ALAIN ASPECT

Experimental Tests of Bell's Inequalities in Atomic Physics

Les rencontres physiciens-mathématiciens de Strasbourg - RCP25, 1992, tome 43
« Conférences de A. Aspect, B. Carter, R. Coquereaux, G.W. Gibbons, Ch. Kassel, Y. Kosman-Schwarzbach, S. Majid, G. Maltsiniotis, P. Pansu, G.A. Vilkovisky, Z. Wojtkowiak », , exp. nº 1, p. 1-26
<http://www.numdam.org/item?id=RCP25_1992__43__1_0>

(Ce texte reprend une publication dans Atomic Physics 8, Editeurs Lindgren, Rosén, Svanberg).

EXPERIMENTAL TESTS OF BELL'S INEQUALITIES IN ATOMIC PHYSICS

Alain Aspect

Institut d'Optique Théorique et Appliquée
Bâtiment 503 - Centre Universitaire d'Orsay - BP 43
91406 ORSAY CEDEX - FRANCE

1 - INTRODUCTION

Bell's Inequalities provide a quantitative criterion to test some reasonable Supplementary Parameters Theories versus Quantum Mechanics. Thanks to Bell[1], the debate about the possibility of completing Quantum Mechanics by an underlying substructure has bee brought into the experimental domain.

The motivations for considering supplementary parameters will be found in the analysis of the famous Einstein-Podolsky-Rosen Gedankenexperiment[2]. Introducing a reasonable Locality Condition, one can derive Bell's theorem, which states

(i) that Local Supplementary Parameters Theories are constrained by Bell's Inequalities :

(ii) that certain predictions of Quantum Mechanics sometimes violate Bell's Inequalities.

We will point out that a fundamental assumption for the conflict is the Locality assumption. We will show that in a more sophisticated version of the E.P.R. thought experiment ("timing experiment"), the Locality Condition may be considered as a consequence of Einstein's Causality, preventing faster-than-light interactions.

The purpose of this discussion is to convince the reader that the formalism leading to Bell's Inequalities is very general and reasonable. What is surprising is that it conflicts with Quantum Mechanics.

As a matter of fact, situations exhibiting such a conflict are very rare, and it was necessary to design special experiments for getting a sensitive test. Atomic physics is the field where the experiments that follow most closely the ideal scheme of the Gedankenexperiment have been carried out. We will review these experiments, and their results.

2 - WHY SUPPLEMENTARY PARAMETERS ? THE EINSTEIN-PODOLSKY-ROSEN-BOHM GEDANKENEXPERIMENT

Experimental scheme

Let us consider the optical variant of the E.P.R. Gedankenexperiment modified by Bohm[3]. A source S emits a pair of photons with different energies, ν_1 and ν_2 , counterpropagating along $\pm O\vec{z}$ (Fig. 1). Suppose that the polarization part of their state vector is :

$$|\Psi(\nu_1,\nu_2)\rangle = (1/\sqrt{2})\Big[|x,x\rangle + |y,y\rangle\Big] \qquad (1)$$

where $|x\rangle$ and $|y\rangle$ are linear polarizations states.

We perform on these photons linear polarization measurements. The analyzer I in orientation $\vec{a}$, followed by two detectors, gives + or - result, corresponding to a linear polarization found parallel or perpendicular to $\vec{a}$. Analyzer II, in orientation $\vec{b}$, acts similarly.*

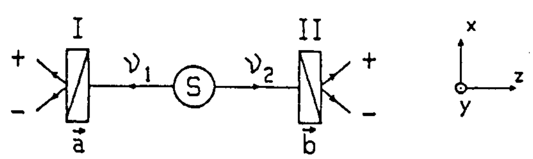

Fig. 1. Einstein-Podolsky-Rosen-Bohm Gedankenexperiment with photons. The two photons ν_1 and ν_2, emitted in the state (1), are analyzed by linear polarizers in orientations $\vec{a}$ and $\vec{b}$. One can measure the probabilities of single or joint detections after the polarizers.

* There is a one-to-one correspondence with the Gedankenexperiment dealing with a pair of 1/2 spin particles, in a singlet state, and analyzed by two Stern-Gerlach filters[3].

It is easy to derive the Quantum Mechanical predictions for these measurements, single or in coincidence.

Let $P_{\pm}(\vec{a})$ be the probability of getting the result $\pm$ for ν_1 ; similarly $P_{\pm}(\vec{b})$ is related to ν_2. Quantum Mechanics predicts :

$$\begin{aligned} P_+(\vec{a}) &= P_-(\vec{a}) = 1/2 \\ P_+(\vec{b}) &= P_-(\vec{b}) = 1/2 \end{aligned} \qquad (2)$$

Let $P_{\pm\pm}(\vec{a},\vec{b})$ be the probability of joint detection of ν_1 in channel $\pm$ of I (in orientation $\vec{a}$), and of ν_2 in channel $\pm$ of II ($\vec{b}$). Quantum Mechanics predicts :

$$\begin{aligned} P_{++}(\vec{a},\vec{b}) &= P_{--}(\vec{a},\vec{b}) = \frac{1}{2}\cos^2(\vec{a},\vec{b}) \\ P_{+-}(\vec{a},\vec{b}) &= P_{-+}(\vec{a},\vec{b}) = \frac{1}{2}\sin^2(\vec{a},\vec{b}) \end{aligned} \qquad (3)$$

Correlations

In the special situation $(\vec{a},\vec{b}) = 0$ one finds

$$P_{++}(\vec{a},\vec{b}) = P_{--}(\vec{a},\vec{b}) = 1/2$$

while

$$P_{+-}(\vec{a},\vec{b}) = P_{-+}(\vec{a},\vec{b}) = 0$$

So, if ν_1 is found in the + channel of I (the probability of which is 50 %), we are sure to find ν_2 in the + channel of II (and similarly for the - channels). There is a strong correlation betwee the results of measurements on ν_1 and ν_2.

A convenient way of displaying these correlations is the polarization correlation coefficient :

$$E(\vec{a},\vec{b}) = P_{++}(\vec{a},\vec{b}) + P_{--}(\vec{a},\vec{b}) - P_{+-}(\vec{a},\vec{b}) - P_{-+}(\vec{a},\vec{b}) \qquad (4)$$

The prediction of Quantum Mechanics is

$$E_{MQ}(\vec{a},\vec{b}) = \cos 2(\vec{a},\vec{b}) \qquad (5)$$

For $(\vec{a},\vec{b}) = 0$, we find $E_{MQ}(0) = 1$, i.e. a complete correlation

Supplementary parameters

Correlations between distant measurements on two systems that have separated may be easily understood in terms of some common properties of the two systems. Let us consider again the correlatio of polarization measurements in the case $(\vec{a},\vec{b}) = 0$. When we find +

for ν_1, we are sure to find + for ν_2. We are thus led to admit that there is some property (Einstein said "an element of physical reality") pertaining to this particular pair, and determining the result ++. For another pair, the results will be -- ; the invoked property is different.

