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Numerical Analysis of the Semi-Linear Heat Equation of Blow-up Type

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§1 Introduction.

Let Ω be a bounded open set in $\mathbb{R}^n$ with a smoo
$\Gamma = \partial\Omega$. We take the continuous problem of

$$(E)\quad \begin{cases} \frac{\partial u}{\partial t} = \Delta u + f(u), & x \in \Omega \\ u(t,x) = 0, & x \in \Gamma \end{cases} \quad t > 0$$
$$u(0,x) = a(x), \quad x \in \Omega.$$

For simplicity we assume that f is twice continuous
differentiable satisfying that

(1) $f(u)$ and $f''(u) \geq 0$ for any $u \in R^1$

and that for some positive γ and C

(2) $f(u) \geq Cu^{1+\gamma}$ as $u \to \infty$.

The initial data a(x) is continuous on $\overline{\Omega}$ vanishing
totality of such functions is denoted by $C_0(\overline{\Omega})$. B
classical argument [6], the solution u(t,x) tends
at a finite time T for some a(x). This fact is cal
blowing-up of solution, and the time T is called th
time or the finite escape time. Fujita studied ext
this problem in [3], [4] and so forth. There a
some works based on different criteria by other aut
example, Tsutsumi [9], [10], Ito [5], among others.

The purpose of this paper is to provide a nume
of (E) by making use of the finite element approxim
lumped mass type, based on Kaplan-Fujita's criterio
of the authors already investigated an algorithm fo
approximation to (E) based on Tsutsumi's criterion
The present work is one of its continuations and ba
recent work of the other author [11]. The finite

he algorithm in [7] will be justified in the paper.

ion 2, a reformulation of Kaplan-Fujita's criterion ented so that it is appropriate to our purpose. ate problem will be stated in Section 3, and an r controlling time steps will be described in The rigorous justification will be done in Sections ally in Section 7 some numerical illustrations will etails of numerical results will be reported

rk that it is straightforward to modify the present e case of the condition (1) holding only $u \geqq 0$ under ion of the initial data $a(x) \geqq 0$.

ld like to express our sincere thanks to Mr. Y. or his help in the computation of our model problem.

§2 Kaplan-Fujita's criterion.

Let λ denote the smallest eigenvalue of $-\Delta$ w Dirichlet boundary condition, and let $\phi(x)$ denote function associated with λ, $\phi(x)$ being normalized

$$\begin{cases} \phi(x) > 0, & x \in \Omega, \\ \int_\Omega \phi(x)dx = 1. \end{cases}$$

Denote by $J(t)$ the inner product of $u(t,x)$ and $\phi(x$

$$J(t) = (u(t,x),\ \phi(x))_{L^2(\Omega)} = \int_\Omega u(t,x)\ \phi$$

<u>Definition 1.</u> The classical solution $u(t,x)$ J-blows up at $t = T$ if and only if

$$\text{(3)} \qquad \begin{cases} u(t,x) \in C([0,T), C_0(\bar\Omega)) \text{ satisfies (E),} \\ \lim_{t\uparrow T} J(t) = \infty . \end{cases}$$

Let J^1 be the largest positive root of the equ

$$-\lambda J + f(J) = 0.$$

If the equation has no positive roots, then let J^1

<u>Proposition 1.</u> The solution $u(t,x)$ J-blows u finite time T if and only if there exists a $t_0 \geqq 0$

$$\text{(4)} \qquad \begin{cases} u(t,x) \in C([0,t_0],\ C_0(\bar\Omega)) \text{ satisfies (E} \\ J(t_0) > J^1. \end{cases}$$

<u>Proof.</u> The necessity of the condition (4) is The sufficiency criterion due to Kaplan is as follo $[0,T)$ be the maximal existence interval of $u(t,x)$. inequality, we have the following differential ineq $J(t)$:

$$\text{(5)} \qquad \frac{d}{dt}J(t) \geqq -\lambda J(t) + f(J(t)) \quad t \in [0,T),$$

on account of the convexity of f and the normalizat f ϕ . This inequality implies first that

$\frac{d}{t}J(t) > 0$ for $t \geq t_0$.

less this were true, there would be $t_1 > t_0$ such

) = 0 and $\frac{d}{dt}J(t) > 0$ for $t_0 \leq t < t_1$. The first

nd the inequality (5) shows

$\geq - \lambda J(t_1) + f(J(t_1))$.

uantity $- \lambda J + f(J)$ is nonnegative for $J \geq J^1$,

$(t_1) \leq J^1 < J(t_0)$.

