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**LOCAL HOLOMORPHIC FACTORIZATION  
FOR FREE CONFORMAL FIELDS <sup>(1)</sup>**

*by*

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**Abstract**

The proof of the Belavin Knizhnik holomorphic factorization theorem is reinvestigated and gives rise to a variation, at fixed non vanishing central charge, in which partition functions are functionals on the relevant infinite dimensional space of complex structures instead of the relevant moduli space.

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## I. Introduction.

Two dimensional conformal models are distinguished by a holomorphic-antiholomorphic factorization property. We shall report here on recent work in collaboration with S. Lazzarini and M. Knecht where this property is reinvestigated in the case of the simplest known Lagrangian systems namely those describing free fields. Resummations of renormalized perturbative expansions à la Feynman in the euclidean framework are used throughout.

In section II, we describe the main features of free fields in infinite euclidean space  $(\mathbb{C})^1$ .

In section III we consider the case of spin  $j$  b-c systems on a compact Riemann surface without boundary.

In section IV we extend the previous analysis to the general case of fields with values sections of a holomorphic vector bundle over a Riemann surface without boundary.

A few concluding remarks are gathered in section V.

## II. Free conformal fields in $\mathbb{C}^1$ .

Conformal fields are naturally coupled to conformal classes of metrics parametrized by Beltrami differentials with coefficients  $\mu$  with respect to the coordinates  $(z, \bar{z})$  in general locally defined in the open sets  $U_\alpha$  of an atlas describing a Riemann surface  $\Sigma$ :

$$ds^2 = \rho |dz + \mu d\bar{z}|^2$$
$$|\mu| < 1. \quad (1)$$

For instance, for a scalar field  $\varphi$ , the classical action is

$$S(\varphi, \mu, \bar{\mu}) = \int_{\Sigma} \frac{d\bar{z} \wedge dz}{2i} \partial_{\bar{z}} \varphi \partial_z \varphi$$
$$= \int_{\Sigma} \frac{d\bar{z} \wedge dz}{2i} (\partial_{\bar{z}} - \mu \partial_z) \varphi \frac{1}{1 - \mu \bar{\mu}} (\partial_z - \bar{\mu} \partial_{\bar{z}}) \varphi \quad (2)$$

where the local coordinates  $Z$  pertaining to the complex structure  $\mu$  are defined by

$$dZ = \lambda (dz + \mu d\bar{z})$$
$$(\bar{\partial} - \mu \partial) \lambda = \lambda \partial \mu \quad (\Leftrightarrow d^2 = 0) \quad (3)$$

We shall often use the abbreviations

$$\begin{aligned}\bar{\partial} &= \partial_{\bar{z}} \text{ or } \bar{\partial} = d\bar{z} \partial_{\bar{z}} \\ \partial &= \partial_z \text{ or } \partial = dz \partial_z\end{aligned}\quad (4)$$

clear from the context.

We have used

$$\partial_{\bar{z}} = \frac{\bar{\partial} - \mu \partial}{\bar{\lambda}(1 - \mu \bar{\mu})} \quad \partial_z = \frac{\partial - \bar{\mu} \bar{\partial}}{\lambda(1 - \mu \bar{\mu})}\quad (5)$$

For a spin  $j$  b-c system, one has

$$\begin{aligned}S(\beta, \gamma; \mu, \bar{\mu}) &= \int \frac{d\bar{z} \wedge dz}{2i} \mathfrak{B}^{(1-j)} \partial_z \mathfrak{C}^{(j)} \\ &= \int \frac{d\bar{z} \wedge dz}{2i} \beta^{(1-j)} (\bar{\partial} - \mu \partial - j \partial \mu) \gamma^{(j)}\end{aligned}\quad (6)$$

where  $\mathfrak{b}^{(1-j)}$  (resp  $\mathfrak{C}^{(j)}$ ) is the coefficient of a  $(1-j)$  (resp  $j$ )–differential  $\mathfrak{b}^{(1-j)} dz^j$  (resp  $\mathfrak{C}^{(j)} dz^j$ ) and

$$\mathfrak{B}^{1-j} = \frac{\beta^{(1-j)}}{\lambda^{1-j}} \quad \mathfrak{C}^j = \frac{\gamma^{(j)}}{\lambda^j}.\quad (7)$$

