted their reading of Galois in the period 1850–70, this comparative analysis of the various local spaces in which

<sup>&</sup>lt;sup>33</sup> See [Dedekind 1981], [Kronecker 1853].

<sup>&</sup>lt;sup>34</sup> On the consequences of the reforms at the beginning of the 20th century on science education, see [Pyenson 1983] and [Olesko 1989]. On numbers: [Goldstein & Schappacher 2007].

<sup>&</sup>lt;sup>35</sup> On this point, see [Rowe 1998], [Ferreirós 1999].

this work of abstraction took place reveals that in each case, it was a specific dimension of Galois's work that was emphasized. A number of factors contributed to make these readings distinctive, viz. the questions that each one of those mathematicians tried to answer, the scholarly tradition into which they inscribed Galois's writings, the results to which he associates them, along with the work routines acquired in his mathematical training, the research practice and mathematical outlooks dominant in his mathematical of Galois. In this way, Galois's first posterity is evidence for the dependence of scholarly practice on local research traditions: mathematicians did not work in the same fashion and did not practice and write the same kind of mathematics in Cambridge, Paris, Göttingen or Berlin. Whether they were adressing a student audience, a local community of mathematicians, or the emerging international community also had an impact on their interpretation of Galois's work.<sup>36</sup>

Furthermore, each mathematician's work of interpretation had several dimensions to it. First it was performed literally on Galois's text: his theorems were reorganised and modified as the re-reading unfolded, and proofs were rewritten. But Galois's objects were also affected; they were redefined by the explicit statements as well as by the procedures of symbolic and graphic manipulation: the group concept, for instance, derived its respective meanings not only from the various definitions given, but most of all through the graphic and computational practices employed. The concept was thus refined by theoretical focalisations as well as by its use and by the practice of proofs; writing a group using letters or as a table, for example, makes a difference for the questions we ask, and for the way it is used in proofs, so that the object forged in this way will ultimately not be the same.

Finally, the various readings concerned Galois's "ideas", i.e., the way in which his successors interpret his writings. This interpretation oriented their techniques of proof without, however, being a matter of the inherent strictly mathematical logic. It was these mathematicians who decided what was most important in Galois's work, explained how to understand it, and how it could serve further research.<sup>37</sup> Galois's writings also become "important" landmarks in the mathematical landscape because of what was *said* about them, rather than what was *done* with them. But even if

 $<sup>^{36}</sup>$  Here we meet the conclusions that are formulated for the Cambridge case in [Warwick 2003].

<sup>&</sup>lt;sup>37</sup> The question of symbolic practices could thus be joined to Corry's useful distinction between "body of knowledge" and "images of knowledge" (see [Corry 1989]).

this distinction turns out to be useful from a heuristic point of view, the difference is not relevant in the work of those mathematicians, in which no explicit line separates the mathematical discourse from the working pratice, or from the discourse on mathematics. As I have suggested elsewhere [Ehrhardt 2007], we may borrow here Maurice Halbwachs' notion of "collective memory" to understand these dynamics. Just as in the case of the collective memory of musicians, which lasts because the group of people carrying it shares not only the same values, but also a specific, rational know-how, the memory of mathematical objects depends at the same time on the internal logic of scientific concepts, on working pratices and know-how, but also systems of representations of the works and of the mathematicians who have created them.<sup>38</sup>

The comparative approach allows to break with a chronological way of writing the history of mathematics which assumes that results succeeding each other in time necessarily "influence" each other. If the first re-readers of Galois worked one after the other in time, they did not do so in concertation and practically did not quote their predecessors. More often than not, their contributions stood alone, each one having emerged and owing its meaning to a space of local relevance. As a result, in the end did more to fragment Galois's text than to unify it. We are confronted, then, not with a chronology, but with a cartography of re-readings of Galois, with each local space having its own proper temporality that has to be spelled out: the median time of all of them is the coexistence of different memories of Galois, not the uniform absorption of his works by a research community.

Yet, continuing our investigation in the long run, beyond the period of the first re-readings, will show that only changing scales, between the local nature of the first re-readings and the capacity of some of them to gain international acceptance—either through teaching or via references to them in other fortcoming research papers—will bring about Galois's lasting posterity. This step from the breaking up into different memories to the sedimentation of knowledge in a unified "Galois theory" is the affair of a longterm development.

<sup>&</sup>lt;sup>38</sup> The collective memory of mathematicians is defined in [Halbwachs 1997, pp. 211–214].

#### C. EHRHARDT

#### 4. COMPETING MEMORIES, OR THE LONGTERM ABSORPTION OF A MATHEMATICAL TEXT

At the turn from the 19th to the 20th century, the publication of quite a few textbooks on Galois theory and on group theory firmly put Evariste Galois at the heart of mathematical activity in Europe and in the United States.<sup>39</sup> It was also during this period that the first lecture courses on these matters were taught and the first historical commentaries and biographies of Galois appeared.<sup>40</sup> Around 1900, just as research in algebra was about to take the structural turn, Galois theory and group theory thus became part of what one may call the *ordinary* scientific knowledge belonging to the common culture of mathematicians, Galois's posterity entered the long period of assimilation into normal science. To analyze such a phenomenon requires answering two questions, both of which concern the method to employ as well as the conceptual framework to adopt for the investigation.

*The first question* concerns the kind of sources we need to take into account. This implies the question of whether the analysis ought to be quantitative or qualitative. It is clearly not sufficient to just look at the works which today appear to have been the most significant contributions of the time. Indeed, mathematics do not progress simply through the innovative works of a handful of excellent mathematicians; progress also depends on the more modest contributions of mathematicians whom history has forgotten but who, without creating anything novel, consolidated with their numerous publications the status of a theory, of the representations that go with it, and in this way gave substance to it. Trying to evaluate from this point of view the scope and the speed of the absorption process means adopting a quantitative approach.<sup>41</sup>

This approach highlights the differences between the situation of 1870–1895 and the preceding period. The first re-readers of Galois did not neatly separate what belonged to "Galois theory", i.e., algebraic equations, from what belonged to the study of groups. Where some of them saw Galois theory as an application of the theory of substitutions, others simply saw the concept of the group as a tool. In the subsequent period,

 $<sup>^{39}\,</sup>$  One may, for instance, refer to [Netto 1882], [Weber 1895], [Borel & Drach 1895].

