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GENERALIZED PICARD LATTICES ARISING FROM HALF-INTEGRAL CONDITIONS

by G. D. MOSTOW (*)

1. Introduction

Set

$$F_{gh}(x_2, \dots, x_{d+1}) = \int_g^h u^{-\mu_0} (u-1)^{-\mu_1} \prod_2^{d+1} (u-x_i)^{-\mu_i} du$$

where $g, h \in \{\infty, 0, 1, x_2, \dots, x_{d+1}\}$. Then for fixed μ_0, \dots, μ_{d+1} , F_{gh} is a multivalued function on the subset M of $(\mathbf{P}^1)^{d+3}$ defined as

$$M = \{(x_i) \mid x_i \neq 0, 1, \infty \text{ and } x_i \neq x_j \text{ for } i \neq j\}.$$

For topological reasons, the \mathbf{C} -linear span of these functions form a $d+1$ dimensional vector space that is invariant under monodromy. Taking $d+1$ such functions as the homogeneous coordinates in projective d -space \mathbf{P}^d , we get a map

$$\hat{w}: \hat{M} \rightarrow \mathbf{P}^d$$

where \hat{M} is the universal covering of the space M . Set

$$\mu_\infty = 2 - (\mu_0 + \mu_1 + \dots + \mu_{d+1}).$$

Assume hereafter that μ_i is real and strictly positive for all i ($0 \leq i \leq d+1$ or $i = \infty$). Let Γ denote the image of $\pi_1(M)$ in $\text{PGL}(d+1, \mathbf{C})$ under the monodromy action. In the preceding paper, the following sufficiency condition was proved:

If for all i, j in $\{\infty, 0, 1, \dots, d+1\}$
(INT): $(1 - \mu_i - \mu_j)^{-1}$ *is an integer for all $i \neq j$ such that $\mu_i + \mu_j < 1$, then Γ is a lattice in the projective unitary group $\text{PU}(d, 1)$.*

In the case $d = 2$, this condition is essentially equivalent to Picard's, and under condition (INT), I call Γ a Picard lattice.

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The purpose of this paper is to relax condition (INT) in case some of the μ_i 's are equal. The main result, proved in § 3, states:

Let $S_1 \subset S = \{\infty, 0, 1, 2, \dots, d+1\}$ and assume that $\mu_s = \mu_t$ for all $s, t \in S_1$. If $\mu_s > 0$ for all $s \in S$ and (μ_s) satisfies the condition $(\Sigma \text{ INT})$: For all $s \neq t$ such that $\mu_s + \mu_t < 1$

$$(1 - \mu_s - \mu_t)^{-1} \text{ is } \begin{cases} \text{an integer if } s \text{ or } t \text{ is not in } S_1, \\ \text{a half-integer if } s, t \in S_1; \end{cases}$$

then Γ is a lattice in $\text{PU}(d, 1)$.

When condition $(\Sigma \text{ INT})$ is satisfied, we define in § 2 a finite extension Γ_Σ of Γ . The lattice Γ_Σ arises from an extension of order $n!$ of the fundamental group $\pi_1(M)$ where $n = \text{card } S_1$. If (μ_s) satisfies condition $(\Sigma \text{ INT})$ but not (INT), then $\Gamma_\Sigma = \Gamma$; if (μ_s) satisfies (INT) too, then Γ_Σ/Γ is the symmetric group on n letters (cf. (3.11)).

In § 4, it is shown that each lattice $\Gamma(p, t)$ of $\text{PU}(2, 1)$ constructed in my paper [2] via three \mathbf{C} -reflections is contained in the lattice Γ_Σ arising from monodromy of a hypergeometric function satisfying condition $(\Sigma \text{ INT})$ for a three element subset S_1 . Conversely, each such lattice Γ_Σ lies in an extension (of order at most 3) of a lattice $\Gamma(p, t)$ for suitable p and t ; in § 6 (p, t) is expressed in terms of $(\mu_s)_{s \in S}$. This $\Gamma(p, t)$ description of Γ applies to most of the 27 Picard lattices, since for 22 of them, at least three of the $(\mu_s)_{s \in S}$ are equal.

In § 5 there is a list of all sequences (μ_1, \dots, μ_N) satisfying condition $(\Sigma \text{ INT})$ but not (INT) for $N > 4$. It is seen that $N \leq 12$; that is, one gets lattices Γ in $\text{PU}(d, 1)$ satisfying condition $(\Sigma \text{ INT})$ for $d \leq 9$ but not for $d > 9$.

The description of Γ_Σ in terms of $\Gamma(p, t)$ makes it possible to give an explicit, fundamental domain for Γ_Σ (cf. [3]) and a two generator presentation for Γ_Σ in case $d = 2$; this fundamental domain is the one described in [2] for $p \leq 5$.

None of the groups $\Gamma(p, t)$ in [2] coincide with a Picard lattice Γ ; the lattice $\Gamma(p, t)$ of [2] is commensurable with a Picard lattice only if p is even (i.e. $p = 4$), in which case $\Gamma/\Gamma \cap \Gamma(p, t)$ has order 1 or 3 and $\Gamma(p, t)/\Gamma \cap \Gamma(p, t)$ has order 6.

2. The Main Theorem

We continue the notation of the preceding paper, referred to hereafter as DM, except that we write $\text{PU}(d, 1)$ for $\text{PU}(1, d)$.

Let $S = S_1 \cup S_2$ be a decomposition of the set S into disjoint subsets and assume that $\mu_s = \mu_t$ for all $s, t \in S_1$. Let Σ denote the permutation group of S_1 . Then Σ operates on P^S by permutation of factors and hence on the set M of injective maps of S into P . It stabilizes the local system L on the family of punctured projective lines over M . The action of Σ on M and $B(\alpha)_M$ descend to an action on Q , Q_{st} , Q_{sst} , and on the

bundle $B(\alpha)_Q$. Consequently, the bundle map $B(\alpha)_Q \rightarrow Q$ descends to a bundle map $B(\alpha)_{Q/\Sigma} \rightarrow Q/\Sigma$. The section w_μ of the bundle $B(\alpha)_Q$ is preserved by Σ ; hence it descends to a section, also denoted w_μ , of the bundle $B(\alpha)_{Q/\Sigma}$.