impossible since

$(t_1) - J(t_0) = \int_{t_0}^{t_1} \frac{dJ(s)}{ds} ds > 0.$

on (6) implies that

$\lambda J(t) + f(J(t)) > 0$ for any $t \geq t_0$,

rn implies

$- t_0 \leq \int_{J(t_0)}^{J(t)} \frac{dJ}{-\lambda J+f(J)}$ for any $t \geq t_0$.

sumed that $f(u) \geq Cu^{1+\gamma}$ for $u \to \infty$, the right hand

is uniformly bounded. Hence we have that the

t,x) blows up at finite T.

ary 2. The blowing-up time T is bounded from above

$\leq t_0 + \int_{J(t_0)}^{\infty} \frac{dJ}{-\lambda J+f(J)}$.

This follows from the estimate (7).

§3 Setting of the approximating problem.

Let $\{ \Omega_h ; h > 0\}$ be the family of polyhedral

domains contained in Ω satisfying

$$(8) \quad \begin{cases} \Omega_h > \Omega_{h'} \quad \text{if } h \leq h', \\ \max_{x \in \Gamma_h} \text{dist}(x, \Gamma) \to 0 \quad \text{as } h \to 0. \end{cases}$$

Here Γ_h is the boundary of Ω_h.

<u>Definition 2.</u> The set $\mathfrak{S}_h = \{S^{(k)}\}$ is a tri

of the polyhedral open domain Ω_h if and only if

(9)
(i) $S^{(k)}$, k=1,2,$\cdots$, are nondegenerat

n-simplices, the number of which

(ii) $\overline{\Omega}_h = \bigcup_{S^{(k)} \in \mathfrak{S}_h} S^{(k)}$,

(iii) the face of $S^{(k)}$ is either a face

n-simplex of $\mathfrak{S}_h$ or else is a port

boundary of Ω_h.

In the following, we shall omit the superscrip

for simplicity of notation. Let $b_0, \cdots, b_n$ denote

(or nodal points) of an n-simplex S. In terms of th

barycentric coordinates ($\lambda_0(x), \cdots, \lambda_n(x)$), $x \in$

barycentric subdivision $B_{b_0(S)}$ of b_0 in S such that

$$B_{b_0(S)} = \{x; 1 \geq \lambda_0(x)/(\lambda_0(x)+ \lambda_i(x)) >$$

The "lumped mass region" B_b associated with node b i

$$B_b = \bigcup_S B_{b(S)}$$

in which $\bigcup_S$ denotes the union with respect to all n-

r vertex. The characteristic function of B_b
(x).

ate renumbering of all nodal points of all
let $b_1, \cdots, b_N$ denote the interior nodal
nd let $b_{N+1}, \cdots, b_{N+M}$ denote the boundary nodal
Define the two functions $\hat{w}_j(x)$ and $\bar{w}_j(x)$, $j=1,\cdots,N$,

) is a linear function on each S satisfying

$$\hat{w}_j(b_k) = \delta_{jk} \quad \text{for } k=1, \cdots, N+M,$$

$$= \bar{w}_{b_j}(x).$$

the sets of linear combinations of $\hat{w}_j(x)$ and
$\cdots$, N, respectively, i.e.,

$$\{\hat{u}_h;\ \hat{u}_h = \sum_{j=1}^{N} \alpha_j \hat{w}_j\},$$

$$\{\bar{u}_h;\ \bar{u}_h = \sum_{j=1}^{N} \alpha_j' \bar{w}_j\}.$$

s $\hat{u}_h \in \hat{V}_h$ and $\bar{u}_h \in \bar{V}_h$ specified by the same
$= \alpha_j'$, $j = 1, 2, \cdots, N$, are said to be
her. Introduce the mappings K_h and K_h^{-1}

$$\bar{u}_h \to \hat{u}_h,$$

$$\hat{u}_h \to \bar{u}_h,$$

$\bar{u}_h \in \bar{V}_h$ are associate each other.
ce X which is the space $C_0(\bar{\Omega})$ normed with
Similarly, introduce the finite dimensional
which are $\hat{V}_h$ and $\bar{V}_h$, respectively, normed with
Define the operator $\tilde{P}_h$ by

$$(\tilde{P}_h u)(x) \equiv \sum_{j=1}^{N} u(b_j)\hat{w}_j(x) \quad \text{for } u \in X.$$

Clearly the mapping $P_h = K_h^{-1}\tilde{P}_h$,

$$P_h:\ X \to X_h,$$

is the projection of X onto X_h.