 $<sup>^{40}</sup>$  The first biography of Galois is [Dupuy 1896], and among the first commemorative texts we may cite [Lie 1895] and [Tannery 1909].

 $<sup>^{41}</sup>$  This quantitative approach is facilitated here by the online database of the *Jahrbuch über die Fortschritte der Mathematik*, and by the fact that the category "algebra" is almost completely stable between 1870 and 1910.

however, the *Jahrbuch* reveals a new representation of Galois theory which turns it into a research area independent from group theory, situating it rather on the side of classical algebra. In this way, Galois's works gained a certain autonomy, but at the same time they lost for the time being a characteristic feature which would often be associated with them later on during the 20th century: their "modernity", in the specific sense that this term acquired with the publication of van der Waerden's book *Moderne Algebra*, i.e., after the recasting of Galois theory in the 1930s at the hands of Emil Artin (1898–1962).<sup>42</sup>

In the same way, the quantitative study of the *Jahrbuch* reveals a new, much more recent collective memory of Galois, if one correlates the occurences of the name "Galois" with the categories of the *Jahrbuch*. These are the beginnings of what today is known as the Galois theory of differential equations. The first papers on the subject were essentially written by three French mathematicians: Emile Picard (1856–1941), Ernest Vessiot (1865–1952), and Jules Drach (1871–1949), who created the new subdiscipline. They cite each other and they all allude to the research of the Norwegian mathematician Sophus Lie as essential inspiration.<sup>43</sup>

But examining the classifications set up by the protagonists themselves, or the occurrences of specific names or terms—such as "Galois", "group", etc.—in publications in specialized journals really tells us nothing about the mathematical practices used within the various categories, about the social spaces from which they arose, or the different meanings that a citation of Galois, or the usage of the group concept may have had. If we are to grasp the whole spectrum of usages and interpretations of Galois's *œuvre*, the quantitative point of view therefore has to be complemented by qualitative studies of as large and "anonymous" a range as possible of contributions.<sup>44</sup>

This double focus reveals the high density of the research conducted on group theory between 1880 and 1910—in sharp contrast with the routine classicism maintained in the theory of equations. So the availability of Galois's work afforded by the first re-readings did not, in the end, turn Galois's ideas on the theory of equations into a fashionable subject. And the appreciation of Galois's *œuvre* came at the price of erasing the very contribution of the first re-readings. The memory of the mathematicians only

<sup>&</sup>lt;sup>42</sup> See [Corry 2004] and [Kiernan 1971].

<sup>&</sup>lt;sup>43</sup> See the contribution by T. Archibald in the present volume.

<sup>&</sup>lt;sup>44</sup> For a deeper reflection on quantitative methods and their application to the case of Number theory, see [Goldstein 1994] and [Goldstein 1999]

kept Galois's name, and finally attributed to him the fully-fledged theory, which was later erected on the basis of the first recastings.

More than that, if all the protagonists naturally alluded to the existence of "Galois theory", this label actually concealed diverging ideas about the status of this theory and of the actual mathematical know-how it involved. The detailed study of the works by Picard, Vessiot, and Drach published between 1883 and 1898, thus allows us to establish the existence of a core of results associated to Galois theory, around which revolved a great variety of problems, research agenda, and representations.<sup>45</sup> If Galois's theorems were the same for all these authors, the same cannot be said of their understanding of Galois's "ideas" or "approach."

In the case of Galois theory, attributing knowledge to the creator of the theory therefore did not imply adopting a uniform practice. Mathematicians refered to Galois theory and agreed on its fundamental theorems, but for matters of meaning, practice, and know-how they remained to a large extent dependent on the particular memory of Galois which they had learnt. Finally, the spread of knowledge and the proliferation of papers claiming to draw on Galois turned Galois theory and the group concept into what the historian Alain Boureau calls a *collective statement*.<sup>46</sup>

The second question suggested by the assimilation of Galois's research is that of the "disciplinarisation" of the theory, and more generally that of the social spaces in which this sort of phenomena ought to be analyzed. The notion of discipline, at the crossroads of "internal" and "external" aspects of science, has been studied extensively and from varying angles.<sup>47</sup> However, if the links between disciplines and teaching have often been stressed, only recently have historians of science taken them as an alternative to the ideotypical topdown model according to which scientific knowledge spreads downards from the laboratory and research activity to the level of textbooks and lecture courses.<sup>48</sup>

Yet, in the case of Galois theory, the consensus about the "core" of the theory did not come before the writing of surveys and textbooks. It was the very publication of such books, the goal of which is to establish

<sup>&</sup>lt;sup>45</sup> See [Picard 1883], [Picard 1895], [Vessiot 1892], [Drach 1895], and [Drach 1898].

 $<sup>^{46}</sup>$  See [Boureau 1989]. A similar conclusion has been obtained for the case of fuzzy logic in [Rosental 2003].

 $<sup>4^7</sup>$  See the survey in [Gauthier 2007, pp. 20–25]. On the distinction between "research field" and "discipline", see [Goldstein & Schappacher 2007]. For a reflection within the domain of "general" history, see [Boutier et al. 2006].

<sup>&</sup>lt;sup>48</sup> Aside from the pioneering [Olesko 1989], one may mention here [Warwick 2003] and [Kaiser 2005].

norms, which enabled the full absorption of Galois's works into the edifice of mathematics. Serret's *Cours d'algèbre supérieure* was important in this respect—it became a standard reference for researchers in this field in the mid 19th century, allowing mathematicians to forge a common scientific language usable for differing practices.