Let Q' denote the subset of Q on which Σ operates freely; Q' is an open dense submanifold of Q . From the flatness of the bundle $B(\alpha)_Q$ over Q we infer the flatness of $B(\alpha)_{Q/\Sigma}$ restricted to Q'/Σ ; this latter bundle is denoted by $B(\alpha)_{Q'/\Sigma}$.

Let o be a base point in Q' , let \bar{o} denote the orbit Σo , and let

$$\theta_\Sigma : \pi_1(Q'/\Sigma, \bar{o}) \rightarrow \text{Aut } B(\alpha)_{\bar{o}}$$

denote the monodromy homomorphism. Then

$$B(\alpha)_{Q'/\Sigma} = \widehat{Q'/\Sigma} \times_{\pi_1(Q'/\Sigma, \bar{o})} B(\alpha)_{\bar{o}} = \widehat{Q'/\Sigma} \times_{\Gamma_\Sigma} B(\alpha)_{\bar{o}}$$

where $\widehat{Q'/\Sigma}$ denotes the simply connected covering space of Q'/Σ , $\Gamma_\Sigma = \pi_1(Q'/\Sigma, \bar{o})/\text{Ker } \theta_\Sigma$, and

$$(2.1) \quad \widehat{Q'/\Sigma} = (\widehat{Q'/\Sigma})/\text{Ker } \theta_\Sigma.$$

Theorem. — Assume that $(\mu_s)_{s \in S}$ satisfies the condition

$$(2.2) \quad (\Sigma \text{ INT}) \text{ For all } s \neq t \text{ in } S \text{ such that } \mu_s + \mu_t < 1, \quad (1 - \mu_s - \mu_t)^{-1} \text{ is} \\ \begin{cases} \text{an integer, if } s \text{ or } t \text{ is not in } S_1, \\ \text{a half-integer, if } s, t \in S_1. \end{cases}$$

Then $\text{Im } \theta_\Sigma$ is a lattice in $\text{PU}(\text{card } S - 3, 1)$.

3. Proof of the theorem

(3.1) The basic idea of the proof is to show that under hypothesis $(\Sigma \text{ INT})$ Q'/Σ plays the same role that Q plays in DM under hypothesis (INT). We begin with some remarks about morphisms of completions of spreads.

(3.2) Let Y_i be a locally connected Hausdorff space ($i = 1, 2$) and Y'_i an open dense connected subset in Y_i . Assume that each point $y \in Y_i$ has a base of open neighborhoods \mathcal{V}_y satisfying

(3.2.1) for V in \mathcal{V}_y , $V \cap Y'_i$ is connected,

(3.2.2) for $V' \subset V''$ in \mathcal{V}_y , $\pi_1(V' \cap Y'_i) \simeq \pi_1(V'' \cap Y'_i)$.

Let $\rho'_i : X'_i \rightarrow Y'_i$ denote a covering map. Considered as a map of X'_i to Y_i , ρ'_i is a spread. Let $\rho_i : X_i \rightarrow Y_i$ denote the completion of ρ'_i ($i = 1, 2$) (cf. DM 8.1). Then X_i and Y_i are locally connected and ρ_i is a complete spread.

Assume in addition that there are maps $\sigma' : X'_1 \rightarrow X'_2$ and $\tau : Y_1 \rightarrow Y_2$ such

that $\rho_2 \sigma' = \tau \rho_1$. Then by (8.1.1) of DM there is a map $\sigma : X_1 \rightarrow X_2$ such that the diagram below is commutative

$$\begin{array}{ccc} X'_1 & \xrightarrow{\sigma'} & X'_2 \\ \downarrow & & \downarrow \\ X_1 & \xrightarrow{\sigma} & X_2 \\ \downarrow \rho_1 & & \downarrow \rho_2 \\ Y_1 & \xrightarrow{\tau} & Y_2 \end{array}$$

Lemma (3.3). — Assume in addition that

(3.3.1) σ' is a surjective covering map,

(3.3.2) τ is an open map,

(3.3.3) for any $y \in Y_1$ and $V \in \mathcal{V}_y$ (cf. (3.2)), V is connected component of $\tau^{-1}\tau(V)$.

Then the map σ is open and surjective.

Proof. — Let V be an open connected set in Y_1 small enough so that V is a connected component of $\tau^{-1}\tau(V)$ (cf. (3.3.3)). In order to prove that σ is open, it suffices, by definition of a spread, to prove that for any connected component $\rho_1^{-1}(V)^c$ of $\rho_1^{-1}(V)$, $\sigma(\rho_1^{-1}(V)^c)$ coincides with a connected component of $\rho_2^{-1}\tau(V)$.

Commutativity of the diagram and surjectivity of σ' yields

$$\rho_2^{-1}\tau(V) \cap X'_2 = \sigma'(\rho_1^{-1}\tau^{-1}\tau(V) \cap X'_1).$$

Set $C'_1 = (\rho_1^{-1}\tau^{-1}\tau(V) \cap X'_1)^c$, the connected component of $\rho_1^{-1}\tau^{-1}\tau(V) \cap X'_1$ contained in $[\rho_1^{-1}\tau^{-1}\tau(V)]^c$, the connected component of $\rho_1^{-1}\tau^{-1}\tau(V)$ which contains $\rho_1^{-1}(V)^c$. We have $\rho_2 \sigma'(C'_1) = \tau(V) \cap \rho_2(X'_2)$. Inasmuch as σ' , ρ'_1 and ρ'_2 are covering maps, $\sigma'(C'_1)$ coincides with a connected component $C'_2 = [\rho_2^{-1}\tau(V) \cap X'_2]^c$ of $\rho_2^{-1}\tau(V) \cap X'_2$, because one sees easily that $\sigma'(C'_1)$ is both open and closed in C'_2 . By definition of the completion of a spread, one deduces at once that

$$\sigma([\rho_1^{-1}\tau^{-1}\tau(V)]^c) = [\rho_2^{-1}\tau(V)]^c,$$

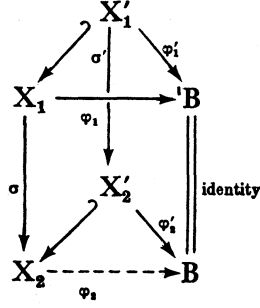
the latter denoting the connected component of $\rho_2^{-1}\tau(V)$ containing $\sigma(\rho_1^{-1}(V)^c)$. But

$$[\rho_1^{-1}\tau^{-1}\tau(V)]^c \subset \rho_1^{-1}[\tau^{-1}\tau(V)]^c = \rho_1^{-1}(V)^c,$$

the last equality by (3.3.3). Consequently $\sigma(\rho_1^{-1}(V)^c) = [\rho_2^{-1}\tau(V)]^c$. Hence σ is open. Verification that σ is surjective is direct. This completes the proof.

