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I. Introduction

In 1956 Wightman [1] has formulated a set of axioms for the describtion of relativistic quantum field theory. Since 1962 it was known [2] that these axioms can be re-formulated in terms of positive linear functionals on a certain $*$-algebra of test-functions. Since not very much was known about general topological algebras, this setting did not attract many physicists. In addition the so called linear program was quite successful during the first years of axiomatic field theory and thus people neglected the non-linear aspects of this theory. Since nowadays the linear program does not lead very often to new results it seems to be desirable to enter into a systematic study of this non-linear aspects of this theory. Some people did work on this subject e.g. W. Wyss [3], G. Lassner and A. Uhlmann [4]. I entered this subject last year. The connection between Wightman's axioms and algebras of test-functions and first results I represented in the Haifa summer-school [5].

These notes will deal with the algebra in question. In particular the structure of the positive cone of this algebra.

II. Notations

We denote by $\mathcal{S}_0$ the space of numbers and by $\mathcal{S}_n$ the space of strongly decreasing C_∞ functions on R^{4n} with the usual topology and by $\mathcal{S}^N = \sum_{i=0}^{N} \oplus \mathcal{S}_i$ the finite direct sums of such spaces. We put $\underline{\mathcal{S}} = \sum_{i=0}^{\infty} \oplus \mathcal{S}_i$

with the usual direct sum topology which implies that $\underline{\mathcal{S}}$ is also the inductive limit of the $\mathcal{S}^N$.

The direct product of functions

$$(f(x_1, \ldots x_n), \ g(y_1 \ldots y_m)) \rightarrow f(x_1, \ldots x_n) \, g(x_{n+1}, \ldots x_{n+m})$$

extends by linearity to a product on $\underline{\mathcal{S}}$. Thus $\underline{\mathcal{S}}$ becomes an algebra under this product. Furthermore the mapping

$$(f^*)(x_1 \ldots x_n) = \overline{f(x_n \ldots x_1)}$$

defines an involution on $\underline{\mathcal{S}}$. With these operations $\underline{\mathcal{S}}$ has the structure of a $*$-algebra. As topological vector-space $\underline{\mathcal{S}}$ is a (LF)-space. The involution is a continuous operation, but the product $(f, g) \rightarrow f \cdot g$ is only separately continuous. This, however, is sufficient to conclude that the closure of an ideal or of a sub-algebra is again an ideal or a sub-algebra.

By $\underline{\mathcal{S}}_h$ we denote the real elements of $\underline{\mathcal{S}}$ and by $\underline{\mathcal{S}}^+$ the positive cone

$$\underline{\mathcal{S}}^+ = \left\{ \sum_{\text{convergent}} f_i^* \cdot f_i \right\}$$

By $\underline{\mathcal{S}}'$ we denote ~~convergent~~ the dual-space of $\underline{\mathcal{S}}$.

$$\underline{\mathcal{S}}' = \prod_{i=0}^{\infty} \mathcal{S}_i'$$

and by $\underline{\mathcal{S}}'^+$ the set of continuous linear positive functionals on $\underline{\mathcal{S}}$ i.e.:

$$\underline{\mathcal{S}}'^+ = \left\{ T \in \underline{\mathcal{S}}' \, ; \, (T, f^* \cdot f) \geq 0 , \ \forall f \in \underline{\mathcal{S}} \right\}$$

Denote by α_a the translation automorphism of $\underline{\mathcal{S}}$ defined by $\alpha_a 1 = 1$, $(\alpha_a f)(x_1 \ldots x_n) = f(x_1 + a, \ldots x_n + a)$. α_a is a continuous $*$-automorphism of $\underline{\mathcal{S}}$. Define the spectral ideal Sp to be the smallest closed left-ideal containing all elements of the form

$$\int g(a) \, \alpha_a f \, da$$

$f \in \underline{\mathcal{S}}$, $g(a) \in \mathcal{S}_1$ with $(\mathcal{F} g)(p) = 0$ for p in the foreward light-cone ($\mathcal{F}$ denotes the Fourier operator). Furthermore define the locality ideal I_c to be the smallest closed twosided ideal containing all elements of the form

$$f(x_1) \, g(x_2) - g(x_1) \, f(x_2)$$

$f, g \in \mathcal{S}_1$ and supp f is spacelike separated to supp g.