But this process gained much more momentum towards the turn of the century when algebra textbooks and courses really developed. In France, for instance, three textbooks were published virtually simultaneously in 1895/96, in a context of the curricular institutionalisation of Galois theory, which had been written into the canon of the agrégation exam, the lecture courses of the *licence* curiculum, and doctoral studies.<sup>49</sup> The books by Vogt, Drach, and Picard employed the same theorical toolkit. For instance, Jordan's and Kronecker's work were now included in a research tradition that bore Galois's name, and every presentation of this theory, however heterodox, remained inside this conceptual framework. Galois theory has its "hard core", one which was not specific to the French textbooks quoted, but was actually identical to what one could find in German or Italian books. It was constitued by the foundations of the theory of substitutions (but without going as far into it as Jordan did), but also by the notion of adjunction, which was barely mentioned in Galois's memoir, and by the concept of rationality domain, which was absent from Galois's works and harkened back to Kronecker's research.

Still, while Vogt presented Galois theory somewhat like a new theory of equations, Drach actually turned it into the foundation of algebra; as for Picard, he developed it only in connection with differential equations and remained as close as possible to Galois's memoir on equations. The differences between these textbooks by Vogt, Drach, and Picard (but also between them and those by Netto and Weber) are evidence for the fact that authors still had a large degree of interpretative freedom on elements of the theory that were not part of the "core." This freedom affected the reception of Galois's works: one could be as faithful to the original proofs as Picard, but one could also decide to retain only the statements without reproducing the main stages of the proofs. It also remained possible to modify the statements themselves. This freedom was also obvious in the fact that mathemtaticians were free to choose the re-reading of Galois they preferred, and then to borrow from it. The protagonists were thus still free

<sup>&</sup>lt;sup>49</sup> See [Vogt 1895], [Picard 1895], and [Borel, Drach 1895]. In July 1898 for example, the Paris candidates for the *certificat d'analyze supérieure de la licence* were asked: "How does one call the group of an algebraic equation? Prove its fundamental property."

#### C. EHRHARDT

to articulate their own presentations of Galois theory around a small number of required elements. This choice depended on a number of elements such as their image of mathematics, the results they wanted to emphasize, the audiences addressed, or on the representations that they associated with the works and ideas of Evariste Galois.

Finally, the writers of university textbooks not only played the passive role of collectors of research ideas which they cobbled into their teaching, they also created mathematical knowledge in that when they introduced students to Galois theory, they offered an organisation and a hierarchy of its constitutive elements which were anything but established within the initial, fragmented landscape of local memories. In this way, these authors structured the mathematical field; they redistributed symbolic capital between the authors, they defined which objects are legitimate, which orientations took precedence, and they enabled the constitution of a community of specialists who had received the same kind of training. Mathematical content and practice thus defined the social space corresponding to Galois theory at the end of the 19th century within the mathematical field.

The texbooks eventually brought Galois theory to new audiences. First to students, thanks to treatises like Netto's *Substitutionstheorie* or Weber's *Lehrbuch der Algebra* paved the way to a new way of teaching, especially in the United States and in Italy.<sup>50</sup> Second, the textbooks also affected non-specialized scientific readers through book reviews.<sup>51</sup> At the turn of the century, Galois theory had not only become a legitimate object of mathematics, it was also an elemental piece of knowledge in the training of future mathematicians, and for any claim to "mathematical culture."

Moreover, these synthetic books became standard working tools for the mathematicians. As they probably enjoyed a wider circulation than monographs or articles in scientific journals and were written in a more accessible way, they made a broad dialogue possible, in turn allowing for new developments associated to Galois theory. For example, the books [Borel & Drach 1895] and [Vogt 1895] were reviewed in international specialized

196

<sup>&</sup>lt;sup>50</sup> The reference is to [Netto 1882] and [Weber 1895]. On the use of these books for teaching, see [Martini 1999], [Cajori 1890, p. 265], [Morton Stewart 1896, p. 18, 26], and [Bulletin of the University of Wisconsin 1902, p. 33].

<sup>&</sup>lt;sup>51</sup> For example, reviews of [Borel & Drach 1895] and [Vogt 1895] are to be found in journals for students of the classes préparatoires, such as Mathesis, the Journal de mathématiques spéciales and the Revue de mathématiques spéciales, but also in the Revue générale des sciences pures et appliquées, and in Nature.

journals<sup>52</sup>, and they were also used in research articles shortly after their publication.<sup>53</sup>

For this reason, these textbooks were more than teaching tools, they were also *media*. A common language for Galois theory could be distilled through them, and they finally brought about a consensus over the results and theorems of this theory. Thus, in spite of slight local variations that persisted as to the practice and know-how associated to the theory, it can be said that "Galois theory" was effectively established at the beginning of the 20th century.

Galois's posterity, however, is not at all restricted to the mathematical field. And this is why it prompts the following question: how mathematics relates to the public space, beyond mathematics education.<sup>54</sup> Indeed, the thought that there may have been an approach or certain ideas typical for Galois that go beyond the plain content of his writings, with a relevance outreaching the theory of equations, was still only marginal at the end of the 19th century. The posterity of Galois did not surpass a modernized version of the theory of equations before 1895. But at the turn of the century, the restructuring of the field of Galois theory and the success of group theory ushered in various attempts to recover Galois's symbolic legacy, thus heralding a mythification of Galois.

The commemoration of the centennial of *École normale supérieure* (ENS) in 1895 was indeed marked by the first speeches about Galois which did not hinge on mathematical production. They focused on the question of how one could be a worthy successor of Galois, either in terms of the historical continuity of mathematical progress,<sup>55</sup> or in terms of sharing the same cultural mold.<sup>56</sup> It is particularly noteworthy that it was the same small

<sup>&</sup>lt;sup>52</sup> Archiv der Mathematik und Physik 15 (1897), pp. 20–21; Zeitschrift für Mathematik und Physik 17 (1896), pp. 1–2 and pp. 18–20; Jornal de sciencias mathematicas e astronomicas 12 (1896), p. 142; El progreso matematico (1894), pp. 34–37; Bulletin of the American Mathematical Society 6, n° 8 (1900), pp. 344–348 and 3, n° 3 (1896), pp. 97–105.

<sup>&</sup>lt;sup>53</sup> See [Dickson 1896–1897], [Miller 1899–1900], [Schottenfels 1899–1900].