Remark (3.4). — By taking $X'_2 = Y'_1$, $X_2 = Y_1$, $\sigma = \rho_1$, $\rho_2 = \text{identity}$, (3.3) implies that the map ρ_1 is open and surjective if Y'_1 is connected and X'_1 is not empty.

Lemma (3.5). — Let $\sigma' : X'_1 \rightarrow X'_2$ and $\sigma : X_1 \rightarrow X_2$ be as in (3.3), and let $\varphi_1 : X_1 \rightarrow B$ be a continuous map. Then the commutative diagram of solid arrows



can be completed as shown.

Proof. — By (3.3), the map σ is a surjective open map. Given $q \in X_2$, it suffices to prove that $\varphi_1(\sigma^{-1}q)$ is a single point, i.e. the map φ_1 descends to a continuous map φ_2 of X_2 .

Let $p \in \sigma^{-1}q$, let U be a connected neighborhood of q in X_2 and let $\sigma^{-1}(U)^c$ denote the connected component of p in $\sigma^{-1}(U)$. Then

$$\varphi_1(p) = \lim_{x \in \sigma^{-1}(U)^c \cap X_1'} \varphi_1'(x) = \lim_{x \in \sigma^{-1}(U)^c \cap X_1'} \varphi_2'(\sigma(x)) = \lim_{y \in U \cap X_2} \varphi_2'(y)$$

since $\sigma(\sigma^{-1}(U)^c \cap X_1') = U \cap X_2'$ because σ' is a surjective covering map. It follows at once that $\varphi_1(p)$ is independent of the choice of p in $\sigma^{-1}q$.

(3.6) We shall apply (3.2) with $X_1' = \tilde{Q}' = \hat{Q}'/\text{Ker } \theta$, the smallest covering space of Q' on which the monodromy acts trivially, $Y_1 = Q_{sst}$ or Q_{st} and $X_1 = \tilde{Q}_{sst}$ or \tilde{Q}_{st} , the completion of X_1' over Y_1 , $X_2' = \tilde{Q}'/\Sigma$, the space defined in (2.1), $Y_2 = Q_{sst}/\Sigma$ or Q_{st}/Σ , and X_2 the completion of X_2' over Y_2 . We write \tilde{Q}_{sst}/Σ (resp. \tilde{Q}_{st}/Σ) for X_2 . In both cases the map τ is the orbit map $x \mapsto \Sigma x$, and $\sigma' : \tilde{Q}' \rightarrow \tilde{Q}'/\Sigma$ is the lift of τ given by the map $\hat{Q}'/\text{Ker } \theta \rightarrow \hat{Q}'/\text{Ker } \theta_\Sigma$.

Remark. — $Q - Q'$ is a finite union of subvarieties some of which may be of \mathbb{C} -codimension 1 in Q . Although $\pi_1(Q', o) \rightarrow \pi_1(Q, o)$ and $\hat{Q}' \rightarrow \hat{Q}$ may fail to be injective, $\tilde{Q}' \rightarrow \tilde{Q}$ is injective, because $\text{Ker } \pi_1(Q', o) \rightarrow \pi_1(Q, o)$ lies in $\text{Ker } \theta$; this last assertion follows immediately from the fact that the map $\omega_\mu : \tilde{Q} \rightarrow B^+(\alpha)_o$ is étale (DM Proposition (3.9)). In particular, \tilde{Q} is the completion of \tilde{Q}' over Q . Here the simply connected \hat{Q}' is identified with \hat{Q}'/Σ via $\hat{\sigma}'$, the lift of σ' :

$$(3.6.1) \quad \begin{array}{ccc} \hat{Q}' & \xrightarrow{\hat{\sigma}'} & \hat{Q}'/\Sigma \\ \downarrow & & \downarrow \\ \hat{Q}'/\text{Ker } \theta = \tilde{Q}' & \xrightarrow{\sigma'} & \tilde{Q}'/\Sigma = \hat{Q}'/\text{Ker } \theta_\Sigma \\ \downarrow & & \downarrow \\ Q' & \xrightarrow{\tau} & Q'/\Sigma \end{array}$$

θ is the monodromy homomorphism of $\pi_1(Q', o)$ to $\text{Aut } B(\alpha)_o$, $\pi_1(Q', o)$ is identified with a subgroup of $\pi_1(Q'/\Sigma, \bar{o})$; $\pi_1(Q'/\Sigma, \bar{o})$ thereby acts on the space \tilde{Q}' and thus $\text{Ker } \theta_\Sigma \cap \pi_1(Q', o) = \text{Ker } \theta$. It is perfectly clear that the hypotheses of (3.2), (3.3) and (3.5) are satisfied, and that (3.2) and (3.5) are applicable.

(3.7) Let \mathcal{E}_1 denote the set of all stable partitions T of S such that
 $\text{card } T = \text{card } S - 1$.

By definition each $T \in \mathcal{E}_1$ has only one element in each coset except for a single coset with two elements $\{s, t\}$ satisfying $\mu_s + \mu_t < 1$. As in DM, Q_T denotes the subset of all $y \in P^S$ such that for any $s_1, s_2 \in S$, $y(s_1) = y(s_2)$ if and only if s_1, s_2 are in the same coset. For each $T \in \mathcal{E}_1$ let Q'_T denote the subset of elements in Q_T which are fixed by no elements of Σ other than the permutation of the two elements occurring in the same coset of T in case they are both in S_1 . Set

$$Q'_1 = Q' \cup \coprod_{T \in \mathcal{E}_1} Q'_T.$$

The degree of the orbit map $Q' \rightarrow Q'/\Sigma$ is $\text{card } \Sigma$, but locally in Q'_1 around a point of Q'_T , the degree of orbit map is 2. Clearly $Q_{\text{sst}} - Q'_1$ is a subvariety, $Q'_1 - Q'$ is a smooth divisor in Q'_1 , and the same is true for their images in Q_{sst}/Σ , even though Q_{sst}/Σ may have singularities. In fact, Q'_1/Σ is an open non-singular subvariety of the variety Q_{sst}/Σ .