Such properties, differing from one pair to another one, are not taken into account by the Quantum Mechanical state vector $|\Psi(1,2)>$ which is the same for all pairs. This is why Einstein concluded that Quantum Mechanics is not complete. And this is why such properties are referred to as "supplementary parameters" (sometimes called "hidden-variables").

As a conclusion, one can hope to "understand" the E.P.R. correlations by such a classical-looking picture, involving supplementary parameters differing from one pair to another one. It can be hoped to recover the Quantum Mechanical predictions when averaging over the supplementary parameters. It seems that so was Einstein's position[4]. At this stage, a commitment to this view point is just a matter of taste.

Remark. Since Einstein spoke of "an element of the physical reality", some authors call these theories invoking supplementary parameters "Realistic Theories"[5].

3 - BELL'S INEQUALITIES

Formalism

Bell tried to translate into mathematics the preceding discussion, by introducing explicitly supplementary parameters, denoted λ. Their distribution on an ensemble of emitted pairs is specified by a probability distribution $\rho(\lambda)$, such that

$$\rho(\lambda) \geq 0 \text{ and } \int d\lambda\ \rho(\lambda) = 1 \tag{6}$$

For a given pair, characterized by a given λ, the results of measurement will be

$$A(\lambda,\vec{a}) = \begin{cases} +1 \\ \text{or} \\ -1 \end{cases} \quad \text{at analyzer I (orientation } \vec{a}) \qquad\qquad (6')$$

$$B(\lambda,\vec{b}) = \begin{cases} +1 \\ \text{or} \\ -1 \end{cases} \quad \text{at analyzer II (orientation } \vec{b})$$

A particular theory must be able to supply explicitly the functions $\rho(\lambda)$, $A(\lambda,\vec{a})$ and $B(\lambda,\vec{b})$.

It is then easy to express the probabilities of various results.

For instance $P_+(\vec{a}) = \frac{1}{2}\int d\lambda\ \rho(\lambda)\left[A(\lambda,\vec{a}) + 1\right]$ etc...

In particular, we will use the correlation function :

$$E(a,b) = \int d\lambda\ \rho(\lambda)\ A(\lambda,\vec{a})\ B(\lambda,\vec{b}) \tag{7}$$

A (naive) exemple

Let us suppose that the two photons of a pair are emitted with the same linear polarization, defined by its angle λ with $\vec{Ox}$ (Fig. 2)

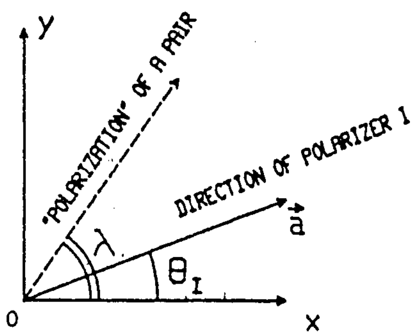

Fig. 2. Our example. Each pair has a "direction of polarization," defined by λ.

The probability distribution is taken to be isotropic :

$\rho(\lambda) = 1/2\pi$

As a simple model for the polarizer I we assume that we get the result + 1 if

$|\Theta_I - \lambda| \leq \frac{\pi}{4}$ or $|\Theta_I - \lambda| \geq 3\frac{\pi}{4}$,

The result - 1 is obtained for

$\frac{\pi}{4} < |\Theta_I - \lambda| < 3\pi/4$

The response can thus be written

$$A(\lambda,\vec{a}) = \frac{\cos 2(\Theta_I - \lambda)}{|\cos 2(\Theta_I - \lambda)|}$$

Similarly

$$B(\lambda,\vec{b}) = \frac{\cos 2(\Theta_{II}-\lambda)}{|\cos 2(\Theta_{II}-\lambda)|}$$

With this model, we find

$P_+(\vec{a}) = P_-(\vec{a}) = P_+(\vec{b}) = P_-(\vec{b}) = 1/2$ which is identical to the Quantum Mechanical result.

As correlation function, we find :

$$E(\vec{a},\vec{b}) = 1 - 4\,\frac{|\Theta_I - \Theta_{II}|}{\pi} = 1 - 4\,\frac{|(\vec{a},\vec{b})|}{\pi}$$

Like the Quantum Mechanical result, $E(\vec{a},\vec{b})$ depends only on the relative angle $(\vec{a},\vec{b})$.

Fig. 3 shows a comparison between this result and the Quantum Mechanical prediction.

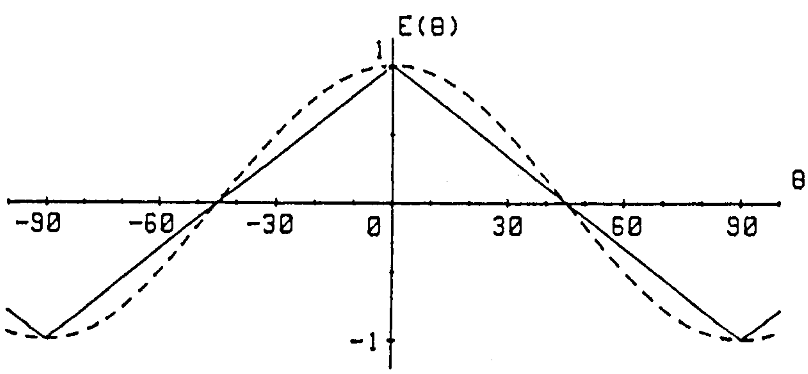

Fig. 3. Polarization correlation coefficient, as a function of the relative orientation of the polarizers.

---- : Calculated by Quantum Mechanics ;

—— : Given by our simple model.

The agreement is not too bad. It might be hoped that some more complicated model will be able to reproduce exactly the Quantum Mechanical predictions.

Bell's discovery is the fact that this search is hopeless.