<sup>&</sup>lt;sup>54</sup> The notion of public space, as it has been framed in [Habermas 1993], focusses on politics and the relation to the state. It therefore does not easily adapt itself to our case of interest. But it does suggest a schematic distinction between different spheres which enjoy relative autonomy and within which mathematics finds its meaning: the sphere of the "professionals" (the mathematicians), that of the "users" (teachers, students, but also scientists from other disciplines), and that of the "amateurs" (be they simply fond of recreational mathematics, or interested in mathematics to bolster up a discourse which is not oriented towards doing mathematics, for instance in philosophy).

<sup>&</sup>lt;sup>55</sup> See [Lie 1895].

<sup>&</sup>lt;sup>56</sup> See [Dupuy 1896].

group of protagonists (essentially Lie, Bertrand, Tannery, and Picard) sharing both a system of mathematical representations and practices, and strong ties to the ENS—who now actively promoted the figure of Galois in numerous publications in the general scientific press between 1895 and 1910.<sup>57</sup> Thus Picard's preface to the new 1897 edition of Galois's *œuvres* was published under the aegis of the *Société mathématique de France* to earn international recognition.<sup>58</sup> Likewise, Tannery's speech at Bourg-la-Reine was reproduced in the general press and would be taken up by literary authors.<sup>59</sup> As a historical construction, Galois's posterity was thus not the fruit of mathematical research on the basis of his writings, but rather of discourses about mathematics conceived for commemorations, and according to a logic of local legitimization of a research field.

As the local scholarly memories of Galois waned, they made room for a new kind of memory, one we may describe as "social" in the sense that it went beyond the boundaries of mathematics and extended into philosophy or literature.<sup>60</sup> The philosophers Louis Couturat (1868–1914) and Maximilien Winter (1871–1935) commented on Galois's writings in the *Revue de métaphysique et de morale*, Léon Brunschvicg (1869–1944) granted him a place in his *Les étapes de la philosophie mathématique*, Alain the philosopher (i.e., Emile-Auguste Chartier, 1868–1951) devoted one of his *propos* to Galois's life, and the poet Victor Segalen (1878–1919) compared him to Rimbaud.<sup>61</sup>

On one hand, this new kind of discourse was part of the construction of the legend of Galois—a legend that some pointed out as early as 1910.<sup>62</sup> Galois as a person was allowed a certain autonomy compared to other mathematicians, since one did not have to be a mathematician to talk about him, and one could find him interesting without the slightest knowledge of mathematics. On the other hand, these discourses did enrich the pre-existing mathematical content. In fact, the representations of

<sup>&</sup>lt;sup>57</sup> Apart from [Lie 1895], see in particular [Picard 1897], [Bertrand 1899], [Tannery 1906], and [Tannery 1909]. It should be pointed out that all these texts have been repeatedly re-edited, in particular in compilations of texts by their authors about mathematics.

<sup>58</sup> See [Picard 1897].

<sup>&</sup>lt;sup>59</sup> See [Tannery 1909].

<sup>&</sup>lt;sup>60</sup> The expression "social memory" is borrowed from [Halbwachs 2008]. It denotes the memory of the biggest possible social group; cf. for this [Brian 2008].

<sup>&</sup>lt;sup>61</sup> See [Couturat 1898], [Winter 1910], [Brunschvig 1912], [Alain 1909], [Segalen 1979, pp. 481–501].

<sup>62</sup> See [Mansion 1910].

Galois as a person and of his "ideas" which were developed at the beginning of the 20th century by mathematicians, philosophers, and historians of mathematics, were legitimized by the success of Galois theory and group theory, at the time, but they in turn legitimized this theory. Indeed, they provide it with a general framework and a meaning, embedding it in a system of knowledge which privileges "generality", or "modernity." These discourses would in due course encourage new generations of mathematicians to get their inspirations from Galois's works, because these are considered to contain "general" ideas, to then do research in domains far removed from Galois's initial centres of interest.

The myth of Galois thus belongs neither to the "inside" nor to the "outside" of mathematics. It exists, still today, simultaneously in multiple social spaces which may be more or less independent from that of mathematical research; spaces where it takes on different meanings and serves different agenda. A mathematician may appeal to Galois in his research articles, and he may equally well give a lecture on his subject to a broad audience, just as a filmmaker may try to put Galois on the screen, and to possibly reach certain mathematicians among his viewers.<sup>63</sup>

#### 5. CONCLUSION

The history of Evariste Galois's posterity thus teaches us that the historian cannot distinguish *a priori*, in a neat and uniform way, between what has to do with mathematical content, the practice of a work of abstraction, the corresponding systems of representations, and the social spaces in which they are developed. The particular context of the analysis of a text and the time frame used to study it cannot be selected before the actual investigation. Instead, they result from an empirical approach, which must seek to recreate the space of relevance in which the actions of the protagonists and the texts they produce become meaningful. The variations of scales, whether geographical or temporal, provide an adequate tool to capture in a single move the immediate nature of the reception of a result, its collective objectivization, and eventually the status of truth which it acquires when it enters the disciplinary matrix—all without losing sight of the logic proper to each one of these steps. Looking at the life of a mathematical text in the *longue durée*, one is in a position to pinpoint

<sup>&</sup>lt;sup>63</sup> One may recall here a public lecture [Connes 2005] that Alain Connes gave at the *Bibliothèque nationale de France*, or else Alexandre Astruc's short film [Astruc 1965], which was awarded a prize at the Cannes International Film Festival in1965.

#### C. EHRHARDT

precisely that which is never captured by studies that favour local aspects of the making of scientific knowledge: the social and cultural process whereby the universal validity of mathematical results is constructed.