Let \tilde{Q}'_1 denote the completion of Q' over Q'_1 and let $(\widetilde{Q'/\Sigma})_1$ denote the completion of $\widetilde{Q'/\Sigma}$ over Q'_1/Σ . Then \tilde{Q}'_1 is a branched cover with branch locus along the disjoint union of \mathbf{C} -codimension 1 submanifolds $\coprod_{T \in \mathcal{E}_1} Q'_T$ and ramification along Q'_T given by the order in \mathbf{R}/\mathbf{Z} of $1 - \mu_s - \mu_t$ where $\{s, t\}$ is the two-element coset of T .

(3.8) Let $\rho: \tilde{Q}'_1 \rightarrow Q'_1$ (resp. $\rho_\Sigma: (\widetilde{Q'/\Sigma})_1 \rightarrow Q'_1/\Sigma$) denote the completion of the covering map $\rho': \tilde{Q}' \rightarrow Q'$ over Q'_1 , (resp. $\rho'_\Sigma: \widetilde{Q'/\Sigma} \rightarrow Q'/\Sigma$ over Q'_1/Σ).

Consider the commutative diagram

$$\begin{array}{ccc} \tilde{Q}'_1 & \xrightarrow{\sigma} & (\widetilde{Q'/\Sigma})_1 \\ \rho \downarrow & & \downarrow \rho_\Sigma \\ Q'_1 & \xrightarrow{\tau} & Q'_1/\Sigma. \end{array}$$

The action of $\frac{\pi_1(Q'/\Sigma, \bar{o})}{\text{Ker } \theta} := \Gamma'_\Sigma$ on \tilde{Q}' extends to \tilde{Q}'_1 by the universal property of completions (cf. DM (8.1.1)) and σ may be regarded as a morphism of Γ'_Σ spaces.

Let $y \in Q'_1 - Q'$ and let V be a neighborhood of y in Q'_1 small enough so that the image of $\pi_1(V \cap Q')$ in $\pi_1(Q', o)$ is the decomposition group D_y of y and the image

of $\pi_1(\tau(V \cap Q'))$ in $\pi_1(Q'/\Sigma, \bar{o})$ is the decomposition group $D_{\tau(y)}$ of $\tau(y)$. We have $y \in Q'_T$ where $T \in \mathcal{E}_1$. As V one can take the product of a disc in Q_r with a disc transversal to Q_r and stable under the permutation of the two-element coset of T . Clearly $Z \cong D_y \hookrightarrow D_{\tau(y)} \cong Z$, the injection being $z \mapsto 2z$. We recall (cf. DM (8.2)) that $\rho^{-1}(y) = \text{Ker } \theta \backslash \pi_1(Q', o)/D_y$, and thus the stabilizer in $\pi_1(Q', o)$ of a point in $\rho^{-1}(y)$ is a conjugate of $D_y \text{Ker } \theta$, and it equals $D_y \text{Ker } \theta$ for a suitable choice base of point o .

Lemma. — Suppose

$$(3.8.1) \quad D_y \text{Ker } \theta_\Sigma \supset D_{\tau(y)}$$

Then any element of $\text{Ker } \theta_\Sigma$ which fixes the point $y \in Q'_T$ fixes each point of $\rho^{-1}(y)$.

Proof. — Let $\tilde{y} \in \rho^{-1}(y)$ and let \tilde{V} denote the connected component of \tilde{y} in $\rho^{-1}(V)$. Since $\sigma' : Q' \rightarrow Q'/\Sigma$ is a covering map, $\sigma(\tilde{V})$ is the connected component of $\sigma(\tilde{y})$ in $\rho_\Sigma^{-1}\tau(V)$. By hypothesis (3.8.1), we can assume that the stabilizer of \tilde{y} in $\pi_1(Q', o)$ contains the stabilizer of $\sigma(\tilde{y})$ in $\pi_1(Q'/\Sigma, \bar{o})$ modulo $\text{Ker } \theta_\Sigma$.

Let h be an element of $\text{Ker } \theta_\Sigma$ with $hy = y$. Then $h\tilde{y} = g\tilde{y}$ with $g \in \pi_1(Q', o)$. Hence $g\sigma(\tilde{y}) = \sigma(h\tilde{y}) = \sigma(\tilde{y})$. Consequently g is in the stabilizer of \tilde{y} in $\pi_1(Q', o) \bmod \theta_\Sigma$. Since $g \in \pi_1(Q', o)$, we get $g = g_1 h_1$ with $h_1 \in \pi_1(Q', o) \cap \text{Ker } \theta_\Sigma = \text{Ker } \theta$ and $g\tilde{y} = g_1 h_1 \tilde{y} = g_1 \tilde{y} = \tilde{y}$. Therefore $h\tilde{y} = g\tilde{y} = \tilde{y}$.

Remark. — From (3.11.1), one can see that (3.8.1) holds if μ satisfies (ΣINT) but not (INT) .