Such actions are invariant under the s-operation associated with infinitesimal diffeomorphisms (i.e. the Lie algebra of vector fields):

$$\begin{aligned}sc &= (c.\partial)c \\ s\mu &= \bar{\partial}C + C \partial\mu - \mu \partial C \\ s\varphi &= (c.\partial)\varphi \\ s\gamma &= (c.\partial)\gamma + j(\partial c + \mu \partial \bar{c})\gamma \\ s\beta &= (c.\partial)\beta + (1-j)(\partial c + \mu \partial \bar{c})\gamma\end{aligned}\quad (8)$$

Here,

$$\begin{aligned}c &= (c, \bar{c}) & (c.\partial) &= c\partial + \bar{c}\bar{\partial} \\ C &= (c + \mu \bar{c}) & \bar{C} &= (\bar{c} + \bar{\mu}c)\end{aligned}\quad (9)$$

and one deduces

$$sC = C \partial C, \quad s^2 = 0 \quad (10)$$

The  $C$  variables, found by C. Becchi<sup>2</sup> are appropriate to the description of the natural complex structure of  $\text{Diff } \Sigma$ . Quantization in  $\mathbb{C}$  can be performed à la Feynman, producing formal power series in  $\mu, \bar{\mu}$ , the renormalization being constrained by the Ward identity stemming from the invariance under  $s$  (Eqs. 8,9), which however is spoiled at the quantum level by the holomorphically split diffeomorphism anomaly according to<sup>2</sup>

$$s \Gamma(\mu, \bar{\mu}) = \gamma \int \frac{d\bar{z} \wedge dz}{2i} C \partial^3 \mu + \text{c.c.} \quad (11)$$

where  $\Gamma(\mu, \bar{\mu})$  is the one loop renormalized vacuum functional, generating correlation functions of the two components of the stress tensor:

$$\Theta = \frac{\delta S}{\delta \mu} \Big|_{\mu=\bar{\mu}=0} \quad \bar{\Theta} = \frac{\delta S}{\delta \bar{\mu}} \Big|_{\mu=\bar{\mu}=0} , \quad (12)$$

and  $\gamma$  is related to the central charge of the model. The Ward identity (Eq.11) fixes the renormalization uniquely and one finds that  $\Gamma(\mu, \bar{\mu})$  holomorphically splits as<sup>2</sup>

$$\Gamma(\mu, \bar{\mu}) = \Gamma(\mu) + \text{c.c.} \quad (13)$$

A.M. Polyakov<sup>3</sup> has proposed a formula:

$$\Gamma(\mu) = \gamma \int \frac{d\bar{z} \wedge dz}{2i} \mu \partial^2 \ln \partial Z \quad (14)$$

with  $Z$  the solution of the Beltrami equation (cf. Eq.3):

$$\begin{aligned} (\bar{\partial} - \mu \partial) Z &= 0 \\ Z &\rightarrow z \\ z &\rightarrow \infty \end{aligned} \quad (15)$$

when  $\mu$  has compact support.

This has been checked to a few orders of renormalized perturbation by S. Lazzarini<sup>4</sup>.

### III Lagrangian quantization on a Riemann surface.

One question is: how much is left of this on a compact Riemann surface without boundary  $\Sigma$ , a question on which conventional quantization schemes say nothing<sup>5,6</sup>. This is a hard question since we know that, if there is a  $\Gamma(\mu)$ , it is not universal and does depend on detailed global properties of the model. A class of candidates has been proposed by R. Zucchini<sup>7</sup> but there remains the question how those are connected to specific models. The existence of such functionals can be derived as follows by a close examination<sup>17</sup> of the Belavin Knizhnik<sup>8-14</sup> proof of the holomorphic factorization theorem over Teichmüller space for zero total central charge. One can either proceed by direct calculation starting from a  $\zeta$ -renormalized definition of the determinant<sup>15</sup> of a certain Laplacian or use the local version of the index theorem for elliptic families due to Bismut Gillet Soule<sup>18</sup> (cf. also Ref.11). There is no way known at present which avoids introducing a metric, positivity being the main ingredient which allows to sum up the perturbative series, and the question is to recover analyticity (in  $\mu$ ), which goes as follows. One gets the following curvature identity for the  $\det \bar{\partial}$  bundle:

