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Addendum to " An Extension of the Theory of Fredholm Determinants "

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Addendum to
"An extension of the theory of Fredholm determinants"
 by D. Ruelle

In deriving (2.12) we have used the inequality

$$\sum_{\vec{b} \in J_k^{(\ell)}} \|\mathfrak{M}_k^{(\ell)} \chi_{\vec{b}}^{\uparrow}\| \leq \text{const } (e^{P+\varepsilon})^{\ell} \quad (2.13)$$

which we shall now prove. Given $\beta > 0$, we let $\varphi_{\omega\beta} = |\varphi_{\omega}| + \beta \|\varphi_{\omega}\|$, and define $\mathfrak{M}_{k\beta}$, $\mathfrak{M}_{k\beta}^{(\ell)}$ with φ_{ω} replaced by $\varphi_{\omega\beta}$. We first check that

$$\|\mathfrak{M}_k^{(\ell)} \chi_{\vec{b}}^{\uparrow}\| \leq C(\beta) \|\mathfrak{M}_{k\beta}^{(\ell)} \chi_{\vec{b}}^{\uparrow}\|_0 \quad (2.14)$$

where $C(\beta)$ does not depend on ℓ . We may indeed write

$$(\mathfrak{M}_k^{(\ell)} \chi_{\vec{b}}^{\uparrow})_j^{\uparrow}(x) = \int \mu(d\omega_1) \dots \mu(d\omega_{\ell})$$

$$\varepsilon \varphi_{\omega_{\ell}}(x) \dots \varphi_{\omega_1}(\psi_{\omega_2} \dots \psi_{\omega_{\ell}} x)$$

where the integral is restricted to those $(\omega_1, \dots, \omega_{\ell})$ for which there is a permutation π such that

$$\pi(v(j_0, \bar{\omega}), \dots, v(j_k, \bar{\omega})) = \vec{b},$$

and $\varepsilon = \text{sign } \pi$. Therefore

$$\|\mathfrak{M}_k^{\ell} \chi_{\vec{b}}^{\uparrow}\|_0 \leq \|\mathfrak{M}_{k\beta}^{\ell} \chi_{\vec{b}}^{\uparrow}\|_0.$$

If $x, y \in X_{j_0} \cap \dots \cap X_{j_k}$, we have also

$$\begin{aligned}
& |(\mathfrak{M}_k^{(\ell)} \chi_b^{\pm})_j^{\pm}(x) - (\mathfrak{M}_k^{(\ell)} \chi_b^{\pm})_j^{\pm}(y)| \leq \int \mu(d\omega_1) \dots \mu(d\omega_\ell) \\
& |\varphi_{\omega_\ell}(x) \dots \varphi_{\omega_1}(\psi_{\omega_2} \dots \psi_{\omega_\ell} x) - \varphi_{\omega_\ell}(y) \dots \varphi_{\omega_1}(\psi_{\omega_2} \dots \psi_{\omega_\ell} y)| \\
& \leq \sum_{i=1}^{\ell} \int \mu(d\omega_1) \dots \mu(d\omega_k) \varphi_{\omega_\ell \beta}(x) \dots \varphi_{\omega_{i+1} \beta}(\psi_{\omega_{i+2}} \dots \psi_{\omega_\ell} x) \\
& \quad |\varphi_{\omega_i}(\psi_{\omega_{i+1}} \dots \psi_{\omega_\ell} x) - \varphi_{\omega_i}(\psi_{\omega_{i+1}} \dots \psi_{\omega_\ell} y)| \\
& \quad \varphi_{\omega_{i-1} \beta}(\psi_{\omega_i} \dots \psi_{\omega_\ell} y) \dots \varphi_{\omega_1 \beta}(\psi_{\omega_2} \dots \psi_{\omega_\ell} y) .
\end{aligned}$$