Lemma (3.9). — Let $S_1 \subset S$, let Σ denote the permutation group of S_1 , and assume that $\mu_s = \mu_t$ for all $s, t \in S_1$. Let s_1, s_2 be distinct elements of S_1 , and let $[s_1, s_2]$ denote the element of $\pi_1(Q'/\Sigma, \bar{o})$ coming from a positive loop in Q'/Σ around the \mathbf{C} -codimension 1 submanifold of Q'_1/Σ lying below the submanifold of Q'_1 on which the s_1 and s_2 coordinates coincide. Suppose that

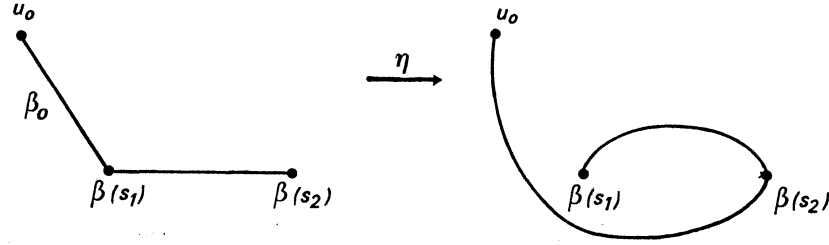
$$(3.9.1) \quad 1 - 2\mu_s = \frac{2}{k}, \quad k \text{ integer, all } s \in S_1.$$

Then

$$\text{order } \theta_\Sigma([s_1, s_2]) = k.$$

Proof. — The proof is very much like the proof of Proposition (9.1.1) in DM. Let T_1 be the tree with vertices $\{s_1, s_2\}$ and let T_2 be a tree with vertices in $S - \{s_1, s_2\}$. Let $\beta : T_1 \sqcup T_2 \rightarrow P$ be an embedding with $\beta|_S = o$, the base point of Q' . Without loss of generality we may assume that $\beta(T_i) \subset D_i$ ($i = 1, 2$) where D_1 and D_2 are discs having disjoint closures. Choose a base $\{\ell(a) \cdot \beta|_a; a \text{ an oriented edge of } T_1 \sqcup T_2\}$ of $H_1^{\text{lf}}(P - o(S), \tilde{\mathbb{L}})$ as in (2.5) of DM. The monodromy, being the result of horizontal transport, is effected by an isotopy η of P_o which is the identity map on $P - D_1$ and turns $(o(s_2), o(s_1))$ into $(o(s_1), o(s_2))$ by one positive half-turn. This isotopy has no effect on $\ell(a) \cdot \beta|_a$ for an oriented edge $a \subset T_2$. To keep track of the change in the sec-

tions of the local system along varying arcs, fix a point $u_0 \in P_0 - D_1$, let β_0 denote the singular chain given by an arc from u_0 to the point $\beta(s_1)$, let a denote the oriented edge from s_1 to s_2 , and let $\ell(\beta_0)$ be an extension of the section $\ell(a)$.



We can assume that the value of $\ell(u_0)$ remains unchanged during the isotopy. We have

$$\begin{aligned} \eta_*(\ell(a) \cdot \beta | a) &= \eta_*(\ell(\beta_0) \cdot \beta_0 + \ell(a) \cdot \beta | a) - \eta_*(\ell(\beta_0) \cdot \beta_0) \\ &= -\alpha_{s_2}^{-1} \ell(a) \cdot \beta | a. \end{aligned}$$

Inasmuch as the local system \tilde{L} is stable under Σ , the monodromy $[s_1, s_2]$ effects on $H_1^u(P - S, \tilde{L})$ a linear transformation with matrix relative to the base $\{\ell(a) \cdot \beta | a; a \text{ an oriented edge of } T_1 \text{ or } T_2\}$

$$\text{diag}(-\alpha_{s_2}^{-1} I, I, \dots, I).$$

By hypothesis, $1 - 2\mu_{s_2} = \frac{2}{k}$ with k an integer. Hence

$$\alpha_{s_2} = \exp 2\pi i \mu_{s_2} = \exp 2\pi i \left(\frac{1}{2} - \frac{1}{k} \right)$$

and $-\alpha_{s_2}^{-1} = \exp \frac{2\pi i}{k}$. From this result follows.

Corollary (3.10). — Let $\mathcal{E}_{1,1}$ denote the set of partitions in \mathcal{E}_1 whose two-element coset lies in S_1 . Assume (3.9.1). Let $\sigma: \tilde{Q}'_1 \rightarrow \tilde{Q}'_1/\Sigma$ be defined as in (3.6) and (3.8). Then

- (1) if k is even, σ is a covering map;
- (2) if k is odd σ has local degree 2 at each point of $\rho^{-1}(Q'_T)$ for all $T \in \mathcal{E}_{1,1}$.

Proof. — The map σ is open and surjective by (3.3). Consider $\rho: \tilde{Q}'_1 \rightarrow Q'_1$ at a point \tilde{y} of $\rho^{-1}(y)$ with $y \in Q'_T$ where $T \in \mathcal{E}_{1,1}$. Then

$$\begin{aligned} \text{local degree of } \rho &= \text{order } \frac{D_y}{D_y \cap \text{Ker } \theta} = \text{order } \frac{D_y \text{ Ker } \theta_\Sigma}{\text{Ker } \theta_\Sigma} \\ &= \text{order } \theta_\Sigma([s_1, s_2]^2) \end{aligned}$$

where $\{s_1, s_2\}$ determines T . Hence

$$\text{local degree of } \rho = \begin{cases} k/2 & \text{if } k \text{ is even,} \\ k & \text{if } k \text{ is odd.} \end{cases}$$

Similarly, the local degree of $\rho_\Sigma: (\widetilde{Q'/\Sigma})_1 \rightarrow Q'_1/\Sigma$ is the order of $\theta_\Sigma([s_1, s_2])$ above any point of $\tau(Q'_T)$, where $\tau: Q'_1 \rightarrow Q'_1/\Sigma$ is the orbit map. Since the local degree of τ at y is 2, one can verify from the commutative diagram of (3.8) the asserted local degree of σ at points of $\rho^{-1}(Q'_T)$ for all $T \in \mathcal{E}_{1,1}$. Since σ is a covering map on \tilde{Q}' , the result follows.