With these notations we have:

Proposition:

The family of Wightman theories is in one to one correspondence with continuous linear functionals W on $\underline{\mathcal{S}}$ with the properties

α) $W(I_c) = 0$ (locality)

β) $W(Sp) = 0$ (spectral condition)

γ) $\alpha_a' W = W$ (translational invariance)

δ) $W \in \underline{\mathcal{S}}'^{+}$ (positivity)

For the proof see [2] or [5] .

This result suggests that a detailed study of the algebra $\underline{\mathcal{S}}$ might give us some insight into the structure of Wightman field theor■ies.

III. Algebraic norms on $\underline{\mathcal{S}}$

The topology on $\underline{\mathcal{S}}$ is given by a family of norms which are not adapted to the algebraic structure of $\underline{\mathcal{S}}$. Therefore we will look for continuous norms on $\underline{\mathcal{S}}$ which are compatible with the algebraic structure of $\underline{\mathcal{S}}$. We define:

III. 1. Definition:

A seminorm p on $\underline{\mathcal{S}}$ is called an algebraic seminorm if it fulfills the relation

$$p\,(f \cdot g) \leq p\,(f) \quad p(g)$$

for all $f,\ g \in \underline{\mathcal{S}}$

It is the aim of this section to exhibit a family of continuous algebraic norms and to show that we can find enough of them in a sense we will define later.

Recall that the topology on $\mathcal{S}_r$ is given by the norms $p_{n,m}$:

$$p_{n,m}\left(f(x_1 \cdots x_r\right) = \sup_{x} \max_{|k_i| \leq m} \left| \prod_{i=1}^{r} (1+|x_i|)^n D_{x_i}^{k_i} f(x_1, \ldots x_r) \right|$$

We define now for every $c > 0$ a norm on $\underline{\mathcal{S}}$ as follows:

III. 2. Definition:

For every $f \in \underline{\mathcal{S}}$

$$f = \left\{f_0,\ f_1, \ldots . f_r \cdots\right\}$$

and every $c > 0$ we define

$$P_{n,m,c}\,(f) = \sum_i c^i \, p_{n,m}(f_i)$$

III. 3. Lemma:

The function $P_{n,m,c} : \underline{\mathcal{S}} \rightarrow \mathbb{R}^+$ defines a continuous algebraic norm on $\underline{\mathcal{S}}$

Proof:

1) Since $p_{n,m}$ are norms on $\mathcal{S}_i$ follows

$$P_{n,m,c}(f+\lambda g) = \sum_i c^i p_{n,m}(f_i+\lambda g_i)$$
$$\leq \sum_i c^i \{ p_{n,m}(f_i) + |\lambda| \, p_{n,m}(g_i) \}$$
$$= P_{n,m,c}(f) + |\lambda| \, P_{n,m,c}(g)$$

This shows $P_{n,m,c}$ is a seminorm.
Now $P_{n,m,c}(f) = 0$ implies

$p_{n,m}(f_i) = 0$ for all i, and since

$p_{n,m}$ are norms follows $f_i = 0$ and hence $f = 0$, consequently $P_{n,m,c}$ is a norm.

2) In order to show that $P_{n,m,c}$ is continuous choose the norm $Q_{\{n_i\},\{m_i\},\{c_i\}}$ on $\underline{\mathcal{S}}$ [1]
with $n_i \geq n$, $m_i \geq m$, $c_i \geq \dot{c}!$ then we get

$$Q_{\{n_i\},\{m_i\},\{c_i\}}(f) \leq 1$$

implies

$$i! \; p_{n,m}(f_i) \leq 1$$

and hence

$$P_{n,m,c}(f) \leq e^c$$

This implies $P_{n,m,c}$ is continuous

3) It remains to show that $P_{n,m,c}$ is an algebraic norm. This is done by the following computation

$$P_{n,m,c}(f\cdot g) = \sum_i c^i p_{n,m}\Big(\sum_{k+l=i} f_k \cdot g_l\Big)$$

$$\leq \sum_i c^i \sum_{k+l=i} p_{n,m}(f_k \cdot g_l)$$

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1) Q is defined by the relation