In terms of the above defined concepts and not
define the operator A_h in X_h by

$$(A_h\bar{u}_h, \bar{v}_h)_{L^2(\Omega_h)} = -(\nabla\hat{u}_h, \nabla\hat{v}_h)_{L^2(\Omega_h)}$$

$$\text{for } \bar{u}_h \in X_h,\ \bar{v}_h \in X_h,$$

$$\hat{u}_h = K_h\bar{u}_h,\ \hat{v}_h = K_h\bar{v}_h,$$

and the nonlinear mapping f in X_h by

$$f(\bar{u}_h) = \sum_{j=1}^{N} f(\alpha_j)\bar{w}_j \quad \text{for } \bar{u}_h = \sum_{j=1}^{N} \alpha_j\bar{w}_j.$$

Let τ denote the ordered set $(\tau_0, \tau_1, \tau_2, \cdots)$
elements $\tau_n > 0$, $n = 0, 1, 2, \cdots$. The set τ will b
time mesh vector. Now we state our appro
scheme.

$$(E_h^{\tau}) \quad \begin{cases} t_{n+1} = t_n + \tau_n,\ t_0 = 0,\quad \tau_n > 0, \\ u_h(t) = u_h(t_n),\ t_n \leqq t < t_{n+1}, \\ \dfrac{u_h(t_{n+1}) - u_h(t_n)}{\tau_n} = A_h u_h(t_n) + f(u_h(t_n) \\ u_h(0) = a_h, \end{cases}$$

where $a_h = P_h a$.

<u>Proposition 3.</u> If $(\nabla\hat{w}_i, \nabla\hat{w}_j)_{L^2(\Omega_h)} \leqq 0$ fo
$1 \leqq i \leqq N$, $1 \leqq j \leqq N+M$, then it holds that the smal
eigenvalue λ_h of $-A_h$ is simple, and that there is

$\phi_h(x)$ normalized as $\phi_h(x) \geqq 0$ $(x \in \Omega_h)$ and
1.

As is well known, the operator A_h is invertible.
ion due to Ciarlet-Raviart [1], the operator
nonnegative element of $L(X_h)$. Namely we have that
for any $u_h \in X_h$ with $u_h \geqq 0$. Consider the matrix
the eigenvalue problem $-A_h\phi_h = \lambda_h\phi_h$. The problem
$-\mathbb{A}\phi = \lambda\mathbb{M}\phi$ where $\mathbb{A}$ and $\mathbb{M}$ are the stiffness
mass matrix, respectively, and ϕ is an N-vector.
enius' Theorem to the nonnegative matrix $(-\mathbb{A})^{-1}\mathbb{M}$,
onclusion.

§4 An algorithm for controlling time steps

Define $J_h(t)$, the discrete analogue to $J(t)$,

$$J_h(t) = (u_h(t,x),\ \phi_h(x))_{L^2(\Omega_h)}.$$

Let $J_h^{\ 1}$ denote the largest positive root of the eq

$$-\lambda_h J + f(J) = 0.$$

If the equation has no positive roots, then let J_h

Define τ_h by the formula,

$$\tau_h = \min_{1 \leqq i \leqq N} \|\bar{w}_i\|^2 / \|\nabla\hat{w}_i\|^2. \tag{10}$$

Choose a fixed value of τ which is not greater tha
Then our algorithm for controlling the time step τ
given by

$$\begin{cases} \tau_0 = \tau, \quad \text{and} \\ \tau_n = \begin{cases} \tau & \text{if } J_h(t_{n-1}) < J_h^{\ 1}, \\ \min\{\tau,\ \dfrac{J_h(t_n) - J_h(t_{n-1})}{-\lambda_h J_h(t_n) + f(J_h(t_n))} \end{cases} \end{cases} \tag{11}$$

for $n = 1, 2, 3, \cdots$.

Fig. 1 shows the general flow chart to calculate u
by $(E_h^{\ \tau})$ with (11).

<u>Definition 4.</u> The solution $u_h(t,x)$ of $(E_h^{\ \tau})$
the time mesh vector obtained by the algorithm d
above, J_h-blows up at $t = T_h$ if and only if

$$T_h = \sum_{n=0}^{\infty} \tau_n < \infty .$$

<u>Proposition 4.</u> The solution $u_h(t,x)$ J_h-blow
finite time T_h if and only if there is a $t_n \geqq 0$ su

$$J_h(t_n) > J_h^{\ 1}. \tag{12}$$

<u>Proof.</u> The necessity of the condition (12)

ciency criterion, in parallel to the proof of [on] 1, is as follows. By Jensen's inequality we have

$$\frac{J_h(t_{k+1}) - J_h(t_k)}{\tau_k} \geq -\lambda_h J_h(t_k) + f(J_h(t_k))$$

for $k = 0, 1, \cdots$.