(3.11) The exact homotopy sequence of the fibration of Q' by Σ orbits gives the exact sequence

$$1 \rightarrow \pi_1(Q', o) \rightarrow \pi_1(Q'/\Sigma, \bar{o}) \rightarrow \Sigma \rightarrow 1.$$

Assume (3.9.1) with k odd. Then, by Lemma (3.9), $\theta_\Sigma([s_1, s_2])$ lies in the group generated by $\theta_\Sigma([s_1, s_2]^2)$ for any 2-element coset $\{s_1, s_2\}$ of a partition in $\mathcal{E}_{1,1}$. It follows at once that

$$\begin{aligned} (3.11.1) \quad & \theta_\Sigma(\pi_1(Q', o)) = \theta_\Sigma(\pi_1(Q'/\Sigma, \bar{o})), \text{ or equivalently} \\ & \pi_1(Q', o) \text{ Ker } \theta_\Sigma = \pi_1(Q'/\Sigma, \bar{o}), \text{ or equivalently,} \\ & \frac{\text{Ker } \theta_\Sigma}{\text{Ker } \theta} \cong \Sigma. \end{aligned}$$

Hence the action of Σ on Q'_1 has a faithful lift to the action of $\frac{\text{Ker } \theta_\Sigma}{\text{Ker } \theta}$ on \tilde{Q}'_1 and to \tilde{Q}_{sst} as well. Thus if k is odd, we may write, by abuse of notation

$$(3.11.1)' \quad \tilde{Q}_{\text{sst}}/\Sigma = \widetilde{Q_{\text{sst}}/\Sigma}.$$

The action of the transposition of two elements of S on \tilde{Q}_{sst} is clear from (3.8).

If on the other hand (3.9.1) holds with k even, then for all $T \in \mathcal{E}_{1,1}$ and $y \in Q'_T$ (under the identification of $\pi_1(Q', o)$ with a subgroup of $\pi_1(Q'/\Sigma, \bar{o})$) $D_{\tau(y)}/D_y = \theta_\Sigma(D_{\tau(y)})/\theta(D_y)$, since each side is isomorphic to $\mathbf{Z}/2\mathbf{Z}$, by Lemma (3.9) for the right side and by the local degree of τ being 2. It follows that $D_{\tau(y)} \cap \text{Ker } \theta_\Sigma \subset D_y$. Hence

$$D_y \cap \text{Ker } \theta = D_{\tau(y)} \cap \text{Ker } \theta_\Sigma.$$

Since these subgroups together with $\{D_y; y \in Q - Q'\}$ generate $\text{Ker } \theta$ and $\text{Ker } \theta_\Sigma$ (because Q_{st} and \tilde{Q}_{st} are simply connected), we get $\text{Ker } \theta = \text{Ker } \theta_\Sigma$. Consequently

$$(3.11.2) \quad \frac{\text{Image } \theta_\Sigma(\pi_1(Q'/\Sigma, \bar{o}))}{\text{Image } \theta(\pi_1(Q', o))} = \Gamma_\Sigma/\Gamma = \Sigma$$

and

$$(3.11.2)' \quad \tilde{Q}_{\text{st}} = \widetilde{Q_{\text{st}}/\Sigma}.$$

Theorem (3.12). — Let S_1 be a subset of S and let Σ denote the permutation group of S_1 . Assume that $(\mu_s)_{s \in S}$ satisfies condition $(\Sigma \text{ INT})$ (cf. (2.21)). Then $\text{Im } \theta_\Sigma$ is a lattice in $\text{PU}(\text{card } S - 3, 1)$.

Proof. — We can assume that S_1 has more than one element. Set $1 - 2\mu_s = \frac{2}{k}$ for $s \in S_1$. By hypothesis (Σ INT), k is an integer. If k is even, then $\text{Im } \theta_\Sigma$ is a finite extension of $\text{Im } \theta$ by (3.11.2) and moreover condition (INT) of DM is satisfied. Hence $\text{Im } \theta$ is a lattice by the main theorem of DM. Thus $\text{Im } \theta_\Sigma$ is a lattice if k is even.

Assume now that k is odd. Set

$$\begin{aligned} U_\Sigma &= Q_{st}/\Sigma, & U_{\Sigma,0} &= Q'/\Sigma, & U_{\Sigma,1} &= Q_1/\Sigma \\ U &= Q_{st}, & U_0 &= Q', & U_1 &= Q_1 \\ \tilde{U}_\Sigma &= \widetilde{Q_{st}/\Sigma}, & \tilde{U}_{\Sigma,0} &= \widetilde{Q'/\Sigma}, & \tilde{U}_{\Sigma,1} &= \widetilde{Q_1/\Sigma} \end{aligned}$$

where $Q_1 = Q \cup \prod_{T \in \mathcal{E}} Q_T$, and $\widetilde{Q_1/\Sigma}$ is the completion of Q'/Σ over Q_1/Σ . By (3.5) we have a commutative diagram

$$(3.12.1) \quad \begin{array}{ccc} \tilde{Q}_{st} & \xrightarrow{w_\mu} & B^+(\alpha)_o \\ \sigma \downarrow & & \parallel \\ \widetilde{Q_{st}/\Sigma} & \xrightarrow{w_\mu} & B^+(\alpha)_o \end{array}$$

Inasmuch as w_μ is étale on \tilde{Q} by Proposition (3.9) of DM, it follows at once that w_μ is étale on $\widetilde{Q/\Sigma}$, the completion of $\widetilde{Q'/\Sigma}$ over Q/Σ and that $\widetilde{Q/\Sigma}$ is non-singular even though Q/Σ may have singularities. As in DM, we take a stratification \mathcal{S} of Q_{st} with strata Q_T where T ranges over the stable partitions of S . Let \mathcal{S}_Σ denote the image of \mathcal{S} under σ . We wish to apply Proposition (10.16.1) of DM to the diagram

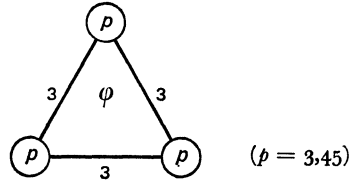
$$\begin{array}{ccccc} \tilde{U}_{\Sigma,0} & \longrightarrow & \tilde{U}_{\Sigma,1} & \longrightarrow & \tilde{U}_\Sigma \xrightarrow{w_\mu} B^+(\alpha)_o \\ \downarrow & & \downarrow & & \downarrow \rho \\ U_{\Sigma,0} & \longrightarrow & U_{\Sigma,1} & \longrightarrow & U_\Sigma \end{array}$$