Analyzing this process in the case of Galois's Mémoire sur les conditions de résolubilité des équations par radicaux also challenges another distinction which is often held to be self-evident: that between the writing of mathematics and the writing of its history. On the one hand, both are based on reading and analyzing existing mathematical texts, and thus necessarily are topical re-castings of these texts, constructed according to the problems and goals germane to each of these investigations.<sup>64</sup> If their methods, their practices, and their ways to argue differ, both types of writing fully contribute to the making and legitimization of results in a longterm perspective. History and mathematics act like filters for the posterity of a result. On the other hand, it is often quite difficult to distinguish in each recasting, between what belongs to the "conceptual discourse", relevant to Galois's research, and what to the "symbolic discourse", concerning the person.<sup>65</sup> How is one to classify, for instance, all those interpretations that try to make Galois's "ideas" explicit? Instead of relving on a pre-established dichotomy between conceptual discourses produced by mathematicians and symbolic discourses produced by non-mathematical protagonists, we need to understand how each author produces both types of discourse at the same time, even if one appears to be subordinate to the other.

In fine, mathematical objects, social contexts, and the systems of representations in which they are embedded may be conveniently distinguished for the analysis; but the reflective control of the historical investigation shows that they are not contradictory. On the contrary, all three wove the canvas on which the posterity of Galois and his work took shape.

#### REFERENCES

ABEL (Niels Henrik)

[1845] Théorèmes posthumes d'Abel, *Nouvelles annales de mathématiques*, 4 (1845), pp. 536–540.

Alain

[1909] Évariste Galois, 1909; La dépêche de Rouen, 10 août 1909.

200

<sup>&</sup>lt;sup>64</sup> We concur here with [Goldstein 1995, pp. 7–9].

<sup>&</sup>lt;sup>65</sup> This distinction is directly inspired by [Schandeler 2000, pp. 9–10, 221–222].

#### ASTRUC (Alexandre)

- [1965] L'Éloge des Mathématiques. Évariste Galois, 1965; short film (30') with José Varela in the role of Galois (extract: http://www.youtube. com/watch?v=FDo02EFxVYs).
- [1994] Évariste Galois, Paris: Flammarion, 1994.

#### AUFFRAY (Jean-Paul)

[2004] Évariste, 1811–1832, le roman d'une vie, Lyon: Aléas, 2004.

#### BELHOSTE (Bruno) & LÜTZEN (Jesper)

[1984] Joseph Liouville et le Collège de France, *Revue d'histoire des sciences*, 37 (1984), pp. 255–304.

#### **BELHOSTE** (Bruno)

- [2003] La formation d'une technocratie. L'École polytechnique et ses élèves de la Révolution au Second Empire, Paris: Belin, 2003.
- [2006] Arago, les journalistes et l'Académie des sciences dans les années 1830, in Harismendy (P.), ed., La France des années 1830 et l'esprit de réforme, Rennes: Presses universitaires de Rennes, 2006, pp. 253–266.

#### BELL (Eric Temple)

[1937] Genius and Stupidity. Galois, in *Men of Mathematics*, New York: Simon and Schuster, 1937, pp. 362–378.

#### BERTRAND (Joseph)

- [1845] Mémoire sur le nombre de valeurs que peut prendre une fonction quand on y permute les lettres qu'elle renferme, *Journal de l'École polytechnique*, 18 (1845), pp. 123–140.
- [July 1899] La vie d'Évariste Galois par P. Dupuy, Journal des savants, 1899, pp. 389-400.

#### **BETTI** (Enrico)

- [1851] Sopra la risolubilità per radicali delle equazioni algebriche irriduttibili di grado primo, *Annali di scienze matematiche e fisiche*, 2 (1851), pp. 5–19.
- [1852] Sulla risoluzione delle equazioni algebriche, Annali di scienze matematiche e fisiche, 3 (1852), pp. 49–115.

#### BOREL (Émile) & DRACH (Jules)

[1895] Introduction à l'étude de la théorie des nombres et de l'algèbre supérieure, Paris: Nony, 1895.

#### BOUREAU (Alain)

[1989] Proposition pour une histoire restreinte des mentalités, Annales, Économies, sociétes, civilisations, 6 (1989), pp. 1491–1504.

#### C. EHRHARDT

BOUTIER (Jean), PASSERON (Jean-Claude) & REVEL (Jacques), eds. [2006] Qu'est ce qu'une discipline?, Paris: Éditions de l'EHESS, 2006.

#### BRIAN (Éric)

- [1994] La mesure de l'État. Administrateurs et géomètres au XVIII<sup>e</sup> siècle, Paris: Albin Michel, 1994.
- [1996] Calepin. Repérage en vue d'une histoire réflexive de l'objectivation, Enquête, *Anthropologie, histoire, sociologie*, 2 (1996), pp. 193–222.
- [2008] Portée du lexique halbwachsien de la mémoire, in Halbwachs (Maurice), ed., La topographie légendaire des évangiles en Terre Sainte. Étude de mémoire collective, Paris: PUF, 2008, pp. 113\*–146\*.

#### BRUNSCHVIG (Léon)

[1912] Les étapes de la philosophie mathématique, Paris: Alcan, 1912.

#### Bulletin of the University of Wisconsin

[1902] Bulletin of the University of Wisconsin, 1902.

#### CAJORI (Florian)

[1890] *The Teaching and History of Mathematics in the United States*, Washington: Government Printing Office, 1890.

#### CAUCHY (Augustin Louis)

- [1815] Sur le nombre de valeurs qu'une fonction peut acquérir lorsqu'on y permute de toutes les manières possibles les quantités qu'elles renferment, *Journal de l'École polytechnique*, 10 (1815), pp. 1–28.
- [1844] *Exercices d'analyse et de physique mathématique*, Paris: Bachelier, 1844; pp. 151–252.

#### CAYLEY (Arthur)

- [1854a] On the theory of groups as depending of the symbolic equation  $\theta^n = 1$ , *Philosophical Magazine*, 7 (1854), pp. 40–47.
- [1854b] On the theory of groups as depending of the symbolic equation  $\theta^n = 1$ , 2nd part, *Philosophical Magazine*, 7 (1854), pp. 408–409.
- [1859] On the theory of groups as depending of the symbolic equation  $\theta^n = 1$ , 3rd part, *Philosophical Magazine*, 18 (1859), pp. 34–37.

#### CHARTIER (Roger)

[1989] Le monde comme représentation, Annales. Histoire, sciences sociales, 6 (1989), pp. 1505–1520.