Inequalities

Let us consider the quantity

$$s = A(\lambda,\vec{a}).B(\lambda,\vec{b})-A(\lambda,\vec{a}).B(\lambda,\vec{b}') + A(\lambda,\vec{a}').B(\lambda,\vec{b})+A(\lambda,\vec{a}').B(\lambda,\vec{b}')$$

$$= A(\lambda,\vec{a})\left[B(\lambda,\vec{b})-B(\lambda,\vec{b}')\right] + A(\lambda,\vec{a}')\left[B(\lambda,\vec{b}) + B(\lambda,\vec{b}')\right] \qquad (8)$$

Remembering that the four numbers A, B take only the values ± 1, we find that

$$s(\lambda,\vec{a},\vec{a}',\vec{b},\vec{b}') = \pm 2$$

The average over λ is therefore included between + 2 and − 2, i.e.

$$-2 \leq \int d\lambda\ \rho(\lambda).s(\lambda,\vec{a},\vec{a}',\vec{b},\vec{b}') \leq 2$$

According to (7), we rewrite this

$$-2 \leq S(\vec{a},\vec{a}',\vec{b},\vec{b}') \leq 2 \qquad (9)$$

with

$$S = E(\vec{a},\vec{b})-E(\vec{a},\vec{b}')+E(\vec{a}',\vec{b})+E(\vec{a}',\vec{b}')$$

These are B.C.H.S.H. inequalities, i.e. Bell's inequalities generalized by Clauser, Horne, Shimony, Holt [6]. They bear upon a combination of four polarization correlation coefficients, measured in four orientations of the polarizers. S is thus a measurable quantity.

4 - CONFLICT WITH QUANTUM MECHANICS

Evidence

Let us take the particular set of orientations of Fig. 4a. Replacing the E's by their Quantum Mechanical values (5) for pairs in state (1), we obtain :

$$S_{MQ} = 2\sqrt{2}$$

This Quantum Mechanical prediction strongly violates the upper limit of inequalities (9). We thus find it impossible to reconcile the formalism defined in (6) and (6') with the predictions of Quantum Mechanics for the particular (E.P.R.-type) state (1).

General study

We look for the greatest conflict, and we derivate S with

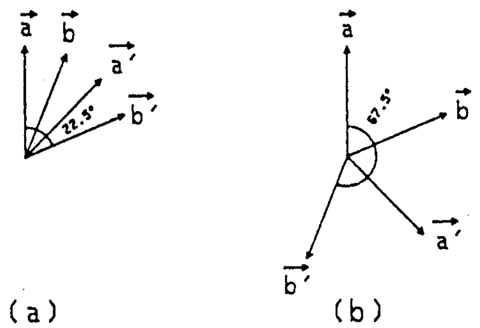

Fig. 4. Orientations yielding the largest conflict between Bell's Inequalities and Quantum Mechanics.

respect to the three angles $(\vec{a},\vec{b})$, $(\vec{b},\vec{a}')$ and $(\vec{a}',\vec{b}')$ (which are independent). S_{MQ} is extremum if

$$(\vec{a},\vec{b}) = (\vec{b},\vec{a}') = (\vec{a}',\vec{b}') = \Theta$$

and it takes the value

$$S_{MQ}(\Theta) = 3 \cos 2\Theta - \cos 6\Theta$$

Derivating now with respect to Θ, we obtain the maximum and minimum values of S_{MQ}.

$$S_{MQ}^{Max} = 2\sqrt{2} \qquad \text{for } \Theta = \pi/8$$

$$S_{MQ}^{Min} = -2\sqrt{2} \qquad \text{for } \Theta = 3\pi/8 \qquad (10)$$

The corresponding orientations are displayed in Fig. 4.

Figure 5 displays the variations of $S_{MQ}(\Theta)$, and the limits given by B.C.H.S.H. inequalities. One sees that the conflict is serious.

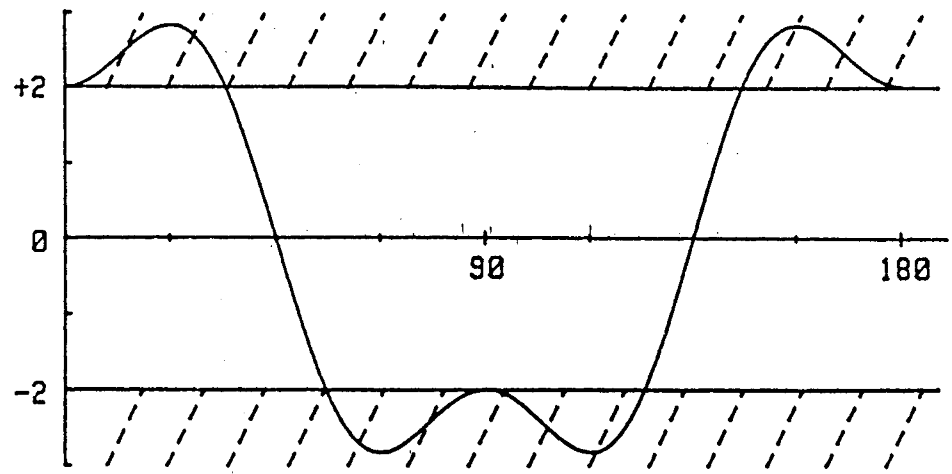

Fig. 5. $S(\Theta)$ as predicted by Quantum Mechanics for pairs in state (1'). The conflict arises in the /// zone.

5 - DISCUSSION OF THE HYPOTHESES

To try to understand which part of the formalism causes this conflict, let us point out the hypotheses implied by formalism (6) and (6'). The supplementary parameters λ have been introduced for explaining the E.P.R. correlations by some common properties of the two photons. This point has already been discussed.

The used formalism is deterministic. When λ is fixed, then the results of measurements $A(\lambda,\vec{a})$ and $B(\lambda,\vec{b})$ are certain, i.e. λ determines the result. It might be thought that it is the reason for the conflict with Quantum Mechanics. But Bell[1], and Clauser and Horne[7] have exhibited Stochastic Supplementary Parameters Theories that are not deterministic, and which nevertheless lead to Bell's Inequalities. The deterministic character does not seem sufficient to lead to a conflict*.

As stressed by Bell, the formalism follows a Locality Condition. The result of measurement at I, $A(\lambda,\vec{a})$, does not depend on the orien-

* This conclusion is not shared by all authors. For instance, A. FINE [8] argues that the stochastic theories of Bell or of Clauser and Horne achieve no further generality, since they can be mimicked by a deterministic theory.

tation $\vec{b}$ of the remote polarizer II, and vice-versa, nor does $\rho(\lambda)$ (i.e. the way in which pairs are emitted) depend on the orientations $\vec{a}$ and $\vec{b}$. Bell's Inequalities no longer hold if we don't make the locality assumption (It is easy to see that the demonstration of § 3 fails with quantities such as $A(\lambda,\vec{a},\vec{b})$ or $\rho(\lambda,\vec{a},\vec{b})$).

As an abstract of this discussion, we can say that Bell's theorem states a conflict between Local Supplementary Parameters Theories and certain Quantum Mechanical predictions. It yields a quantitative criterion for this conflict, that will allow us to design sensitive experiments.