[conv]exity of f and the condition (12) imply that

$$J_h^{1} < J_h(t_n) < J_h(t_{n+1}) < J_h(t_{n+2}) < \cdots .$$

[a]n integer greater than or equal to n+1. Then our implies

$$k \leq \frac{J_h(t_k) - J_h(t_{k-1})}{-\lambda_h J_h(t_k) + f(J_h(t_k))}$$

$$= \int_{J_h(t_{k-1})}^{J_h(t_k)} \frac{dJ}{-\lambda_h J_h(t_k) + f(J_h(t_k))}$$

$$\leq \int_{J_h(t_{k-1})}^{J_h(t_k)} \frac{dJ}{-\lambda_h J + f(J)},$$

last inequality follows from the convexity of f.

[w]e have

$$T - t_n - \tau_n = \sum_{k=n+1}^{\infty} \tau_k$$

$$\leq \int_{J_h(t_n)}^{\infty} \frac{dJ}{-\lambda_h J + f(J)} < \infty .$$

[Coroll]ary 5. The blowing-up time T_h is bounded from

$$T_h \leq t_n + \tau_n + \int_{J_h(t_n)}^{\infty} \frac{dJ}{-\lambda_h J + f(J)} .$$

§5 Convergence of the blowing up time.

Theorem 1. Assume the following two condition[s]

(i) $\lambda_h \to \lambda$ and $\phi_h \to \phi$ in $L^2(\Omega)$ as $h \to 0$.

(ii) Let the solution u of (E) J-blow up at a f[...] T. For any T' < T and for any sufficientl[y] there is a solution $u_h(t)$ of $(E_h^{\tau_h})$ for 0 [...] satisfying $\max_{0\leq t\leq T'} \| u_h(t) - u(t) \|_{L^2(\Omega)} \to$ [...]

Here τ_h is the time mesh vector obtained b[...]

Then it holds that

$$T_h \to T \text{ as } h \to 0$$

provided that $\| \tau_h \|_\infty \to 0$ as $h \to 0$.

Proof. Fix T' < T arbitrarily. Then we have [...] conditions of Theorem 1

(14) $\lim_{h\to 0} J_h(t) = J(t)$

uniformly in $t \in [0,T']$. This implies that $T' \leq \lim$[...]

Since T' is arbitrarily close to T, we have

$$T \leq \liminf_{h\to 0} T_h .$$

Suppose next that $T'' = \limsup_{h\to 0} T_h > T$ would hold. [...] (i) we can find numbers J^2 and h_0 in such a way tha[t] [...] and $\int_{J^2}^{\infty} \frac{dJ}{-\lambda_h J + f(J)} \leq \frac{T'' - T}{2}$ hold for any $h \leq h_0$. [...] there is a number $t' < T$ such that $J_h(t') > J^2$ for [...]

We may assume that $t_n^h \leq t' < t_{n+1}^h < T$ for $h \leq h_0$. [...] $J_h(t') = J_h(t_n^h)$. By Corollary 5, it holds that

$$T_h - t_n^h \leq \tau_n^h + \int_{J_h(t_n^h)}^{\infty} \frac{dJ}{-\lambda_h J + f(J)} .$$

Hence we have

$\leq t_n^h + \tau_n^h + \frac{T'' - T}{2}$

$\leq T + \frac{T'' - T}{2} = T'' - \frac{T'' - T}{2}$.

have $\limsup_{h\to 0} T_h < T'' = \limsup_{h\to 0} T_h$, which is a

.

§6 Convergence of the approximate solution.

Let $\{\tau_h : h > 0\}$ be a sequence tending to zero a to zero, and satisfying $\bar{\tau}_h \leq \tau_h$ where τ_h is defined formula (10). Let T' be a positive number specifie Consider a family $\lambda = \{\boldsymbol{\tau}_h : h > 0\}$ of time mesh vecto satisfying that

$$\| \boldsymbol{\tau}_h \|_\infty = \sup \{\tau : \tau \in \boldsymbol{\tau}_h\} \leq \bar{\tau}_h ,$$

and that

$$\| \boldsymbol{\tau}_h \|_1 = \sum_{\tau \in \boldsymbol{\tau}_h} \tau > T' .$$

We can construct a family of solutions $\{u_h^\lambda(t,x)$: of $(E_h^{\boldsymbol{\tau}})$ choosing $\boldsymbol{\tau}_h$ as the time mesh vector for ea Λ be the totality of the above index λ. Let h(S) diameter of an n-simplex S, and let $\rho(S)$ denote the of diameters of the inscribed spheres of S.