$$\bar{\delta}\delta [\Gamma(\rho, \mu, \bar{\mu}) - \Gamma(\rho, 0, 0)] = \mathcal{A}_F \quad (16)$$

where  $\bar{\delta}$  (resp.  $\delta$ ) is the differentiation with respect to  $\bar{\mu}$  (resp.  $\mu$ ).

$$\Gamma(\rho, \mu, \bar{\mu}) = -\frac{1}{2} \ln \left( \frac{\det' \zeta - \Delta_j}{\det \langle \gamma_m | \gamma_n \rangle_{1 \leq m \leq n \leq N_j} \det \langle \beta_p | \beta_q \rangle_{1 \leq p \leq q \leq N_{1-j}}} \right) \quad (17)^{16}$$

with<sup>15</sup>

$$\ln [\det' \zeta - \Delta_j] = -\frac{d}{ds} \Big|_{s=0} \frac{1}{\Gamma(s)} \int_0^\infty t^{s-1} dt [\text{Tr} (e^{t\Delta_j}) - N_j] \quad (18)$$

$\Delta_j$  being the natural Laplacian for spin  $j$  computed with the metric Eq.1;

$\gamma_m, \gamma_n$  (Resp  $\beta_p, \beta_q$ ) denote a basis in the Kernel of  $\Delta_j$  (resp  $\Delta_{1-j}$ ), of dimension  $N_j$  (resp  $N_{1-j}$ ), assumed to depend holomorphically on  $\mu$ , and  $\langle | \rangle$  denotes the metric canonically associated with that in Eq.1.

One finds the factorization anomaly to be

$$\mathcal{A}_F = -\frac{c_j}{12\pi} \int_{\Sigma} \frac{d\bar{z} \wedge dz}{2i} \left[ \partial + \theta - \frac{(\bar{\partial} + \bar{\theta})\bar{\mu}}{1-\mu\bar{\mu}} + \bar{\mu} \frac{(\partial + \theta)\mu}{1-\mu\bar{\mu}} \right] \delta\mu$$

$$\times \frac{1}{1-\mu\bar{\mu}} \left[ \bar{\partial} + \bar{\theta} - \frac{(\partial + \theta)\mu}{1-\mu\bar{\mu}} + \mu \frac{(\bar{\partial} + \bar{\theta})\bar{\mu}}{1-\mu\bar{\mu}} \right] \delta\bar{\mu} \quad (19)$$

with

$$c_j = 6j^2 - 6j + 1 \quad (20)$$

$$\theta = \partial \ln \rho \quad \bar{\theta} = \bar{\partial} \ln \rho \quad (21)$$

One happily finds

$$\omega_F = -\frac{c_j}{12\pi} \bar{\delta} \delta \Delta\Gamma \quad (22)$$

with<sup>19</sup>

$$\Delta\Gamma = \int_{\Sigma} \frac{d\bar{z} \wedge dz}{2i} \left( \mu(\mathcal{R} - R) + \text{c.c.} - \frac{1}{1-\mu\bar{\mu}} \left[ (\partial + \theta)\mu (\bar{\partial} + \bar{\theta})\bar{\mu} - \frac{1}{2} \bar{\mu} ((\partial + \theta)\mu)^2 - \frac{1}{2} \mu ((\bar{\partial} + \bar{\theta})\bar{\mu})^2 \right] \right) \quad (23)$$

where

$$R = \partial\theta - \frac{1}{2}\theta^2 \quad \bar{R} = \bar{\partial}\bar{\theta} - \frac{1}{2}\bar{\theta}^2 \quad (24)$$

and  $\mathcal{R}$  is the coefficient of a projective connection<sup>20</sup> on  $\Sigma$ . The non trivial fact is that  $\Delta\Gamma$  is local.