6 - GEDANKENEXPERIMENT WITH VARIABLE ANALYZERS : THE LOCALITY CONDITION AS A CONSEQUENCE OF EINSTEIN'S CAUSALITY.

In static experiments, in which the polarizers are held fixed for the whole duration of a run, the Locality Condition must be stated as an assumption. Although highly reasonable, it is not prescribed by any fundamental physical law. To quote J. Bell "the settings of the instruments are made sufficiently in advance to allow them to reach some mutual rapport by exchange of signals with velocity less than or equal to that of light". If such interactions existed, the Locality Condition would no longer hold for static experiments, nor would Bell's Inequalities.

Bell thus insisted upon the importance of "experiments of the type proposed by Bohm and Aharonov [4], in which the settings are changed during the flight of the particles"*. In such a timing-experiment, the locality condition would become a consequence of Einstein's Causality that prevents any faster-than-light influence.

As shown in our 1975 proposal[9], it is sufficient to switch each polarizer's orientation between two particular settings ($\vec{a}$ and $\vec{a}'$ for I, $\vec{b}$ and $\vec{b}'$ for II). It then becomes possible to test experimentally a larger class of Supplementary Parameters Theories: those obeying Einstein's Causality. In such theories, the response of polarizer I at time t is allowed to depend on the orientation $\vec{b}$ (or $\vec{b}'$) of II at time $t - L/C$ (L being the distance between the polarizers). A similar retarded dependence is considered for the way in which pairs are emitted at the source (characterized by the supplementary parameters distribution). For random switching times, with both sides uncorrelated, the predictions of these more general theories are constrained by generalized Bell's Inequalities[9].

On the other hand, it is easy to show that the polarization correlations predicted by Quantum Mechanics depend only on the

* The idea was already expressed in Bohm's book[3].

orientations $\vec{a}$ or $\vec{a}'$ and $\vec{b}$ or $\vec{b}'$ at the very time of the measurements, and do not involve any retardations terms such as L/C. For a suitable choice of the set of orientations $(\vec{a},\vec{a}',\vec{b},\vec{b}')$ - for instance the sets displayed in Fig. 4 - the Quantum Mechanical predictions still conflict with generalized Bell's Inequalities.

Such a timing-experiment with variable analyzers would thus provide a test of Supplementary-Parameters-Theories, obeying Einstein's Causality, versus Quantum Mechanics.

7 - GENERAL CONSIDERATIONS FOR A REAL SENSITIVE EXPERIMENT

Sensitive situations are seldom

Quantum Mechanics has been so much upheld in a great variety of experiments that Bell's Theorem might appear as an impossibility proof of supplementary parameters. However, situations in which this conflict arises (sensitive situations) are rare ; in 1965 none had been realized.

Bell's Inequalities obviously constrain the whole classical physics, i.e. Classical Mechanics and Classical Electrodynamics, which can be expressed according to the formalism (6) and (6'). (For instance, in Classical Mechanics, we can take as λ the initial positions and velocities...). Moreover, in a situation involving two correlated measurements onto two separated subsystems, Quantum Mechanics will very seldom predict a violation of Bell's Inequalities. Without being exhaustive, we can point out to important necessary conditions for a sensitive experiment (according to Quantum Mechanics) :

(i) : the two subsystems must be in a non-factorizing state, such as a singlet state for two spin 1/2 particles, or the similar state (1) for two photons ;

(ii) : for each subsystem, it must be possible to choose the measured quantity among at least two non-commuting observables (such as polarization measurements along directions $\vec{a}$ and $\vec{a}'$ neither parallel nor perpendicular).

As a matter of fact, these are stringent conditions.

Time conditions

As we have seen, the Locality Condition may be derived from Einstein's Causality, if the experiment fulfils some requirements, that can be split in two conditions :

(i) : the measurements onto the 2 subsystems are space-like separated ;

(ii) : the choices of the quantities measured on each subsystem are made at random, and are space-like separated from the measurement on the opposite side. It is obviously much more difficult to fulfil the second condition.

Production of pairs of photons correlated in polarization

As pointed out by C.H.S.H.[6], pairs of photons emitted in suitable atomic radiative cascades are good candidate for a sensitive test. Consider for instance a $(J = 0) \rightarrow (J = 1) \rightarrow (J = 0)$ cascade, in the singlet states of an alkaline earth (Fig. 6). Suppose that we select, with the use of wavelengths filters and collimators, two plane waves of frequencies ν_1 and ν_2 propagating along $-\vec{Oz}$ and $+\vec{Oz}$ (Fig. 7.)

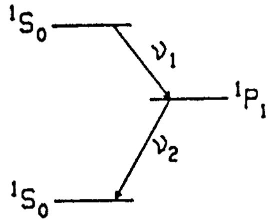

Fig. 6. Radiative cascade emitting pairs of photons correlated in polarization. that only even isotopes can be used[10]).

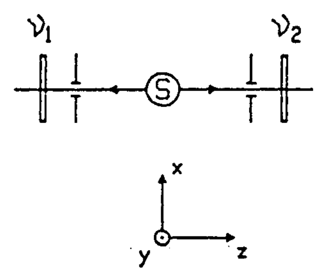

Fig. 7. Ideal configuration (infinitely small solid angles).

It is easy to show, by invoking parity and angular momentum conservation, that the polarization part of the state vector describing the pair (ν_1, ν_2) can be written :

$$(1/\sqrt{2})\left[|R,R\rangle + |L,L\rangle\right] \tag{11}$$

where R and L are circularly polarized states. By expressing $|R\rangle$ and $|L\rangle$ on a linear polarization basis, we obtain the state (1)

$$|\Psi(\nu_1,\nu_2)\rangle = (1/\sqrt{2}) \left[|x,x\rangle + |y,y\rangle\right]$$

We know that such a pair is a good candidate for a sensitive experiment, since corresponding Quantum Mechanical predictions violate Bell's Inequalities.

Real experiment

A real experiment differs from the ideal one in several respects. For instance, the light should be collected in finite solid angles, as large as possible (Fig. 8). One can show[10] that the contrast of the correlation function then decreases, since (5) is replaced by :

$$E_{MQ}(\vec{a},\vec{b}) = F(\vec{u}).\cos 2(\vec{a},\vec{b}) \qquad (12)$$

where $F(\vec{u}) \leq 1$.

Fig. (9) displays F(u) for a $0 \to 1 \to 0$ alkaline-earth cascade (with no hyperfine structure). Fortunately, one can use large angles without great harm. For $u = 32°$ (our experiments), $F(u) = 0.984$.