#### CHARTIER (Roger), ed.

[2003] Pratiques de la lecture, Petite bibliothèque Payot, 2003.

#### CIFOLETTI (Giovanna)

[1998] L'histoire culturelle des mathématiques, in Guesnerie (Roger) & Hartog (François), eds., Des sciences et des techniques: un débat, Paris: EHESS, 1998, pp. 163–170.

#### COCKLE (James)

[1859] Observations on the theory of equations of the fifth degree, *Philosophical Magazine*, 18 (1859), pp. 50–54, 342–344, 508–510; 19 (1860) pp. 197–203.

#### CONNES (Alain)

[2005] La Pensée d'Evariste Galois et le Formalisme moderne, 2005; conférence à la BNF, http://www.alainconnes.org/docs/ galoistext.pdf.

#### Corry (Leo)

- [1989] Linearity and reflexivity in the growth of mathematical knowledge, *Science in Context*, 3(2) (1989), pp. 409–440.
- [2004] Modern Algebra and the Rise of Mathematical Structure, Basel: Birkhäuser, 2nd edition, 2004.

#### COUTURAT (Louis)

[1898] Sur les rapports des nombres et de la grandeur, *Revue de métaphysique et de morale*, 6 (1898), pp. 422–447.

#### CRILLY (Tony)

[2006] Arthur Cayley. Mathematician Laureate of the Victorian Age, Baltimore: John Hopkins University Press, 2006.

#### **CROSLAND** (Maurice)

[1992] Science under Control. The French Academy of Science 1795–1814, Cambridge: Cambridge Univ. Press, 1992.

#### DAHAN-DALMEDICO (Amy)

[1983] Résolubilité des équations par radicaux et premier mémoire d'Évariste Galois, in Walusinski (Gilbert), ed., Présence d'Évariste Galois (1811–1832), Paris: A.P.M.E.P., 1983, pp. 43–49.

#### DALMAS (André)

[1956] Évariste Galois, révolutionnaire et géomètre, Paris: Fasquelle, 1956.

#### FRIEDELMEYER (Jean-Pierre)

[1991] Émergence du concept de groupe à travers le problème de la résolution des équations algébriques, *Brochure APMEP*, 83 (1991).

#### DEDEKIND (Richard)

[1981] Eine Vorlesung über Algebra, in Scharlau (Winfried), ed., Richard Dedekind 1831–1981. Eine Würdigung zu seinem Geburtstag, 1981, pp. 59– 108.

#### DHOMBRES (Jean) & DHOMBRES (Nicole)

[1989] Naissance d'un pouvoir. Sciences et savants en France, 1793–1824, Paris: Payot, 1989.

#### DICKSON (Leonard Eugene)

[1896–1897] The Analytic Representation of Substitutions on a Power of a Prime Number of Letters with a Discussion of the Linear Group, Annals of Mathematics, 11 (1896–1897), pp. 67–120.

#### DRACH (Jules)

- [1895] Sur l'application aux équations différentielles de méthodes analogues à celles de Galois, Comptes rendus hebdomadaires des séances des l'Académie des sciences, 120 (1895), pp. 73–76.
- [1898] Essai sur la théorie générale de l'itération et sur la classification des transcendances, Annales scientifiques de l'École Normale Supérieure, 15 (1898), pp. 243–384.

#### **DUPUY** (Paul)

[1896] La vie d'Évariste Galois, Annales scientifiques de l'École Normale Supérieure, 13 (1896), pp. 197–266; réed. Paris: Jacques Gabay, 1992.

#### DURAND-RICHARD (Marie-José)

[1996] L'école algébrique anglaise: les conditions conceptuelles et institutionnelles d'un calcul symbolique comme fondement de la connaissance, in Goldstein (Catherine), Gray (Jeremy) & Ritter (Jim), eds., L'Europe mathématique. Histoires, mythes, identités Éditions de la Maison des Sciences de l'Homme, 1996, pp. 445–477.

#### EHRHARDT (Caroline)

- [2007] Évariste Galois et la théorie des groupes. Fortune et réélaborations (1811– 1910), Ph.D. Thesis, EHESS, 2007.
- [2008] Évariste Galois, un candidat à l'École préparatoire en 1829, *Revue d'histoire des mathématiques*, 14(2) (2008), pp. 289–328.
- [2010a] A Social History of the "Galois Affair" at the Paris Academy of Science (1831), *Science in Context*, 23(1) (2010), pp. 91–119.
- [2010b] Histoire sociale des mathématiques, *Revue de synthèse*, 131(4) (2010), pp. 489–493.
- [2010c] La naissance posthume d'Évariste Galois, *Revue de synthèse*, 131(4) (2010), pp. 543–568.
- [2011a] Évariste Galois. La fabrication d'une icône mathématique, Paris: Éditions de l'EHESS, 2011.

- [2011b] D'un mémoire mathématique à une théorie. Réélaborations des travaux d'Evariste Galois au XIX<sup>e</sup> siècle, Paris: Éditions du CTHS, 2011.
- [2011c] The quarrels between Joseph Liouville and Guillaume Libri. Recurrent feund at the French Academy of Sciences in the middle of the Nineteenth Century, *Historia Mathematica*, 38 (2011), pp. 389–414.

#### Ferreirós (José)

[1999] Labyrinth of Thought. A History of Set Theory and its Role in Modern Mathematics, Basel: Birkhäuser, 1999.

#### GALOIS (Évariste)

- [1846] Œuvres mathématiques d'Évariste Galois, *Journal de mathématiques pures et appliquées*, 11 (1846), pp. 381–444; rééd. Paris: Jaques Gabay, 1989.
- [1962/1997] Écrits et mémoires mathématiques. Édition critique intégrale des manuscrits et publications d'Évariste Galois, Paris: Gauthier-Villars, 1962/1997; rééd. Paris: Jaques Gabay, 1997.

#### GALUZZI (Massimo)

[2001] Galois's Note on the Approximative Solution of Numerical Equations (1830), *Archive for History of Exact Sciences*, 56 (2001), pp. 29–37.