Thus one reaches the conclusion that

$$\bar{\delta} \delta \left( \Gamma + \frac{c_j}{12\pi} \Delta\Gamma \right) = 0 \quad (25)$$

and one can prove that  $\Gamma + \frac{c_j}{12\pi} \Delta\Gamma$  does not depend on  $\rho$  any more and fulfills a Ward identity

close to Eq.11 with the right hand side replaced by

$$\frac{c_j}{24\pi} \int_{\Sigma} \frac{d\bar{z} \wedge dz}{2i} C(\partial^3 + 2\mathcal{R}\partial + \mathcal{R}')\mu \quad (26)$$

provided the zero modes evolve covariantly. Holomorphic factorization follows from Eq.25, but at the present level it is far from unique. It depends on the existence of basis in the Kernels of  $\Delta_j, \Delta_{1-j}$ , covariant under diffeomorphisms, if one wants the correct Ward identity (cf. Eq.1) to hold (cf. ref. 11).

#### IV Free fields sections of a holomorphic bundle.

This generalizes the case treated in the previous section where the holomorphic bundle  $V$  was the  $j$  th tensor power of the holomorphic cotangent bundle<sup>22</sup>.

The action is

$$\begin{aligned} S(\psi, \tilde{\psi}; \mu, \bar{\mathcal{A}}, \text{c.c.}) &= \int \frac{d\bar{z} \wedge dz}{2i} \tilde{\psi}_z \partial_{\bar{z}} \psi \\ &= \int \frac{d\bar{z} \wedge dz}{2i} \tilde{\psi}_z (\bar{\partial} - \mu \partial + \bar{\mathcal{A}}) \psi \end{aligned} \quad (27)$$

where  $\psi$  is a smooth section of  $V$  and  $\tilde{\psi}_z dz$ , a smooth section of the dual  $\tilde{V}$  of  $V$  tensored with the holomorphic cotangent bundle. One defines  $\psi, \tilde{\psi}$  by

$$\psi = \varrho^{\rho-1} \psi \quad \tilde{\psi}_z = \frac{\tilde{\psi}_z}{\lambda} \varrho^\rho \quad (28)$$

with

$$(\partial_{\bar{z}} + \mathcal{A}_{\bar{z}}) \varrho^\rho = 0 \quad (29)$$

i.e.

$$(\bar{\partial} - \mu \partial + \bar{\mathcal{A}}) \varrho^\rho = 0 \quad (30)$$

where

$$\mathcal{A}_{\bar{z}} = \frac{\bar{\mathcal{A}}}{\lambda(1-\mu\bar{\mu})} \quad (31)$$

It is convenient to introduce a background connection  $A dz$  and define

$$\bar{A} = \bar{\mathcal{A}} + \mu A \quad (32)$$

which has the advantage to transform homogeneously between two holomorphic charts. Parameter space is defined in terms of holomorphic coordinates  $\mu$  and  $\bar{\mathcal{A}}$  or  $\bar{A}$ .

We introduce a family of metrics

$$E = \varrho^{\rho^\dagger} e \varrho^\rho \quad (33)$$

on the fiber, and

$$\rho_{z\bar{z}} = \frac{\rho}{\lambda\bar{\lambda}} \quad (34)$$

where  $e$  and  $\rho$  are fixed metrics, and look at

$$\begin{aligned} \Gamma(\rho, e; \mu, \bar{\mathcal{A}}, \text{c.c.}) &= -\frac{1}{2} \left[ \log \det'_{\xi} \Delta_{\rho, e, \mu, \bar{\mathcal{A}}, \text{c.c.}}^V \right. \\ &\quad \left. - \log \det \|\langle \phi_\alpha | \phi_\beta \rangle_V\| - \log \det \|\langle \tilde{\phi}_\alpha | \tilde{\phi}_\beta \rangle_{\tilde{V} \otimes K}\| \right] \end{aligned} \quad (35)$$