All of the hypothesis of Proposition (10.16.1) descend from U_i to $U_{\Sigma,i}$ except possibly the assertion in 1(e): $w_\mu|_{\tilde{U}_{\Sigma,1}}$ is a local homeomorphism. This last condition follows directly at all points except those in $\sigma(Q_T)$ with $T \in \mathcal{E}_{1,1}$. However, at such points we use in diagram (3.12.1) that σ has local degree 2 by Corollary (3.10). Consequently at $\sigma(Q_T)$ with $T \in \mathcal{E}_{1,1}$, the map $w_\mu: \widetilde{Q_{st}/\Sigma} \rightarrow B^+(\alpha)_o$ has local degree $\frac{1}{2}$ (the degree of $w_\mu: \tilde{Q}_{st} \rightarrow B^+(\alpha)_o$ at Q_T). The computation in DM § 9 shows that $w_\mu|_{\tilde{U}_{\Sigma,1}}$ has local degree 1 at points of $\sigma(Q_T)$ for $T \in \mathcal{E}_{1,1}$. By Proposition (10.16.1), $w_\mu: \tilde{U}_\Sigma \rightarrow B^+(\alpha)_o$ is a local homeomorphism. The proof of Theorem (10.18.2) of DM applies verbatim to yield that $\tilde{w}_\mu: \widetilde{Q_{sst}/\Sigma} \rightarrow \bar{B}^+(\alpha)$ is a homeomorphism onto an open subset

of $\bar{B}^+(\alpha)_{\bar{o}}$ in the DM (5.4) topology and maps $\widetilde{Q_{st}/\Sigma}$ homeomorphically onto $B^+(\alpha)_{\bar{o}}$. The image is a lattice, by the same reasoning as in DM. This completes the proof.

4. RCP

In [2], there is a geometric construction of a fundamental domain for groups $\Gamma(\varphi)$ in $PU(2, 1)$ generated by \mathbf{C} -reflections on a 3 dimensional complex vector space $V(\varphi)$ with Coxeter diagram



and ibid p. 248 there is a list of the groups $\Gamma(\varphi)$ which satisfy the condition (CD2) ensuring discreteness. Let $A\Gamma(\varphi)$ denote the group obtained by adjoining to $\Gamma(\varphi)$ the group of cyclic permutations of its generators. Then $\text{card}(A\Gamma(\varphi)/\Gamma(\varphi)) = 1$ or 3.

Theorem. — Let $d = 2$, $\mu_0 = \mu_1 = \mu_2$, and let Σ denote the symmetric group on $\{0, 1, 2\}$. Then each of the groups $A\Gamma(\varphi)$ satisfying condition (CD2) coincides with the group Γ_{Σ} for suitable $\{\mu_i \mid i = 0, \dots, 4\}$ satisfying condition $(\Sigma \text{ INT})$.

Proof. — Set $\eta = e^{\pi i/p}$, $\rho = \text{order } \bar{\eta}i\varphi^3$, $\sigma = \text{order } \bar{\eta}i\bar{\varphi}^3$, $t = \frac{1}{\pi} \arg \varphi^3$. The list of $\Gamma(\varphi)$ is specified by the values of t, ρ, σ with $0 \leq t < 3 \left(\frac{1}{2} - \frac{1}{p} \right)$. We write $k_{ij} = (1 - \mu_i - \mu_j)^{-1}$, $0 \leq i < j \leq 4$ and $\Gamma(p, t) = \Gamma(\varphi)$.

$$\begin{aligned} \text{Set } \mu_0 &= \frac{1}{2} - \frac{1}{p}, \\ k_{03} &= \rho, \\ k_{04} &= \begin{cases} \sigma & \text{if } 0 \leq t \leq \frac{1}{2} - \frac{1}{p}, \\ -\sigma & \text{if } \frac{1}{2} - \frac{1}{p} < t < 3 \left(\frac{1}{2} - \frac{1}{p} \right), \end{cases} \\ t &= \frac{1}{k_{03}} - \frac{1}{k_{04}}. \end{aligned}$$

By a lengthy but straightforward calculation (cf. [3]), the map $R_1(\varphi) \mapsto \theta_{\Sigma}([01])$, $R_2(\varphi) \mapsto \theta_{\Sigma}([12])$, $R_3(\varphi) \mapsto \theta_{\Sigma}([20])$ yields an isomorphism of $A\Gamma(\varphi)$ onto Γ_{Σ} induced by an isometry of $V(\varphi)$ onto $(H^1(P_o, L), \psi)$. (For a geometric proof, cf. [4]).

We list the groups $\Gamma(\varphi)$ and the corresponding (μ_1) . From p. 248 of [2] we see where $A\Gamma(\varphi)/\Gamma(\varphi)$ has order 1 or 3. In the last five cases, $A\Gamma$ contains a Picard lattice as a subgroup of index 6 by (3.11.2). In the last column, write $A\Gamma$ if $\Gamma_{\Sigma} \neq \Gamma(p, t)$.

#	p	k_{03}	k_{04}	t	μ_0	μ_3	μ_4	Arith	$\Gamma_\Sigma = \Gamma$ or $A\Gamma$
1	3	12	12	0	1/6	9/12	9/12		$A\Gamma$
2	3	10	15	1/30	1/6	22/30	23/30	NA	Γ
3	3	9	18	1/18	1/6	13/18	14/18		$A\Gamma$
4	3	8	24	1/12	1/6	17/24	19/24	NA	Γ
5	3	7	42	5/42	1/6	29/42	34/42	NA	Γ
6	3	6	∞	1/6	1/6	4/6	5/6		$A\Gamma$
7	3	5	-30	7/30	1/6	19/30	26/30		Γ
8	3	4	-12	1/3	1/6	7/12	11/12		Γ
9	5	5	10	1/10	3/10	5/10	6/10		Γ
10	5	4	20	1/5	3/10	9/20	13/20	NA	Γ
11	5	3	-30	11/30	3/10	11/30	22/30	NA	$A\Gamma$
12	5	2	-5	7/10	3/10	2/10	9/10		Γ
13	4	8	8	0	1/4	5/8	5/8		Γ
14	4	6	12	1/12	1/4	7/12	8/12	NA	$A\Gamma$
15	4	5	20	3/20	1/4	11/20	14/20	NA	Γ
16	4	4	∞	1/4	1/4	2/4	3/4		Γ
17	4	3	-12	5/12	1/4	5/12	10/12		$A\Gamma$

5. Lattices Γ_Σ in $PU(N-3, 1)$ for $N \geq 5$ satisfying $(\Sigma \text{ INT})$, p odd

(5.1) $N > 5$.