Theorem 2. Assume (8) and the following thre

(i) $(\nabla \hat{w}_i, \nabla \hat{w}_j) \leq 0$ for $i \neq j$, $1 \leq i \leq N$, $1 \leq j$

(ii) $\max_{S \in \mathfrak{S}_h} h(S) \leq h$

(iii) $\inf_h \min_{S \in \mathfrak{S}_h} \rho(S)/h(S) = \mu > 0$.

If the unique classical solution u(t,x) of (E) exis $t \in [0,T']$ then

$$\lim_{h\to 0} \max_{0 \leq t \leq T'} \max_{x \in \Omega_h} |u_h^\lambda(t,x) - u(t,x)| = 0$$

uniformly in $\lambda \in \Lambda$.

Proof. This is a slight variant of Theorem 1. in which the uniform dependence of λ was not discuss checks its proof, the present result can be obtaine expression

$$u_h^\lambda(t) = U_h^\lambda(t,0)a_h + \int_0 \hat{U}_h^\lambda(t,s)f(u_h^\lambda(s))ds$$

$u_h^\lambda(t,x)$ as an X_h-valued step function where $U_h^\lambda(t,s)$) are suitably defined approximating operators of

It is to be noted that the proof of Theorem 1.2 in es that the families of operator valued sequences $h > 0\}_{0\leq s\leq t\leq T', \lambda\in\Lambda}$ and $\{\hat{U}_h^\lambda(t,s): h > 0\}_{0\leq s\leq t\leq T', \lambda\in\Lambda}$ to $e^{(t-s)A}$ as h tends to 0 uniformly with respect to ers s, t, and λ. Here, the operator A is the generator -group $e^{(t-s)A}$ in $X = C_0(\bar{\Omega})$ corresponding to the heat Ω with the Dirichlet boundary condition. This fact validity of the present Theorem.

Although the twice differentiability of f was [11], the local Lipshitz continuity is sufficient sent conclusion apart from the assurance of the tence of the smooth solution of (E). See [8].

establish the condition (ii) of Theorem 1 to our t) obtained by the algorithm described in Section 4. Theorem 2, it suffices to show that for any fixed can choose h_0 in such a way that $u_h(t)$ never thin the interval $[0,T']$ if $h \leq h_0$. This fact lied by Theorem 2. In fact, let h_0 be such that

$$\max_{t\leq T'} \max_{x\in\Omega} | u_h^\lambda(t,x) - u(t,x) | \leq 1$$

Λ and $h \leq h_0$ in the situation of Theorem 2. This

implies that there is a finite number M satisfying

$$\sup_{0\leq t\leq T',\ \lambda\in\Lambda, h\leq h_0} \| u_h^\lambda(t)\|_{L^2(\Omega)} = M < \infty. \tag{15}$$

Assume that there is a solution $u_h(t)$ J_h-blowing up $t = T_h < T'$. Then there is a mesh point t_n such th $\| u_h(t_n) \| > M$ since $\|\phi_h\| \, \|u_h(t)\| \geq J_h(\phi_n) \uparrow \infty$. Thi the condition (15), since there is a λ containing t mesh vector in the form

$$\tau_h = (\tau_0, \tau_1, \ldots, \tau_{n-1}, \tau_{n-1}, \ldots)$$

where τ_j, $0 \leq j \leq n-1$, are the mesh lengths determine algorithm.

It is seemingly well known that the condition Theorem 1 holds under the same conditions of Theore skip its proof though we have not known literat containing its proof.

Finally we remark the following two Propositio literature which concern the conditions of Theorem

<u>Proposition 6.</u> (Ciarlet-Raviart [1].) De

$$\sigma_S = \max_{i\neq j} \{(\nabla\lambda_i, \nabla\lambda_j)_{R^n} / |\nabla\lambda_i|_{R^n} \cdot |\nabla\lambda_j|_{R^n}\}$$

in which $\lambda_i = \lambda_i(x)$ is the barycentric coordinate o with respect to vertex P_i of S, and $(\, , \,)_{R^n}$ and $|$ respectively denote the Euclidean scalar product an norm in R^n. If $\sigma_S \leq 0$ for any $S \in S_h$, then Conditi Theorem 2 holds.