where  $\phi_\alpha$  (resp  $\tilde{\phi}_\alpha$ ), assumed to be holomorphic in  $\mu, \bar{\mathcal{A}}$  span the kernel of the  $\bar{\partial}$  operators on  $V$  (resp  $\tilde{V} \otimes K$ ), and the scalar products as well as the Laplacians are computed with the metrics Eqs. 33, 34. One then derives<sup>18,21</sup>

$$\begin{aligned} &\bar{\delta} \delta \left[ \Gamma(\rho, e; \mu, \bar{\mathcal{A}}, \text{c.c.}) - \Gamma(\rho, e; 0, 0, \text{c.c.}) \right] \\ &= -\frac{1}{8i\pi} \int_{\Sigma} \text{Tr} \left[ \left( \epsilon + \frac{1}{2} \kappa \mathbf{1} \right)^2 - \frac{1}{12} (\kappa \mathbf{1})^2 \right]^{2,2} \end{aligned} \quad (36)$$

where  $\bar{\delta}$  resp  $\delta$  is the differential with respect to  $\bar{\mu}, \bar{\mathcal{A}}^\dagger$  (resp  $\mu, \bar{\mathcal{A}}$ ) and  $\epsilon, \kappa$  are certain curvatures, the upper script selecting out the component of degree 2 in the parameters and of degree 2 in  $d\bar{z} dz$ . The term  $\int [\kappa^2]^{2,2}$  has been called  $\mathcal{O}_F$  in the previous section. The first term reads

$$\begin{aligned} -\frac{1}{2} \int_{\Sigma} \text{Tr} \left[ \left( \epsilon + \frac{1}{2} \kappa \mathbf{1} \right)^2 \right]^{2,2} &= \int_{\Sigma} \frac{d\bar{z} \wedge dz}{2i} (1 - \mu \bar{\mu}) \text{Tr} \delta \left( \frac{\tilde{\alpha} - \mu \tilde{e}^{-1} \tilde{\alpha}^\dagger \tilde{e}}{1 - \mu \bar{\mu}} \right) \\ &\quad \times \bar{\delta} \left( \frac{\tilde{e}^{-1} \tilde{\alpha}^\dagger \tilde{e} - \bar{\mu} \tilde{\alpha}}{1 - \mu \bar{\mu}} \right) \end{aligned} \quad (37)$$

where

$$\tilde{e} = e \rho^{1/2}, \quad \tilde{\alpha} = \bar{\alpha} + \frac{1}{2} \partial \mu + \mu (\tilde{e}^{-1} \partial \tilde{e}) \quad (38)$$

One finds

$$-\frac{1}{4i} \int_{\Sigma} \text{Tr} \left( (\epsilon + \frac{1}{2} \kappa \mathbf{1})^2 \right)^{2,2} = \bar{\delta} \delta \tilde{\Delta} \Gamma \quad (39)$$

where the local counterterm  $\Delta \Gamma$  is given by

$$\tilde{\Delta} \Gamma = \int \frac{d\bar{z} \wedge dz}{2i} \text{Tr} \frac{1}{1-\mu \bar{\mu}} \left[ \tilde{\alpha} \tilde{e}^{-1} \tilde{\alpha}^\dagger \tilde{e} - \frac{1}{2} \bar{\mu} \tilde{\alpha}^2 - \frac{1}{2} \mu \tilde{e}^{-1} \tilde{e} \tilde{\alpha}^\dagger \tilde{e} \right] \quad (40)$$

Hence

$$\bar{\delta} \delta \Gamma_{\text{fact}} = 0 \quad (41)$$

Where

$$\begin{aligned} \Gamma_{\text{fact}} &= \Gamma(\rho, e; \mu, \bar{A}, \text{c.c.}) - \Gamma(\rho, e; 0, 0, \text{c.c.}) \\ &\quad - \frac{1}{2\pi} \tilde{\Delta} \Gamma + \frac{1}{24\pi} \Delta \Gamma \text{Tr} \mathbf{1} + \frac{1}{2\pi} \tilde{\Delta} \Gamma_{\ell} \text{Tr} \mathbf{1} \end{aligned} \quad (42)$$