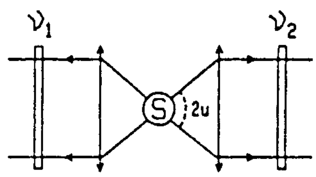

Fig. 8. Realistic configuration, with finite solid angles.

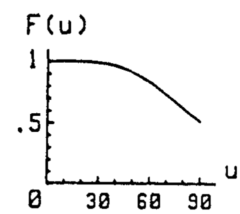

Fig. 9. F(u) for a 0 -1 - 0 cascade.

All other inefficiencies - polarizers defects, accidental birefringences etc... - will similarly lead to a decrease of the correlation function $E(\vec{a},\vec{b})$. The function $S_{MQ}(\Theta)$ (Fig. 5) is then multiplied by a factor less than 1, and the conflict with Bell's Inequalities decreases, or even vanishes.

Therefore, an actual experiment must be carefully designed and every auxiliary effect must be evaluated. Everything must be perfectly controlled since one can assume that a forgotten effect would similarly lead to a decrease of the conflict (one knows for instance that hyperfine structure dramatically decreases F(u), so that only even isotopes can be used [10]).

8 - PREVIOUS EXPERIMENTS (1970-1976)[5,11]

The C.H.S.H. paper[6] in 1969 had shown the possibility of sensitive experiments in atomic physics. Two groups began to build an experiment. Following the C.H.S.H. proposition, they used a simpler experimental scheme, involving one-channel polarizers.

Experiments with one channel polarizer

In this simplified experimental scheme, one uses polarizers that transmit light polarized parallel to $\vec{a}$ (or $\vec{b}$), and blocks the orthogonal one. One thus only detects the + results, and the coincidence measurements only yield $N_{++}(\vec{a},\vec{b})$.

Auxilliary runs are performed with one or both polarizers removed (we denote ∞ the "orientation" of a removed polarizers). We can write relations such as :

$$N(\infty,\infty) = N_{++}(\vec{a},\vec{b}) + N_{+-}(\vec{a},\vec{b}) + N_{-+}(\vec{a},\vec{b}) + N_{--}(\vec{a},\vec{b})$$

$$N_{++}(a,\infty) = N_{++}(\vec{a},\vec{b}) + N_{+-}(\vec{a},\vec{b})$$

etc...

By substitution into inequalities (9), one gets new B.C.H.S.H. inequalities

$$-1 \leq S' \leq 0 \qquad (13)$$

with

$$S' = (1/N(\infty,\infty)) \left[N(\vec{a},\vec{b})-N(\vec{a},\vec{b}')+N(\vec{a}',\vec{b})+N(\vec{a}',\vec{b}')-N(\vec{a}',\infty)-N(\infty,\vec{b})\right]$$

(we omitted the subscripts ++)

For the same orientation sets as previously (Fig. 4), the Quantum Mechanical predictions violate ineq. (13) :

$$\left.\begin{array}{lll} S'^{Max}_{MQ} = \dfrac{\sqrt{2}-1}{2} & \text{for} & \Theta = \pi/8 \\ S'^{Min}_{MQ} = \dfrac{-\sqrt{2}-1}{2} & \text{for} & \Theta = 3\pi/8 \end{array}\right. \qquad (14)$$

The derivation of ineq. (13) requires a supplementary assumption. Since the detection efficiencies are low (due to small angular acceptance and low photomultipliers efficiencies), the probabilities involved in the $E(\vec{a},\vec{b})$ (Eq. (4)) must be redefined on the ensemble of pairs that would be detected with polarizers removed. This procedure is valid only if one assumes a reasonable hypothesis about the detectors. The C.H.S.H. assumption states that, "given that a pair of photons emerges from the polarizers, the probability of their joint detection is independent of the polarizer orientations" (or of their removal) [6]. Clauser and Horne have exhibited another assumption[7], leading to the same inequalities.*

Results

In the Berkeley experiment (Clauser and Freedman[12]), the $4p^2\ ^1S_0 - 4s4p\,^1P_1 - 4s^2\ ^1S_0$ cascade of Calcium was excited by ultraviolet absorption towards a $^1P_1$ upper state. Since the signal was weak, and spurious cascades occurred, it took more than 200 hours of measurement for a significant result. The experiment upheld Quantum Mechanics, and violated inequalities (13) by several standard deviations.

At the same time, in Harvard, Holt and Pipkin[11] found a result in disagreement with Quantum Mechanical predictions, and in agreement with Bell's Inequalities. They excited the $9^1P_1 \rightarrow 7^3P_1 \rightarrow 6^3P_0$ cascade in Mercury 200 by an electron beam. The data accumulation lasted 150 hours.

Clauser[13] repeated their experiment in Mercury 202. He found an agreement with Quantum Mechanics, and a violation of Bell's Inequalities.

In 1976, in Houston, Fry and Thompson[14] used the $7^3S_1 \rightarrow 6^3P_1 \rightarrow 6^3S_0$ cascade in Mercury 200. Their selective excitation involved a C.W. single-line-laser. The signal was several order of magnitude larger than in previous experiments, allowing them to collect the data in a period of 80 minutes. Their result was in excellent agreement with Quantum Mechanics and violated generalized Bell's inequalities by 4 standard deviations.

9 - ORSAY EXPERIMENTS (1980-1982)

The source

Since our aim was to use more sophisticated experimental schemes, we had first to build a high-efficiency and very stable and well controlled source. This was carried out (Fig. 10) by a two-

---

* Although these assumptions are reasonable, let us mention that there exist supplementary-parameters theories that do not obey them. From the view point of supplementary-parameters theories, there is no way for experimentally testing these assumptions[5].

photon-excitation of the $4p^2\ {}^1S_0$ - $4s4p\ {}^1P_1$ - $4s^2\ {}^1S_0$ cascade of calcium[15]. This cascade is very well suited to coincidence counting experiments since the lifetime τ_r of the intermediate level is rather short (5ns). If one can reach an excitation rate of about $1/\tau_r$, then an optimum signal-to-noise ratio for this cascade is attained.

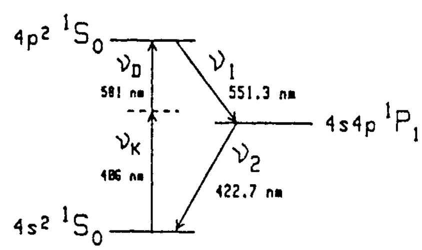

Fig. 10. Two-photon excitation of the chosen cascade in Calcium.