$$Q_{\{n_i\}\{m_i\}\{c_i\}}(f) = \sup_i c_i \, p_{n_i,m_i}(f_i)$$

$$= \sum_i c^i \sum_{k+l=i} p_{n,m}(f_k)\cdot p_{n,m}(g_l)$$

$$= \sum_i \sum_{k+l=i} c^k p_{n,m}(f_k)\, c^l p_{n,m}(g_l)$$

$$= \Big(\sum_k c^k p_{n,m}(f_k)\Big)\Big(\sum_l c^l p_{n,m}(g_l)\Big)$$

$$= P_{n,m,c}(f)\cdot P_{n,m,c}(g)$$

It is clear that these algebraic norms do not define the same topology on $\underline{\mathcal{S}}$ than the family of norms $Q_{\{n_i\}\{m_i\}\{C_i\}}$ but we get

III. 4. Lemma:

The two families of norms $\{P_{n,m,c}\}$ and $\{Q_{\{n_i\},\{m_i\},\{c_i\}}\}$ define on $\mathcal{S}^N = \sum_{i=0}^{N} \oplus \mathcal{S}_i$, the same topology.

Proof: Since we know that the $P_{n,m,c}$ are continuous norms it remains to show that on $\mathcal{S}^N$ the norms $Q_{\{n_i\},\{m_i\},\{c_i\}}$ are continuous with respect to the topology definded by the P's. Assume $Q_{\{n_i\},\{m_i\},\{c_i\}}$ are given then exists $b > o$ with

$$b^i \leq c_i \quad \text{for} \quad i = o,1 \dots N.$$

Choose now $n \leq n_i$ and $m \leq m_i$ for $i = 1 \dots N$
then we get for $f \in \mathcal{S}^N$

$P_{n,m,b}(f) \leq 1$ implies

$b^i\, p_{n,m,}(f_i) \leq 1$ and consequently

$c_i\, p_{n_i,\, m_i}(f_i) \leq 1$ which implies

$Q_{\{n_i\},\{m_i\},\{c_i\}}(f) \leq 1$

This implies that on $\mathcal{S}^N$ the Q norms are P continuous.
From this follows

III. 5. Corollary:

The set of P continuous linear forms on $\underline{\mathcal{S}}$ is dense in $\underline{\mathcal{S}}'$.

Proof: Since the norms $P_{n,m,c}$ define on $\mathcal{S}^N$ the topology of $\underline{\mathcal{S}}$, we find that every $T \in \underline{\mathcal{S}}'$ which has only a finite number of non vanishing components is continuous with respect to the P's. But this set is dense in $\underline{\mathcal{S}}'$ since the topology in $\underline{\mathcal{S}}'$ is the topology of component-wise convergence.

Consider now $\underline{\mathcal{S}}$ furnished with one of the algebraic norms $P_{n,m,c}$ then $\underline{\mathcal{S}}$ becomes a normed $*$ -algebra with identity. Its completion is then a Banach $*$ -algebra with identity.

Now to every Banach $*$ -algebra A with identity exists a unique C^*-algebra B which is called the enveloping C^*-algebra (see J. Dixmier [6] § 2,7). If $\|\cdot\|$ denotes the norm of the Banach $*$ -algebra A and $\|\cdot\|_1$ the norm of the enveloping C^* -algebra B then we have $\|x\|_1 \leq \|x\|$ for all $x \in A$. This implies that $\|\cdot\|_1$ is a continuous seminorm on A. This seminorm $\|\cdot\|_1$ is a norm on A exactly if A has no radical.

For elements $x = x^* \in A$ the C^*-norm is given by the equation

$$\|x\|_1 = \sup \{ |\omega(x)| \; ; \; \omega \in A'^{+} \, , \; \omega(1) = 1 \}$$

This implies that A and B have the same set of continuous positive linear functionals.

We want now to introduce on $\underline{\mathcal{S}}$ the corresponding family of seminorms.