We may assume that $\|\varphi_\omega\|$ is bounded uniformly with respect to ω (this can be achieved by a change of the measure μ). We may then write

$$\begin{aligned}
& |\varphi_{\omega_i}(\psi_{\omega_{i+1}} \dots \psi_{\omega_\ell} x) - \varphi_{\omega_i}(\psi_{\omega_{i+1}} \dots \psi_{\omega_\ell} y)| \\
& \leq \|\varphi_{\omega_i}\| (\theta^{\ell-i} d(x,y))^\alpha \leq \text{const } \varphi_{\omega_i \beta}(\psi_{\omega_{i+1}} \dots \psi_{\omega_\ell} x) \theta^{\alpha(\ell-i)} d(x,y)^\alpha
\end{aligned}$$

and similarly

$$\begin{aligned}
& \varphi_{\omega_r \beta}(\psi_{\omega_{r+1}} \dots \psi_{\omega_\ell} y) \\
& \leq \varphi_{\omega_r \beta}(\psi_{\omega_{r+1}} \dots \psi_{\omega_\ell} x) (1 + \text{const } \theta^{\alpha(\ell-r)}) . \quad (2.15)
\end{aligned}$$

Therefore

$$\frac{|(\mathfrak{M}_k^{(\ell)} \chi_b^{\pm})_j^{\pm}(x) - (\mathfrak{M}_k^{(\ell)} \chi_b^{\pm})_j^{\pm}(y)|}{d(x,y)^\alpha} \leq \text{const } \|\mathfrak{M}_{k\beta}^{(\ell)} \chi_b^{\pm}\|_0$$

and (2.14) follows.

From (2.15) we also obtain

$$(\mathfrak{M}_{k\beta}^{(\ell)} \chi_b^{\pm})_j^{\pm}(y) \leq C'(\beta) (\mathfrak{M}_{k\beta}^{(\ell)} \chi_b^{\pm})_j^{\pm}(x)$$

where $C'(\beta)$ does not depend on ℓ . Therefore

$$\begin{aligned} \sum_{\vec{b} \in J_k^{(\ell)}} \|\mathfrak{M}_{k\beta}^{(\ell)} \chi_b^{\pm}\|_0 &\leq C'(\beta) \sum_{\vec{j}} \sup_x |(\mathfrak{M}_{k\beta}^{(\ell)} 1)_j^{\pm}(x)| \\ &\leq C''(\beta) \|\mathfrak{M}_{k\beta}^{\ell} 1\|_0 \end{aligned}$$

and finally

$$\begin{aligned} \sum_{\vec{b} \in J_k^{(\ell)}} \|\mathfrak{M}_k^{(\ell)} \chi_b^{\pm}\| &\leq C(\beta) C''(\beta) \|\mathfrak{M}_{k\beta}^{\ell} 1\|_0 \\ &\leq C'''(\beta) (e^{P(\beta)+\varepsilon/2})^{\ell} \end{aligned}$$

where $e^{P(\beta)}$ is the spectral radius of $\mathfrak{M}_{k\beta}$. Note that $\mathfrak{M}_{k\beta}$ is close in norm to $|\mathfrak{M}_k|$ for β small :

$$\|\mathfrak{M}_{k\beta} - |\mathfrak{M}_k|\| \leq \beta \int \mu(d\omega) \|\varphi_{\omega}\|.$$

Using the upper semicontinuity of the spectral radius we may thus choose β such that

$$\sum_{\vec{b} \in J_k^{(\ell)}} \|\mathfrak{M}_k^{(\ell)} \chi_b^{\pm}\| \leq C'''(\beta) (e^{P+\varepsilon})^{\ell}$$

i.e., (2.13) holds as announced.

A similar argument may be used to obtain the inequality

$$\sum_{b \in J^{(\ell)}} \|\mathfrak{M}^{(\ell)}((\cdot - x(b))^n \chi_b)\| \leq \text{const}(e^{P+\varepsilon})^\ell (\theta^\ell)^{|\mathbf{n}|}$$

which is needed in the proof of (3.11).