We have achieved this optimum rate with the use of a Krypton laser (λ_K = 406.7 nm) and a dye laser (λ_D = 581 nm) tuned to resonance for the two-photon process. Both lasers are single-mode operated. They have parallel polarizations.

They are focused onto a Calcium atomic beam (laser beam waists about 50 μm).

Two feedback loops provide the required stability of the source (better than 0.5 % for several hours) : the first loop controls the wavelength of the tunable laser to ensure the maximum fluorescence signal ; a second loop controls the power of one laser and compensates all the fluctuations.

With a few tens of milliwatts from each laser, the cascade rate is about $N = 4 \times 10^7\ s^{-1}$. An increase beyond this rate would not significantly improve the signal-to-noise ratio for coincidence counting, since the accidental coincidence rate increases as N^2, while the true coincidence rate increases as N.

Detection - Coincidence counting

The fluorescent light is collected by large-aperture aspherical lenses, followed by a set of lenses and the polarizers.

The photomultipliers feed the coincidence-counting electronics, that includes a time-to-amplitude converter and a multichannel analyzer, yielding the time-delay spectrum of the two-photon detections (Fig. 11). This spectrum involves a flat background due to accidental coincidences (i.e. between photons emitted by different atoms). True coincidences yield a peak around the null-delay, with an exponential decrease (time constant τ_r).

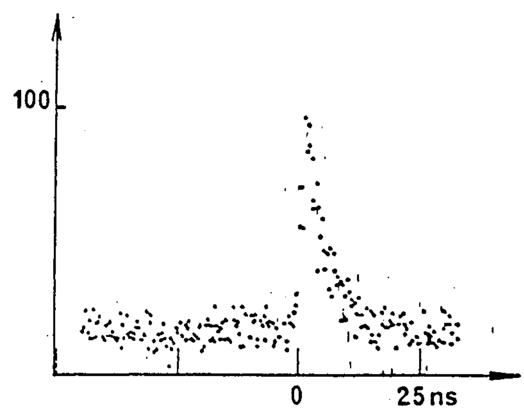

Fig. 11. Time-delay spectrum. Number of detected pairs as a function of the delay between the detections of two photons.

The true-coincidence signal is thus taken as the signal in the peak.

Additionally, a standard coincidence circuit with a 19 ns coincidence window monitors the rate of coincidences around null delay, while a delayed-coincidence channel monitors the accidental rate. It is then possible to check that the true coincidence rate obtained by substraction is equal to the signal in the peak of the time-delay spectrum.

In the second and third experiments, we have used a fourfold coincidence system, involving a fourfold multichannel analyzer and four double-coincidence circuits. The data were automatically gathered and processed by a computer.

Experiment with one-channel polarizers[16]

Our first experiment was carried out using one-channel-pile-of-plates polarizers, made of ten glass-plates at Brewster angle.

Thanks to our high-efficiency source, the statistical accuracy was better than 2 % in a 100 s run (with polarizers removed). This allowed us to perform various checks.

The test of Bell's inequalities has yielded

$$S'_{exp} = 0.126 \pm 0.014 \tag{15}$$

violating inequalities (13) by 9 standard deviations, and in good agreement with the Quantum Mechanical predictions (for our polarizers and solid angles) :

$$S_{MQ} = 0.118 \pm 0.005$$

(this error accounts for uncertainty in the measurements of the polarizers efficiencies).

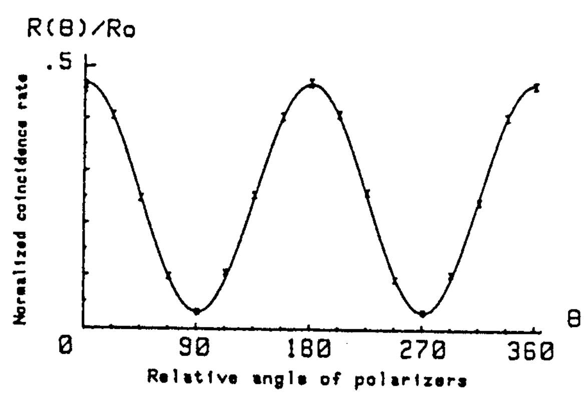

Fig. 12. Experiment with one channel polarizers : Normalized coincidence rate as a function of the relative polarizers orientation. Indicated errors are ± 1 standard deviation. The solid curve is not a fit to the data but the prediction by Quantum Mechanics.

The agreement between the experimental data and the Quantum Mechanical predictions has been checked in a full 360° range of orientations (Fig. 12).

In order to fulfil the first time-condition (§7) we have repeated these measurements with the polarizers at 6.5 m from the source. At such a distance (four coherence-lengths of the wave packet associated with the lifetime τ_r) the detection events are space-like separated. No modification of the experimental results was observed.

Experiment with two-channel analyzers[17]

With single-channel polarizers, the measurements of polarization are inherently incomplete. When a pair has been emitted, if no count is obtained at one of the photomultipliers, there is no way to know if "it has been missed" by the detector or if it has been blocked by the polarizer (only the later case corresponds to a result $-$ for the measurement). This is why one had to resort to auxilliary experiments, and indirect reasoning, in order to test Bell's inequalities.

With the use of two-channel polarizers, we have performed an experiment following much more closely the ideal scheme of Fig. 1.* Our polarizers were polarizing cubes transmitting one polarization (parallel to $\vec{a}$, or respectively to $\vec{b}$) and reflecting the orthogonal one. Such a polarization splitter, and the two corresponding photomultipliers, are mounted in a rotatable mechanism. This device (polarimeter) yields + and − results for linear polarization measurements along $\vec{a}$ (respectively $\vec{b}$). It is an optical analog of a Stern-Gerlach filter for spin 1/2 particles.

With polarimeters I and II in orientations $\vec{a}$ and $\vec{b}$, and the fourfold coincidence counting system, we are able to measure in a single run the four coincidence rates $R_{\pm\pm}(\vec{a},\vec{b})$. We then get directly the correlation coefficient for the measurement along $\vec{a}$ and $\vec{b}$:

$$E(\vec{a},\vec{b}) = \frac{R_{++}(\vec{a},\vec{b}) + R_{--}(\vec{a},\vec{b}) - R_{+-}(\vec{a},\vec{b}) - R_{-+}(\vec{a},\vec{b})}{R_{++}(\vec{a},\vec{b}) + R_{--}(\vec{a},\vec{b}) + R_{+-}(\vec{a},\vec{b}) + R_{--}(\vec{a},\vec{b})} \qquad (16)$$

It is then sufficient to repeat the same measurement for three other orientations, and the B.C.H.S.H. inequality (9) can directly be tested.

* A similar experiment, using calcite polarizers, has been undertaken at the University of Catania, Italy.

