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## TOPOLOGICAL ENTROPY AND HOMOLOGY

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Let  $X$  be a compact metric space and  $f: X \rightarrow X$  a continuous map.

Definition.- (Bowen)

A  $(k, \delta)$  spanning set for  $f$  is a finite set  $Y \subset X$  s.t.

$\forall x \in X \exists y \in Y$  s.t.  $d(f^j x, f^j y) < \delta$  for  $0 \leq j < k$ .

$$h(f, \delta) = \limsup_{k \rightarrow \infty} \frac{1}{k} \log \text{card}(\text{minimal } (k, \delta) \text{ spanning set})$$

The topological entropy is defined as :

$$h(f) = \sup_{\delta > 0} h(f, \delta)$$

$h(f)$  thus describes the exponential growth rate of the number of types of  $f$ -orbits, or roughly speaking the amount  $f$  mixes up the points of  $X$ .

$h(f) = \sup_{\mu} h_{\mu}(f)$  where  $\mu$  ranges over all normalized Borel invariant measures (Dinaburg).

Conjecture.- (Shub)

Let  $f: M \rightarrow M$  be a diffeomorphism of a smooth compact manifold  $M$ . Then  $h(f) \geq \log \text{sp } f_*$  where  $\text{sp } f_*$  is the largest modulus of any eigenvalue of the induced map of homology  $f_*: H_*(M; \mathbb{R}) \rightarrow H_*(M; \mathbb{R})$ .

$H_i(M; \mathbb{R})$  is a finite dimensional real vector space describing the  $i$  dimensional structure of  $M$ ,  $f$  describes the action of  $f$  on this structure and so the conjecture says that a diffeomorphism that mixes the  $i$  dimensional structure of  $M$  for some  $i$  must also have at least the corresponding amount of entropy I.e. the homology of  $f$  detects some of the dynamics of  $f$ .

The conjecture has been proved for diffeomorphisms satisfying Axiom A and the no cycle property (Shub and Williams). Hence it holds for a  $C^0$  open and

dense set of diffeomorphisms of  $M$ . For  $C^0$  maps the inequality fails even on the manifold  $S^2$ . However, any  $C^0$  map of  $M$  satisfies  $h(f) \geq \log |\lambda|$  for each eigenvalue  $\lambda$  of  $f_*$  in  $H_1(M; \mathbb{R})$  the first homology group (Manning).

For eigenvalues in higher groups differentiable hypotheses would usually be required. In this direction Misiurewicz and Przytycki showed recently that any  $C^1$  map  $f: S^2 \rightarrow S^2$  satisfies  $\exp h(f) \geq |\deg(f)|$ .

#### REFERENCE

- A. MANNING. Topological Entropy and the First Homology Group, in Dynamical Systems Warwick (1974), Lecture Notes in Mathematics vol. 468, Springer, (1975).