#### GAUTHIER (Sébastien)

[2007] *La géométrie des nombres comme discipline*, Ph.D. Thesis, Université Pierre et Marie Curie, Paris 6, 2007.

#### GOLDSTEIN (Catherine)

- [1994] La théorie des nombres dans les notes aux Comptes rendus de l'Académie des sciences (1870–1914): un premier examen, *Rivista di* storia della scienza, 2(2) (1994), pp. 137–160.
- [1995] Un théorème de Fermat et ses lecteurs, Saint-Denis: Presses universitaires de Vincennes, 1995.
- [1999] Sur la question des méthodes quantitatives en histoire des mathématiques: le cas de la théorie des nombres en France (1870-1914), Acta historiae rerum nec non technicarum, 3 (1999), pp. 187–214.

#### GOLDSTEIN (Catherine) & SCHAPPACHER (Norbert)

[2007] A book in search of a discipline, in Goldstein (Catherine), Schappacher (Norbert) & Schwermer (Joachim), eds., *The Shaping of Arithmetic after C. F. Gauss's* Disquisitiones Arithmeticae, Heidelberg/Berlin: Springer, 2007, pp. 3–65.

#### HABERMAS (Jürgen)

[1993] L'espace public, Paris: Payot, 1993; 1st ed. 1962.

#### HALBWACHS (Maurice)

- [1997] La mémoire collective, Paris: Albin Michel, 1997; 1st ed. PUF, 1950.
- [2008] La topographie légendaire des évangiles en Terre Sainte. Étude de mémoire collective, Paris: PUF, 2008; 1st ed. Paris: Bibliothèque de philosophie contemporaine, 1941.

#### HENRY (Charles)

[1879] Manuscripts de Sophie Germain. Documents inédits, *Revue philosophique de la France et de l'étranger*, 8 (1879), pp. 619–641.

#### HIRANO (Yoichi)

[1984] La diffusion des idées de Galois et le rôle de Camille Jordan (1838–1922). La formation de la théorie des groupes et l'apport de Camille Jordan, Ph.D. Thesis, EHESS, 1984.

#### HUNGER-PARSHALL (Karen)

- [1998] James Joseph Sylvester. Life and Work in Letters, Oxford: Oxford Univ. Press, 1998.
- [2006] James Joseph Sylvester. Jewish Mathematician in a Victorian World, Baltimore: John Hopkins University Press, 2006.

#### INFANTOZZI (Carlos Alberto)

[1968] Sur la mort d'Évariste Galois, Revue d'histoire des sciences, 21 (1968), pp. 157-160.

#### INFELD (Leopold)

[1948] Whom the Gods Love. The Story of Evariste Galois, New York: Whittlesey House, 1948.

#### JORDAN (Camille)

[1970] Traité des substitutions et des équations algébriques, Paris: Gauthier-Villars, 1970.

#### KAISER (David), ed.

[2005] Pedagogy and the Practice of Science. Historical and Contemporary Perspectives, Cambridge: MIT Press, 2005.

#### KIERNAN (Melvin B.)

[1971] The Development of Galois Theory from Lagrange to Artin, *Archive for History of Exact Sciences*, 8 (1971), pp. 40–152.

#### KIRKMAN (Thomas)

[1860] On the theory of groups and many-valued functions, Memoirs and Proceedings of the Manchester Literary and Philosophical Society, 1 (1860), pp. 274–398.

#### KRONECKER (Leopold)

[1853] Über die algebraisch auflösbaren Gleichungen, Monatsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin, 1853, pp. 365–374.

#### LACROIX (Sylvestre-François) & POISSON (Siméon-Denis)

[1828–1831] Rapport sur le mémoire de M. Galois relatif aux Conditions de résolubilité par radicaux (séance du 4 juillet 1831), Procès verbaux des séances de l'Académie des sciences, 9 (1828–1831), pp. 660–661.

#### LAGRANGE (Joseph-Louis)

[1808] Traité de la résolution des équations numériques de tous les degrés, Paris: Duprat, 1808.

#### LEPETIT (Bernard)

[1999] Carnets de croquis. Sur la connaissance historique, Paris: Albin Michel, 1999.

#### LIE (Sophus)

[1895] Influence de Galois sur le développement des mathématiques, in Dupuy (Paul), ed., Le centenaire de l'École normale, Paris: Hachette, 1895.

#### MANSION (Paul)

[1910] La légende de Galois, Annales de la Société Scientifique de Bruxelles, 1910, pp. 104–105.

#### MARGAIRAZ (Dominique)

[2005] *François de Neufchâteau. Biographie intellectuelle*, Paris: Publications de la Sorbonne, 2005.

#### MARTINI (Laura)

[1999] The First Lectures in Italy on Galois Theory: Bologna, 1886-1887, *Historia Mathematica*, 26 (1999), pp. 201–223.

#### MILLER (George Abram)

[1899–1900] Note on the theory of substitutions, Annals of Mathematics, 1 (1899–1900), pp. 71–73.

#### MORTON STEWART (Charles)

[1896] Twenty-first Annual Report of the President of the Johns Hopkins University, 1896.

#### NETTO (Eugen)

[1882] Substitutionentheorie und ihre Anwendung auf die Algebra, Leipzig: Teubner, 1882.

#### Nový (Luboš)

[1968] L'école algébrique anglaise, *Revue de synthèse*, 84 (1968), pp. 211–222.

#### OLESKO (Kathryn), ed.

[1989] Science in Germany: the intersection of institutional and intellectual issues, Osiris, vol. 5, University of Chicago Press, 1989.

#### PERROT (Jean-Claude)

- [1992] Quelques préliminaires à l'intelligence des textes économiques, in Une histoire intellectuelle de l'économie politique aux XVII<sup>e</sup> et XVIII<sup>e</sup> siècles, Paris: EHESS, 1992.
- [1998] Histoire des sciences, histoire concrète de l'abstraction, in Guesnerie (Roger) & Hartog (François), eds., Des sciences et des techniques: un débat, Paris: EHESS, 1998, pp. 25–37.