This procedure is sound if the measured values (16) of the correlation coefficients can be taken equal to the definition (4), i.e. if we assume that the ensemble of actually detected pairs is a faithful sample of all emitted pairs. This assumption is very reasonable with our very symmetrical scheme, where the two measurements +1 and -1 are treated in the same way (the detection efficiencies in both channels of a polarimeter are equal). Moreover, we have checked that the sum of the four coincidence rates $R_{\pm\pm}(\vec{a},\vec{b})$ is constant when changing the orientations, although each rate strongly varies. The size of the selected sample of pairs is thus found constant.

The experiment has been done at the set of orientations of Fig. 4, for which the greatest conflict is predicted. We have found

$$S_{exp} = 2.697 \pm 0.015 \qquad (17)$$

violating the inequalities (9) ($|S| \leq 2$) by more than 40 standard deviations ! This result is in excellent agreement with the predictions by Quantum Mechanics (for our polarizers and solid angles) :

$$S_{MQ} = 2.70 \pm 0.05$$

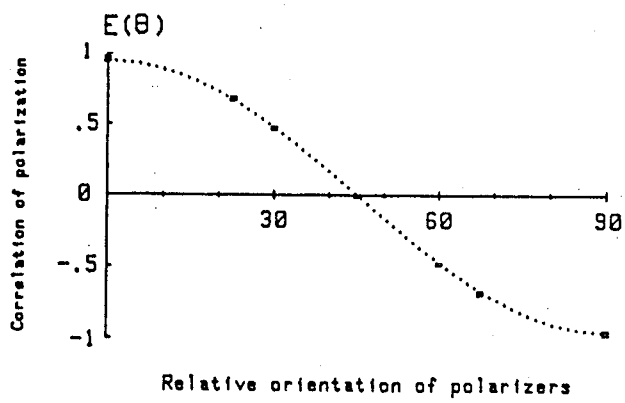

Fig. 13. Experiment with two-channels polarizers : Correlation of polarizations as a function of the relative angle of the polarimeters. The indicated errors are ± 2 standard deviations. The dashed curve is not a fit to the data, but Quantum Mechanical predictions for the actual experiment. For ideal polarizers, the curve would reach the values ± 1.

The uncertainty of S_{MQ} accounts for a slight lack of symmetry of both channels of a polarizer ($\pm$ 1%). The effect of these dissymmetries has been computed and cannot create a variation of S_{MQ} greater than 2%.

We have also performed measurements of $E(\vec{a},\vec{b})$ in various orientations, for a direct comparison with the predictions of Quantum Mechanics (Fig. 13). The agreement is clearly excellent.

Timing experiment[18]

As stressed in § 6, an ideal E.P.R. type experiment would involve the possibility of switching a random times the orientation of each polarizer. We have done a step towards such an ideal experiment by using the modified scheme displayed in Fig. 14.

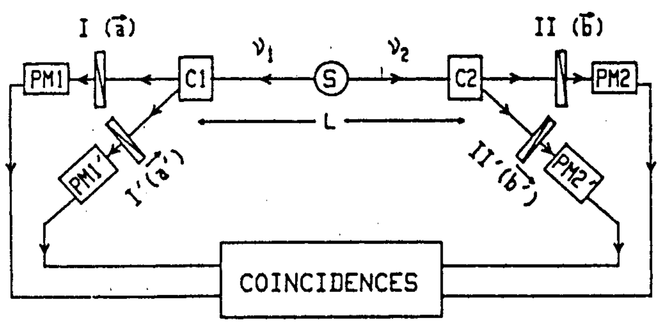

Fig. 14. Timing-experiment with optical switches. (C_1 and C_2). A switching occurs each 10 ns. The two switches are independently driven.

Each (single-channel) polarizer is replaced by a setup involving a switching device followed by two polarizers in two different orientations : $\vec{a}$ and $\vec{a}'$ on side I, $\vec{b}$ and $\vec{b}'$ on side II. The optical switch is able to rapidly redirect the incident light from one polarizer to the other one. Each setup is thus equivalent to a variable polarizer switched between two orientations. The distance L between the two switches is 12 m.

The switching of the light is effected by acousto-optical interaction of the light with an ultrasonic standing wave in water. The incidence angle (Bragg angle) and the acoustic power are adjusted for a complete switching between the 0^{th} and 1^{st} order of diffraction. At an acoustical frequency of 25 MHz, the

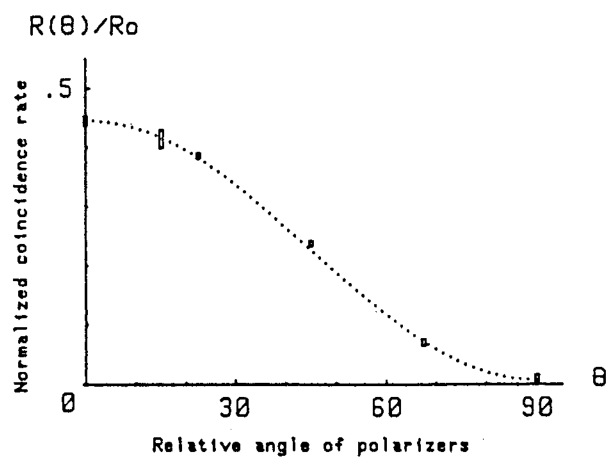

Fig. 15. Timing experiment : average normalized coincidence rate as a function of the relative orientation of the polarizers. Indicated errors are ± 1 standard deviation. The dashed curve is not a fit to the data but the predictions by Quantum Mechanics for the actual experiment.

switching frequency is 50 MHz. A change of orientation of the equivalent variable polarizer then occurs each 10 ns. Since this period (10 ns) as well as the lifetime τ_r (5 ns) are small compared to L/C (40 ns), a detection event on one side and the corresponding change of orientation on the other side are separated by a space-like interval. The second time-condition is thus partially fulfilled.

With the large beams used in the experiment, the commutation was not complete, since the incidence angle was not exactly the Bragg angle. Instead of being 0, the minimum of transmitted light in each channel was 20 %.

Since we had to reduce the divergence of the beams, the detected coincidence rates were weaker by an order of magnitude than in our previous experiments. Accordingly, the duration of data accumulation was longer.

The test of Bell's Inequalities (13) involves a total of 8 000 s of data accumulation with the 4 polarizers in the orientations of Fig. 4. A total of 16 000 s was devoted to auxilliary calibration measurements with half or all polarizers removed.