<u>Proposition 7.</u> (Fujii [2].) Define κ_h by

$$\kappa_h = \min_{S\in S_h} \kappa_S$$

is given as

$= \min_{b_i \varepsilon S}$ dist (b_i, the face of S not containing b_i)

stimated as

$\geq \kappa_h{}^2/(n+1)$.

§7 Numerical Examples.

As an numerical illustration, we take a spatia

1 dimensional problem with $f(u) = u^2$. Let Ω = (0,

$S_h = \{[jh, (j+1)h]: 0 \leq j < 2^{-N}\}$ for $h = 2^{-N}$ with po

integer N. Then our approximate equation ($E_h{}^\tau$) is

the usual explicit difference scheme in which $\frac{d^2}{dx^2}u($

approximated by the central difference: $\frac{u(x+h)-2u(x}{h^2}$

As an initial value we take the function

$u(x) = A \sin \pi x$.

In Figure 2, a comparison between the controll

mesh algorithm and the fixed time mesh algorithm is

the case of $h = 2^{-3}$, $\tau_0 = 2^{-7}$ and A = 12. We denot

and ×, the values of u(t,x) at x = 1/2, calculated

controlling mesh algorithm and the fixed mesh algo

respectively. We obtained $t_n = 0.307$ as the approx

of T_h when the controlling mesh algorithm stopped b

condition of $\tau_n < 2^{-20}$, whereas the fixed mesh algo

worked until $t_n = 0.367$ when the calculation became

because of machine overflow ($u_{max} > 10^{65}$).

In Figure 3, the convergence of u_h

as $h \to 0$ is shown for A = 12, with $\tau_0 = 2^{-1}h^2$. The

criterion is that $\tau_n < 2^{-20}$. Seemingly the round o

might effect the calculation in the case $h = 2^{-6}$.

Our calculation was performed by the HITAC 8250, a

sized machine, in single precision arithmetic with

mantissa.

Finally in Figure 4, we present the result of

e threshold of blowing up by changing the coef-
f the initial data a(x), in the case of $h = 2^{-4}$
Numerically the solution decays exponentially
17, and blows up at finite time if $A \geq 11.47418$.
ecalling that if $J(0) > J^1$ the solution of (E)
inite time. In our sample problem, $J(0) > J^1$
to the condition of $A > 4\pi = 12.56637$.