#### PICARD (Émile)

- [1883] Sur les groupes de transformation des équations différentielles linéaires, Comptes rendus hebdomadaires des séances des l'Académie des sciences, 46 (1883), pp. 1131–1134.
- [1895] Sur les groupes de transformation des équations différentielles linéaires, *Mathematische Annalen*, 46 (1895), pp. 161–166.
- [1896] Traité d'analyse, vol. 3, Gauthier-Villars, 1896.
- [1897] Introduction, in Œuvres mathématiques d'Evariste Galois, publiées sous les auspices de la Société mathématiques de France, Paris: Gauthier-Villars, 1897.

#### PYENSON (Lewis)

[1983] Neohumanism and the Persistence of Pure Mathematics in Wilhelmian Germany, Philadelphia: American Philosophical Society, 1983.

#### RADLOFF (Ivo)

[2002] Évariste Galois, Principles and Applications, *Historia Mathematica*, 29 (2002), pp. 114–137.

#### RASPAIL (François-Vincent)

[1830] Coteries scientifiques, Annales des sciences d'observations, 3 (1830), pp. 150–152.

#### REVEL (Jacques), ed.

[1996] Jeux d'échelles. La micro-analyse à l'expérience, Paris: EHESS/Gallimard, 1996.

#### **ROSENTAL** (Claude)

[2003] La trame de l'évidence. Sociologie de la démonstration en logique, Paris: PUF, 2003.

#### ROCHE (Daniel)

[1988] De l'histoire sociale à l'histoire des cultures: le métier que je fais, in Les Républicains des lettres. Gens de culture et Lumières au XVIII<sup>e</sup> siècle, Paris: Fayard, 1988, pp. 7–21.

#### ROTHMAN (Tony)

[1989] Genius and Biographers: the Fictionalization of Évariste Galois, *American Mathematical Monthly*, 89(2) (1989), pp. 84–106.

#### ROWE (David E.)

[1998] Mathematics in Berlin, 1810–1933, in Begehr (H.G.W.), Koch (H.), Kramer (J.), Schappacher (N.) & Thiele (E.-J.), eds., *Mathematics in Berlin*, Berlin: Birkhäuser, 1998, pp. 9–26.

#### SAIGEY (Jacques)

[1829] Abel. Nécrologie, Annales des sciences d'observations, 2 (1829), pp. 317– 321.

#### SCHANDELER (Jean-Pierre)

[2000] Les interprétations de Condorcet. Symboles et concepts (1794–1894), Oxford: Voltaire Foundation, 2000.

#### SCHOTTENFELS (Ida May)

[1899–1900] Two non-isomorphic simple groups of the same order 20160, Annals of Mathematics, 1 (1899–1900), pp. 147–152.

#### SEGALEN (Victor)

[1979] *Le double Rimbaud*, Montpellier: Fata Morgana, 1979; 1st ed. Mercure de France, 15 April 1906.

#### SERRET (Joseph-Alfred)

- [1849] Cours d'algèbre supérieure, Paris: Bachelier, 1849.
- [1850] Sur le nombre de valeurs que peut prendre une fonction quand on y permute les lettres qu'elle renferme, *Journal de mathématiques pures et appliquées*, 15 (1850), pp. 1–44.
- [1854] Cours d'algèbre supérieure, Paris: Bachelier, 2nd edition, 1854.
- [1866] Cours d'algèbre supérieure, Paris: Gauthier-Villars, 3rd edition, 1866.

#### TANNERY (Jules)

- [1906] Manuscrits et papiers inédits d'Évariste Galois, Bulletin des sciences mathématiques, 30 (1906), pp. 226–248, 255–263; 31 (1907), pp. 275– 308.
- [1909] La vie et l'œuvre d'Evariste Galois, *Revue scientifique*, July 1909, 1909, pp. 129–132.

TATON (René)

- [1947] Les relations scientifiques d'Évariste Galois avec les mathématiciens de son temps, *Revue d'histoire des sciences*, 1 (1947), pp. 114–130.
- [1971] Sur les relations mathématiques d'Augustin Cauchy et Évariste Galois, *Revue d'histoire des sciences*, 24 (1971), pp. 123–148.
- [1983] Évariste Galois et ses contemporains, in Walusinski (Gilbert), ed., Présence d'Évariste Galois (1811–1832), Paris: Éditions de l'Association des Professeurs de Mathématiques, 1983, pp. 5–11.
- [1993] Évariste Galois et ses biographes. De l'histoire aux légendes, Sciences et Techniques en Perspective, 26 (1993), pp. 155–172.

#### TOTI RIGATELLI (Laura)

- [1989] La Mente Algebrica. Storia dello sviluppo della teoria di Galois nel XIX secolo, Bramante: Busto Arsizio, 1989.
- [1996] Évariste Galois (1811–1832), Basel: Birkhäuser, 1996.

#### VERDIER (Norbert), ed.

[2003] Évariste Galois, le mathématicien maudit, Les génies de la science, Paris: Pour la science, 2003.

#### VESSIOT (Ernest)

[1892] Sur l'intégration des équations différentielles linéaires, Annales scientifiques de l'École Normale Supérieure, 9 (1892), pp. 197–280.

#### VOGT (Henri)

[1895] Leçons sur la résolution algébrique des équations, Paris: Nony, 1895.

#### WANTZEL (Pierre Laurent)

[1845] De l'impossibilité de résoudre toutes les équations algébriques avec des radicaux, *Nouvelles annales de mathématiques*, 4 (1845), pp. 57–66.

#### WARWICK (Andrew)

[2003] Masters of Theory. Cambridge and the Rise of Mathematical Physics, Chicago: The University of Chicago Press, 2003.

#### WEBER (Heinrich)

[1895] Lehrbuch der Algebra, Braunschweig: Vieweg und Sohn, 1895.

#### WINTER (Maximilien)

[1910] Les caractères de l'algèbre moderne, *Revue de métaphysique et de morale*, 18(4) (1910), pp. 491–529.

#### WUSSING (Hans)

[1984] The Genesis of the Abstract Group Concept, Cambridge (Mass.): MIT Press, 1984.