In order to compensate the effects of systematic drifts, the data accumulation was alternated between the various configurations each 400 s. The average yields

$$S'_{exp} = 0.101 \pm 0.020 \qquad (18)$$

violating inequ. (13) by 5 standard deviations, and in good agreement with the Quantum Mechanics predictions

$$S_{MQ} = 0.113 \pm 0.005$$

Another run has been carried out for a direct comparison with Quantum Mechanics. Fig. 15 exhibits an excellent agreement.

According to these results, Supplementary-Parameters Theories obeying Einstein's Causality seem to be untenable. To escape this conclusion, one might argue that the switching was not complete. However, a large fraction of the pairs undergoes forced switching. If Bell's Inequalities were obeyed by these pairs, it is hard to believe that we would not have observed a significant discrepancy between our results and the Quantum Mechanical predictions.

Our experiments differs from the ideal scheme in another respect : the switching are not truly at random, since the acousto-optical switches are driven by quasi-periodic generators. Nevertheless, the two generators on the two sides function in a completely uncorrelated way, especially considering their frequency drifts.

10 - CONCLUSION

Our last experiment (timing-experiment), as well as the previous ones, has some technical imperfections. Some loopholes thus remain open for the advocates of Supplementary-Parameters Theories obeying Einstein's Causality. Improved experiments will probably become feasible in the future[19], but we already have an impressive agreement with Quantum Mechanics. Supplementary Parameters Theories obeying Einstein's Causality and compatible with our results appear somewhat artificial, since the experimental results would have to change dramatically (disagreement with Quantum Mechanics) with certain technical improvements(such as an increase of the efficiencies of the photomultipliers)*.

According to Bell[20], we are thus forced to admit :

(i) either that there are, at the level of the supplementary parameters, faster-than-light influences**;

(ii)or to renounce an explanation in terms of supplementary parameters.

The second position seems a priori more confortable. But, to quote Mermin[22], "I challenge the reader to suggest any... other way to account for what happens" (i.e. the observed strong correlations).

I hope that even those who are not committed to such discussions will be convinced that Einstein has pointed out one of the most extraordinary property of Quantum Mechanics. We must thank J. Bell to have provided us with the possibility of experimentally evidencing this property.

---

* Another far-fetched issue is to admit that the two switches, although they look randomly driven (in an ideal experiment) are in fact correlated with each other, and also with the pairs. We then have to admit that the whole world is completely entangled, and that there is no possibility of a free choice of what we decide to measure !

**We must emphasize that in a timing experiment - even ideal - these hypothetical faster-than-light influences cannot be controlled for practical telegraphy[21].

REFERENCES

1. J.S. Bell, On the Einstein-Podolsky-Rosen Paradox, Physics 1 : 195 (1964).
   J.S. Bell, Introduction to the Hidden-Variable Question, in : "Foundations of Quantum Mechanics", B. d'Espagnat ed., Academic, N.Y. (1972).

2. A. Einstein, B. Podolsky and N. Rosen, Can Quantum-Mechanical description of physical reality be considered complete ?, Phys. Review 47 : 777 (1935). See also Bohr's answer :
   N. Bohr, Can Quantum-Mechanical description of physical reality be considered complete ?, Phys. Rev. 48 : 696 (1935).

3. D. Bohm, "Quantum Theory", Prentice-Hall, Englewoods Cliffs, N.J. (1951).

4. D. Bohm and Y. Aharonov, Discussion of Experimental Proof for the paradox of Einstein, Rosen and Podolsky, Phys. Rev. 108 : 1070 (1957).

5. J.F. Clauser and A. Shimony, Bell's Theorem : Experimental Tests and Implications, Rep. Progr. Phys. 41 : 1881 (1978).

6. J.F. Clauser, M.A. Horne, A. Shimony and R.A. Holt, Proposed experiment to test local hidden-variable theories, Phys. Rev. Lett. 23 : 880 (1969).

7. J.F. Clauser, and M.A. Horne, Experimental consequences of objective local theories, Phys. Rev. D 10 : 526 (1974).

8. A. Fine, Hidden Variables, Joint Probability, and the Bell Inequalities, Phys. Rev. Lett. 48 : 291 (1982).

9. A. Aspect, Proposed Experiment to Test Separable Hidden-Variable Theories, Phys. Lett. 54 A : 117 (1975).
   A. Aspect, Proposed Experiment to test the nonseparability of Quantum Mechanics, Phys. Rev. D 14 : 1944 (1976).

10. E.S. Fry, Two-Photon Correlations in Atomic Transitions, Phys. Rev. A 8 : 1219 (1973).

11. F.M. Pipkin, Atomic Physics Tests of the Basic Concepts in Quantum Mechanics, in : "Advances in Atomic and Molecular Physics", D.R. Bates and B. Bederson, ed., Academic (1978).

12. S.J. Freedman and J.F. Clauser, Experimental test of local hidden-variable theories, Phys. Rev. Lett. 28 : 938 (1972).

13. J.F. Clauser, Experimental Investigation of a Polarization Correlation Anomaly, Phys. Rev. Lett. 36 : 1223 (1976).

14. E.S. Fry, and R.C. Thompson, Experimental Test of Local Hidden-Variable Theories, Phys. Rev. Lett. 37 : 465 (1976).

15. A. Aspect, C. Imbert, and G. Roger, Absolute Measurement of an Atomic Cascade Rate Using a Two Photon Coincidence Technique. Application to the $4p^{2}\,{}^{1}S_{0}$ - $4s4p\ {}^{1}P_{1}$ - $4s^{2}\,{}^{1}S_{0}$ Cascade of Calcium excited by a Two Photon Absorption, Opt. Comm. 34 : 46 (1980).

16. A. Aspect, P. Grangier and G. Roger, Experimental Tests of Realistic Local Theories via Bell's Theorem, Phys. Rev. Lett. 47 : 460 (1981).

17. A. Aspect, P. Grangier and G. Roger, Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment : A New Violation of Bell's Inequalities, Phys. Rev. Lett. 49 : 91 (1982).

18. A. Aspect, J. Dalibard and G. Roger, Experimental Test of Bell's Inequalities Using Variable Analyzers, Phys. Rev. Lett. 49, 1804 (1982)

19. T.K. Lo, and A. Shimony, Proposed Molecular Test of Local Hidden-Variables Theories, Phys. Rev. A 23 : 3003 (1981)

20. J.S. Bell, Atomic-cascade Photons and Quantum-Mechanical Nonlocality, Comments on Atom. Mol. Phys. 9 : 121 (1980).

21. A. Aspect, Expériences basées sur les Inégalités de Bell, J. Physique Colloque C2 : 63 (1981).

22. N.D. Mermin, Bringing home the atomic work : Quantum mysteries for anybody, Am. J. Phys. 49 : 940 (1981).