III. 6. Definition

Let $P_{n,m,c}$ an algebraic norm on $\underline{\mathcal{S}}$ than we denote by $P^*_{n,m,c}$ the seminorm defined by the enveloping C^*-algebra of $\{ \underline{\mathcal{S}}, P_{n,m,c} \}$.
From this definition follows directly

III. 7. Lemma:

a) The seminorms $P^*_{n,m,c}$ are continuous

b) Every $P^*_{n,m,c}$ continuous positive linear functional is $P_{n,m,c}$ continuous and vice versa

c) Every hermitian $P^*_{n,m,c}$-continuous linear functional is the difference of two $P^*_{n,m,c}$-continuous positive linear functionals.

Proof: a and b follows from the definition of the enveloping C^*-algebra. Statement c follows from the fact that on a C^*-algebra every real continuous linear functional is the difference of two continuous positive functionals (see J. Dixmier [6] Corollaire 2.6.4).

Next we want to investigate the seminorms $P^*_{n,m,c}$ in more detail. To this end we need some preparatory results.

III. 8. Lemma

Let $\mathcal{S}^N = \sum_{i=0}^{N} \oplus \mathcal{S}_i$ and $|k_i| \leq k \quad i=1\ldots N$

Then we get for every $f \in \mathcal{S}^N$

$$\left(\frac{c}{2}\right)^N \left| \prod_{i=1}^{N} (1+|x_i|)^n D^{k_i}_{x_i} f_N(x_1, \ldots, x_N) \right| \leq 4\, P^*_{n,k,c}(f)$$

Proof: Define a continuous $*$-homomorphism β of $\mathcal{S}$ into the $(N+1)\times(N+1)$ matrices by means of the distribution

$$t_{i,k} = t^*_{k,i} \in \mathcal{S}_1' \quad ; \quad (\beta f_n)_{i,k} = \left(\sum_{j_1 \ldots j_{n-1}} t_{i,j_1} \times t_{j_1,j_2} \times \cdots \times t_{j_{n-1},k},\, f_n\right)$$

with

$$t_{i,i} = 0; \quad t_{i,i+j} = 0,\ j>1; \quad t_{i,i+1} = \frac{c}{2}(1+|x_i|)^n \delta^{k_i}_{x_i}.$$

$$(\delta^{k_i}_{x_i}, f) = D^{k_i} f(x_i)$$

We get

$$\|(t_{ik}, f)\| \leq c\, P_{n,k}(f), \quad f \in \mathcal{S}_1$$

From this follows $\|\beta(f)\| \leq P_{n,k,c}(f)$ and therefore for any state ω

we get

$$|\omega(\beta(f))| \leq \|\beta(f)\| \leq P_{n,k,c}(f)$$

Since $\omega \circ \beta$ is $P^*_{n,k,c}$ continuous and a state follows:

$$|\omega\circ\beta(f)| \leq P^*_{n,k,c}(f)$$

Since this holds for every state ω follows by polarisation:

$$|\beta_{i,j}(f)| \leq 4\, P^*_{n,k,c}(f).$$

Choosing now i = 1 and j = N+1 we get

$$\left(\frac{c}{2}\right)^N \left| \prod_{i=1}^{N} (1+|x_i|)^n \, D^{k_i}_{x_i} \, f_N(x_1,\dots x_N)\right| \leq 4\, P^*_{n,k,c}(f).$$

With this result we get

III. 9. <u>Theorem:</u>

a) Every seminorm $P^*_{n,m,c}$ is a norm

b) The two families of norms

$$\left\{P_{n,m,c}\right\} \quad \text{and} \quad \left\{P^*_{n,m,c}\right\}$$

induce on $\mathcal{S}^N = \sum_{i=0}^{N} \oplus \mathcal{S}_i$ the same topology.

<u>Proof:</u>

a) Assume $f \in \underline{\mathcal{S}}$ and $P^*_{n,m,c}(f) = 0$. If $f \neq 0$ then exists a highest component $f_k \neq 0$. But we get from Lemma III.8.

$$\left(\frac{c}{2}\right)^k p_{n,m}(f_k) \leq 4\, P^*_{n,m,c}(f) = 0$$

This contradicts the assumption $f \neq 0$

b) From Lemma III.8. we get for $f \in \mathcal{S}^N$

$$P_{n,m,c}(f_N) \leq 4\, P^*_{n,m,2c}(f)$$

and hence

$$P_{n,m,c}(f_{N-1}) \leq 4\, P^*_{n,m,2c}(f-f_N)$$

$$\leq 4\, P^*_{n,m,2c}(f) + 4^2\, P^*_{n,m,4c}(f)$$

Iteratin this equation again we get :

$$P_{n,m,c}(f_{N-j}) \leq \sum_{i=0}^{j} \binom{i}{j} 4^{i+1} P^{*}_{n,m,2^{i+1}c}(f)$$