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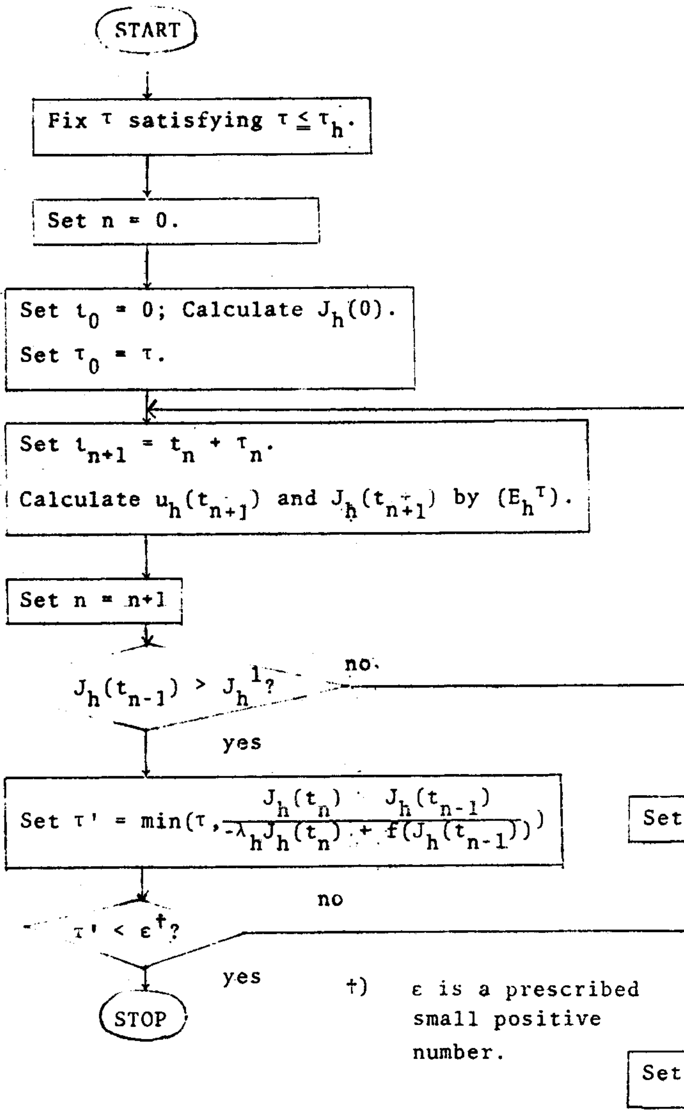
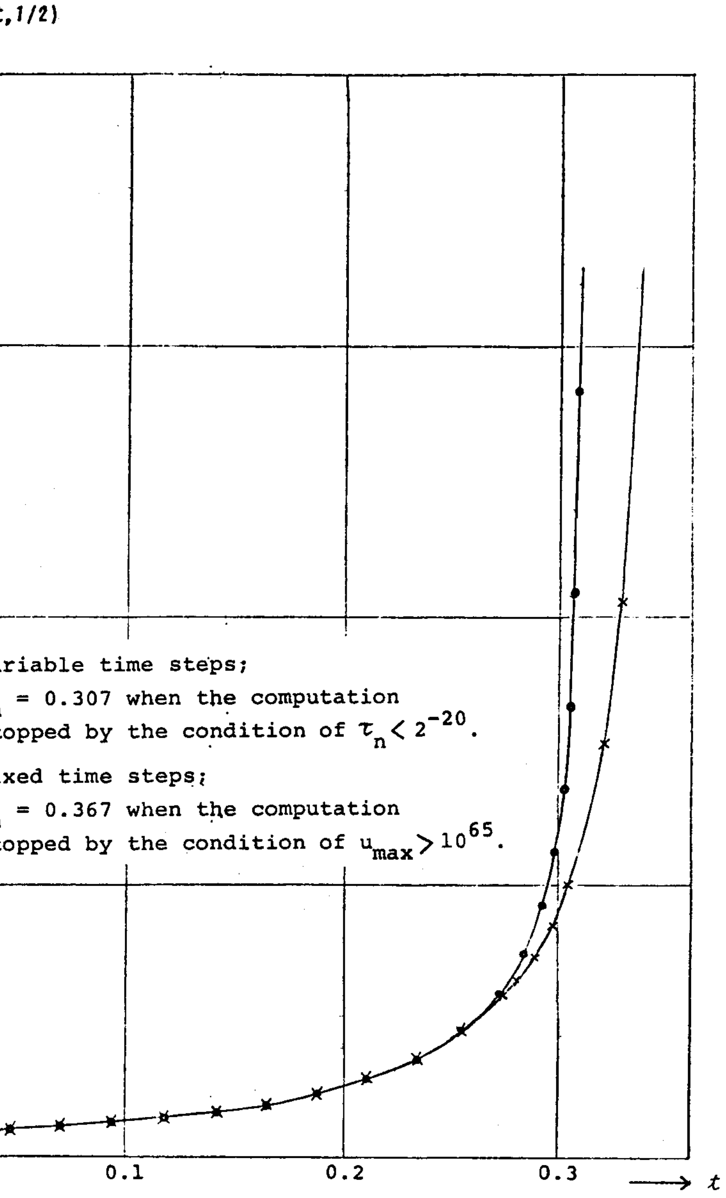


Fig. 1 Flow chart for comput
by controlled time st



puted value of u by the variable time-step algorithm
fixed-time step algorithm in the case of $a(x) = 12\sin\pi x$.
ers: $h = 2^{-3}$ and $\tau_0 = 2^{-7}$.