The additional counterterm  $\tilde{\Delta} \Gamma_{\ell}$ , linear in the parameters has been inserted to insure that  $\Gamma_{\text{fact}}$  is independent of the metrics  $\rho, e$ :

$$\tilde{\Delta} \Gamma_{\ell} = \int_{\Sigma} \frac{d\bar{z} \wedge dz}{2i} \text{Tr} \left( \tilde{e}^{-1} \partial \tilde{e} - \bar{A} \right) \left( \bar{A} + \frac{\mu}{2} (\tilde{e}^{-1} \partial \tilde{e} - \bar{A}) \right) \quad (43)$$

$\bar{A}$  is a background connection on  $V \otimes K^{-1/2}$ . There remains to check that  $\Gamma_{\text{fact}}$  fulfills the correct analogous Ward identities for Diffeomorphisms and gauge transformations. The corresponding s operation reads:

$$\begin{aligned} s\psi &= c. (\partial + A)\psi + H\psi \\ s\tilde{\psi} &= c. (\partial - A^T)\tilde{\psi} + (\partial c + \mu \partial \bar{c})\tilde{\psi} - H^T \tilde{\psi} \\ s\bar{A} &= C\bar{F} - (\bar{\partial} - \mu \partial)H - [\bar{A} - \mu A, H] \\ sA &= 0 \\ sH &= C\partial H + C[A, H] + H^2 \end{aligned} \quad (44)$$

$$\begin{aligned} s\mu &= \bar{\partial}C + C\partial\mu - \mu\partial C \\ sC &= C\partial C \end{aligned} \quad (45)$$

where

$$F = \partial\bar{A} - \bar{\partial}A + [A, \bar{A}] \quad (46)$$

Of course,

$$s^2 = 0 \quad (47)$$

One finds correctly the two types of anomalies associated with  $\Gamma_{\text{fact}}$  provided the zero modes evolve covariantly.

$$\begin{aligned} s\Gamma_{\text{fact}} &= -\frac{\text{Tr}\mathbf{1}}{24\pi} \int \frac{d\bar{z}\wedge dz}{2i} C (\partial^3 + 2\mathcal{R}\partial + \mathcal{R}')\mu \\ &\quad + \frac{1}{2\pi} \int \frac{d\bar{z}\wedge dz}{2i} \text{Tr} \tilde{H} \tilde{F} \end{aligned} \quad (48)$$

where the  $\sim$  sign indicates one works with  $V \otimes K^{-1/2}$ . The particular case where  $V=K^j$  (solved in the preceding section) is recovered by putting

$$\begin{aligned} \tilde{\mathcal{A}} &= \left(-j + \frac{1}{2}\right)\partial\mu; \quad \tilde{A} = \left(j - \frac{1}{2}\right)A \\ \tilde{H} &= \left(j - \frac{1}{2}\right) \left[ (\Omega + \partial c + \mu\partial\bar{c} - cA - \bar{c}\bar{A}) \right] \end{aligned} \quad (49)$$

and inserting an additional counterterm

$$\text{Tr}\mathbf{1} \frac{\left(j - \frac{1}{2}\right)^2}{2\pi} \int_{\Sigma} \frac{d\bar{z}\wedge dz}{2i} \mu \left( \partial A + \frac{1}{2} A^2 + \mathcal{R} \right) + \text{c.c.} \quad (50)$$

which eliminates the  $A$  dependence at the benefit of  $\mathcal{R}$ .

The lack of uniqueness of the holomorphic factorization which follows from Eq.41 is even more severe than in the conventional case dealt with in section III and has to be further analyzed in terms of the properties of the bundle V.

## V. Concluding remarks.

Locality provides a bridge between summed up renormalized perturbation theory à la Feynman, in its euclidean version, and refined versions of the index theorem for elliptic families. The introduction of metrics, which, thanks to positivity allow the resummation is however compatible with analyticity.

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