Summing over j we obtain for all $f \in \mathcal{S}^N$

$$P_{n,m,c}(f) \leq 4 \sum_{i=0}^{N} 8^{i} P^{*}_{n,m,2^{i+1}c}(f)$$

This proves statement b.

IV. Consequences of the algebraic semi-norms

The existence of a family of algebraic semi-norms which induce on $\mathcal{S}^N$ the original topology has some important consequences for the structure of the algebra $\underline{\mathcal{S}}$.

IV. 1. Theorem:

a) $\underline{\mathcal{S}}'^{+} - \underline{\mathcal{S}}'^{+}$ is dense in $\underline{\mathcal{S}}'_h$

b) $\overline{\underline{\mathcal{S}}^{+}} \cap - \overline{\underline{\mathcal{S}}^{+}} = \{0\}$

c) $\underline{\mathcal{S}}'^{+} \cap - \underline{\mathcal{S}}'^{+} = \{0\}$

d) $\overline{\underline{\mathcal{S}}^{+}} - \overline{\underline{\mathcal{S}}^{+}} = \underline{\mathcal{S}}_h$

Remark: J. Yngvason has given an example showing that not every real linear functional on $\underline{\mathcal{S}}$ is the difference of two positive ones.

Proof:

d) follows from the fact that $\underline{\mathcal{S}}$ contains the identity. this implies for $f = f^*$ the equation

$$f = \frac{1}{4}\left\{(1+f)^2 - (1-f)^2\right\}.$$

c) follows from d) by duality.

a) is a consequence of Theorem III.9. The real linear functionals on $\underline{\mathcal{S}}$ with only a finite number of components are continuous with respect to some C^*-norm

$P^*_{n,m,c}$ and hence the difference of two positive functionals. But since these elements are dense in $\underline{\mathcal{S}}'_h$ follows a).
b) follows from a) again by duality.
A further consequence is given in the following

IV. 2. Lemma:

The product from $\mathcal{S}^N \times \mathcal{S}^M \to \mathcal{S}^{N+M}$ is simultaneously continuous.

Proof: This follows from the fact that the topologies on these spaces are given by algebraic norms.

The product defines a linear map P from $\mathcal{S}^N \otimes_\pi \mathcal{S}^N \to \mathcal{S}^{2N}$, Since this mapping is continuous we can extend it to complete tensor product $\mathcal{S}^N \tilde{\otimes}_\pi \mathcal{S}^N$. Let K be the kernel of this map then we have an imbedding of $\mathcal{S}^N \tilde{\otimes}_\pi \mathcal{S}^N / K$ in $\mathcal{S}^{2N}$. Since $\mathcal{S}^N \tilde{\otimes}_\pi \mathcal{S}^N$ is a nuclear Frechet space follows that $\mathcal{S}^N \tilde{\otimes}_\pi \mathcal{S}^N / K$ is a complete Frechet space. From the well known fact that $\mathcal{S}_n \tilde{\otimes}_\pi \mathcal{S}_m$ is isomorphic to $\mathcal{S}_{n+m}$ follows that $\mathcal{S}^N \tilde{\otimes}_\pi \mathcal{S}^N / K$ must be isomorphic to $\mathcal{S}^{2N}$.

Denote by D = $\mathrm{cl}\{\sum f_i^* \otimes f_i\}$ in $\mathcal{S}^N \tilde{\otimes}_\pi \mathcal{S}^N$ then we know from the definition of $\underline{\mathcal{S}}^+$ that P D = $\underline{\mathcal{S}}^+ \cap \mathcal{S}^{2N}$. From this follows that $\underline{\mathcal{S}}^+ \cap \mathcal{S}^{2N}$ is closed if and only if D + K is a closed cone.

The proof of this fact needs some preparation.