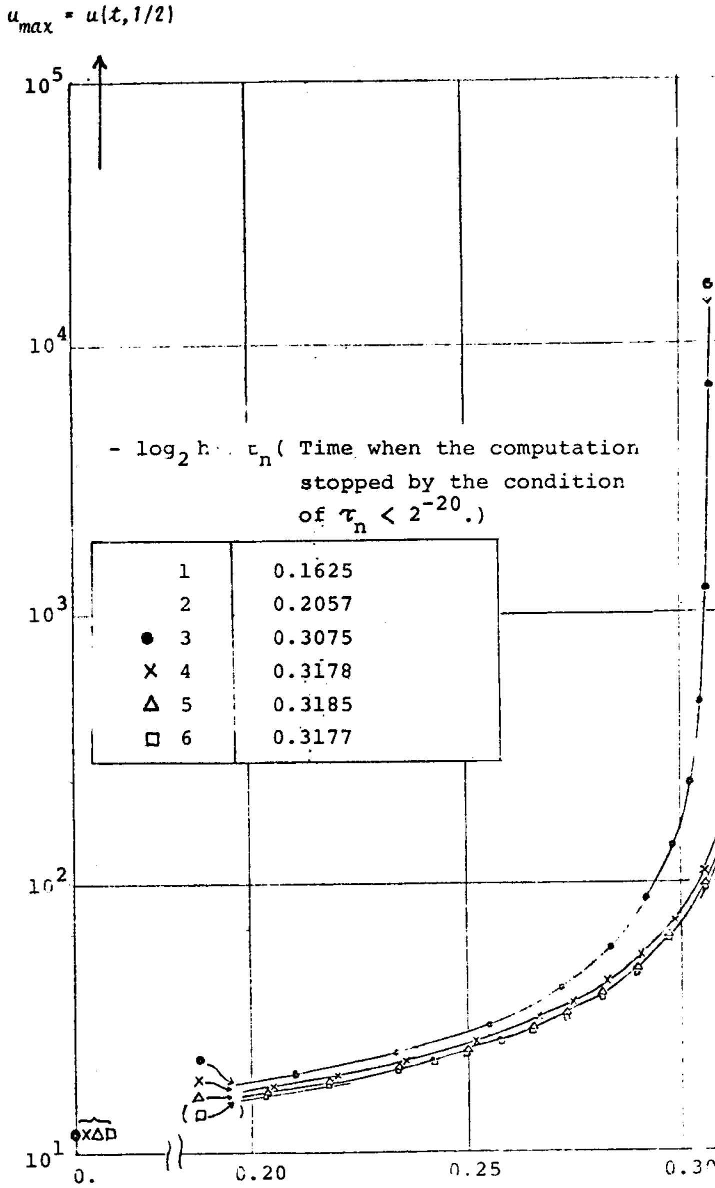
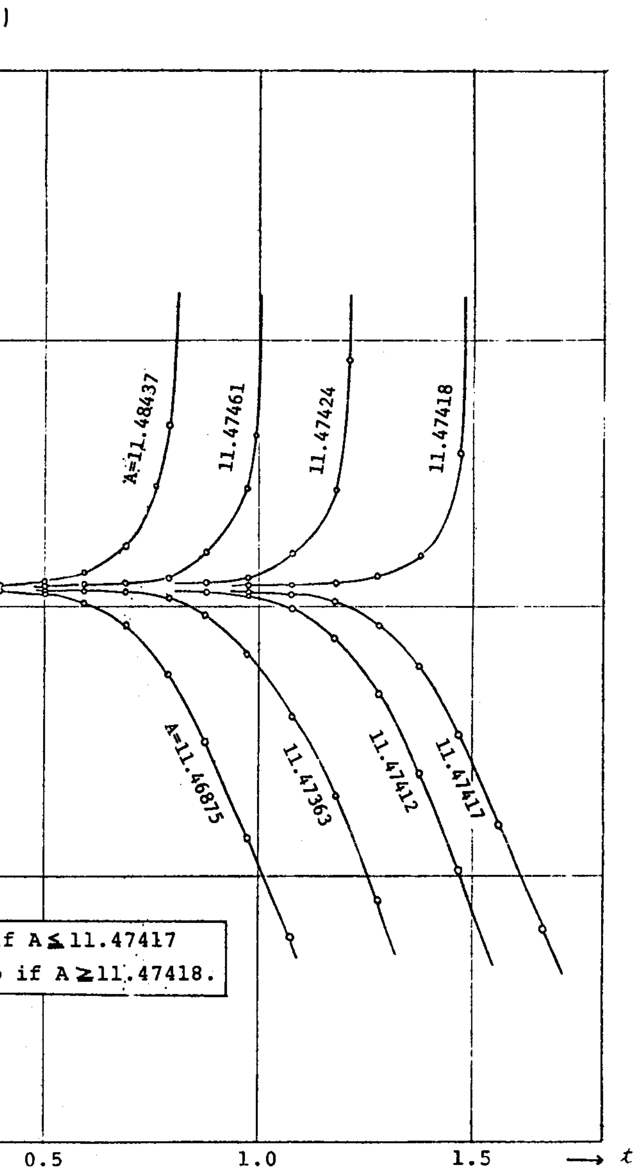


Fig.3 Convergence of profile of u as $h \rightarrow 0$ in the
$a(x) = 12\sin\pi x$.



al search for the threshold of blowing-up in the
$a(x) = A \sin \pi x$.
ers: $h = 2^{-4}$ and $\tau_0 = 2^{-9}$.