IV. 3. Lemma:

Let P be the product map from $\mathcal{S}^N \tilde{\otimes}_\pi \mathcal{S}^N$ onto $\mathcal{S}^{2N}$ then we get for any bounded set $B \subset \underline{\mathcal{S}}^+ \cap \mathcal{S}^{2N}$

$$P^{-1} B \cap D$$

is a bounded set in D.

Proof Let P^*_α be a C^*-norm on $\underline{\mathcal{S}}$ then we get $\sup_{x \in B} P^*_\alpha(x) = M_\alpha < \infty$

Since P^*_α is a C^*-norm follows that every linear functional T

with $P_\alpha^{*\prime}(T) \leq 1$ can be written as $T = T_1 - T_2 + i(T_3 - T_4)$
with $P_\alpha^{*\prime}(T_i) \leq P_\alpha^{*\prime}(T)$
From this follows for $\sum f_i^* \otimes f_i \in P^{-1} B$

$$\sup \sum_i (T, f_i^* \cdot f_i) \leq \sum_i \sum_{j=1}^{4} (T_j, f_i^* \cdot f_i) \leq 4 M_\alpha$$

$P_\alpha^{*\prime}(T) \leq 1$

This shows that the set of sequences
$\{ \{f_i^* \cdot f_i\}, \sum f_i^* \cdot f_i \in B$ is a bounded set in the ε-topology of the weak summable sequences.

Since $\mathcal{S}^N$ is a nuclear space follows that the ε and π topology on this spaces are the same (see e.g.A.Pietsch [7] Satz 4.2.2.) and hence we get

$$\sum_i P_\alpha^*(f_i^* \cdot f_i) \leq M_\alpha' < \infty.$$

This implies

$$\sum_i P_\alpha^*(f_i^*) \quad P_\alpha^*(f_i) < M_\alpha'$$

and consequently follows for any semi-norm $\pi(P_\alpha^*, P_\alpha^*)$

$$\sup_{y \in P^{-1} B \cap D} \pi(P_\alpha^*, P_\alpha^*)(y) < M_\alpha' < \infty$$

This shows $P^{-1} B \cap D$ is bounded .

IV. 4. <u>Lemma</u> D + K is a closed cone.

<u>Proof</u> Let $x \in cl\{D + K\}$. Since $\mathcal{S}^N \tilde{\otimes}_\pi \mathcal{S}^N$ is a (F)-space exists a sequence $x_n \in D + K$ which converges to x. Since a converging sequence is bounded follows $P\{x_n\}$ is bounded in $\underline{\mathcal{S}}^+ \cap \mathcal{S}^{2N}$. Consequently by the previous lemma follows that

$$\{x_n\} + K \cap D \text{ is a bounded set.}$$

Since x is the limit of the x_n follows that for every neighbourhood U we get

$$(x + K + U) \cap (\{x_n\} + K \cap D) \neq \emptyset$$

Since $\mathcal{S}^N$ is nuclear, follows that $\mathcal{S}^N \tilde{\otimes}_\pi \mathcal{S}^N$ is nuclear([7] Satz 5.4.2). This implies that all bounded sets are pre-compact ([7] Satz 4.4.7).

This implies that

$$x + K \cap cl\{\{x_n\} + K \cap D\} \neq \emptyset$$

Since D itself is closed follows $x \in D + K$.
Collecting these results we get

IV. 5. <u>Proposition:</u>

The intersection of the positive come $\underline{\mathcal{S}}^{+}$ with $\mathcal{S}^{2N}$ is closed.

<u>Proof</u>. Since P is continuous follows

$$P^{-1} cl\{\underline{\mathcal{S}}^{+} \cap \mathcal{S}^{2N}\} = cl\{D + K\} = D + K$$

This implies $\underline{\mathcal{S}}^{+} \cap \mathcal{S}^{2N}$ is closed.

Using the Knowlegde of a dense set of explizitly given positive linear functionals one can show that the positive cone $\underline{\mathcal{S}}^{+}$ is closed if and only if all $\underline{\mathcal{S}}^{+} \cap \mathcal{S}^{2N}$ are closed. Hence the last proposition shows that $\underline{\mathcal{S}}^{+}$ is closed. Up to now I did not find a direct proof of this fact and therefore I will not bring this result in this note.

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