There are groups Γ_Σ satisfying condition $(\Sigma \text{ INT})$ only for $N \leq 12$. We list all cases with $6 \leq N \leq 12$, p odd. All are arithmetic. For $p = 3$, all are centralizers of a subgroup of the first one except for $\left(\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{7}{12}, \frac{7}{12}\right)$.

N	p	μ_0	Multiplicity of μ_0	Remaining μ_i
12	3	$\frac{1}{6}$	12	
11	3	$\frac{1}{6}$	10	$\frac{2}{6}$
	3	$\frac{1}{6}$	9	$\frac{3}{6}, \frac{3}{6}$
10	3	$\frac{1}{6}$	8	$\frac{2}{6}, \frac{2}{6}$
9	3	$\frac{1}{6}$	8	$\frac{4}{3}$

N	p	μ_0	Multiplicity of μ_0	Remaining μ
8	3	$\frac{1}{6}$	7	$\frac{2}{6} \frac{3}{6}$
	3	$\frac{1}{6}$	6	$\frac{2}{6} \frac{2}{6} \frac{2}{6}$
	3	$\frac{1}{6}$	7	$\frac{5}{6}$
	3	$\frac{1}{6}$	6	$\frac{4}{6} \frac{2}{6}$
	3	$\frac{1}{6}$	6	$\frac{3}{6} \frac{3}{6}$
7	3	$\frac{1}{6}$	5	$\frac{2}{6} \frac{2}{6} \frac{3}{6}$
	3	$\frac{1}{6}$	5	$\frac{3}{6} \frac{4}{6}$
	3	$\frac{1}{6}$	5	$\frac{2}{6} \frac{5}{6}$
	3	$\frac{1}{6}$	5	$\frac{7}{12} \frac{7}{12}$
	3	$\frac{1}{6}$	4	$\frac{2}{6} \frac{2}{6} \frac{4}{6}$
	3	$\frac{1}{6}$	4	$\frac{2}{6} \frac{3}{6} \frac{3}{6}$
	5	$\frac{3}{10}$	6	$\frac{2}{10}$
6	3	$\frac{1}{6}$	4	$\frac{4}{6} \frac{4}{6}$
	3	$\frac{1}{6}$	4	$\frac{3}{6} \frac{5}{6}$
	3	$\frac{1}{6}$	3	$\frac{2}{6} \frac{3}{6} \frac{4}{6}$
	3	$\frac{1}{6}$	3	$\frac{3}{6} \frac{3}{6} \frac{3}{6}$
	5	$\frac{3}{10}$	5	$\frac{5}{10}$
	5	$\frac{3}{10}$	4	$\frac{2}{10} \frac{6}{10}$

(5.2) $N = 5$.

In addition to lattices listed in § 4 which satisfy condition (Σ INT) but not condition (INT), we have the following.

p	μ_0	Multiplicity	Remaining μ_i	Arith
5	$\frac{3}{10}$	4	$\frac{8}{10}$	
5	$\frac{3}{10}$	2	$\frac{9}{20}, \frac{9}{20}, \frac{1}{2}$	NA
7	$\frac{5}{14}$	4	$\frac{8}{14}$	
9	$\frac{7}{18}$	4	$\frac{8}{18}$	NA
	$\frac{7}{18}$	3	$\frac{5}{18}, \frac{10}{18}$	NA

The lattice corresponding to $\mu = \left(\frac{3}{10}, \frac{3}{10}, \frac{3}{10}, \frac{3}{10}, \frac{8}{10}\right)$ deserves mention.

1. Let M_{st} denote the subset of μ -stable points in $(\mathbf{P}^1)^5$ and let $\pi: M_{st} \rightarrow Q_{st}$ denote the map to PGL orbits. The group Σ_4 of permutations on the first four coordinates descends to an action on P_{st} . We have

$$\begin{aligned}(x_1, x_2, 1, 0, \infty) &\equiv (1 - x_1, 1 - x_2, 0, 1, \infty) \bmod \text{PGL} \\ &\equiv \sigma(1 - x_2, 1 - x_1, 1, 0, \infty) \bmod \text{PGL}\end{aligned}$$

where σ denotes the permutation $(1, 2)(3, 4)$. Hence σ fixes each point of the line $L = \{\pi(x, 1 - x, 1, 0, \infty) : x \neq \infty\}$ and this line punctured at $x = 0, \frac{1}{2}, 1$ lies in the set $Q - Q'$ (cf. Remark of (3.6)). In this example, Q_{st} is the projective plane and σ descends to the involution $[x_1, x_2, 1] \rightarrow [1 - x_2, 1 - x_1, 1]$ in the line $x_1 + x_2 = 1$.

2. The lattice Γ_μ is the lattice $\Gamma\left(5, \frac{1}{2}\right)$ of [2] by the result in § 4 above. On the other hand, it is proved in [2] that $\Gamma\left(5, \frac{1}{2}\right)$ is isomorphic to $\Gamma\left(5, \frac{7}{10}\right)$. Using the result in § 4, $\Gamma\left(5, \frac{7}{10}\right)$ coincides with the group Γ_ν , $\nu = \left(\frac{3}{10}, \frac{3}{10}, \frac{3}{10}, \frac{2}{10}, \frac{9}{10}\right)$. Consequently, $\Gamma_\mu \cong \Gamma_\nu$. It is clear that Γ_ν contains a complex reflection of order 2, a fact that is not so obvious for Γ_μ . The existence of this reflection in Γ_μ is related to the involution in the line L above.

We take this opportunity to insert 3 errata for the proof that $\Gamma\left(5, \frac{1}{2}\right) \cong \Gamma\left(5, \frac{7}{10}\right)$ in [2]:

Read on page 273, Equation (21.1): $\dots - \alpha\varphi \frac{1 - \eta + 2\eta^{-2}}{1 + \eta + \bar{\eta}}$

line 12: Γ_{12} not F_{12}

line 13: \dots subgroup of $\Gamma \cap \text{PU}(2)$.

6. $\text{A}\Gamma(\varphi)$ as extensions of Picard lattices in $\text{PU}(2, 1)$

The 27 Picard lattices are listed in (14.3) of DM. For all except five of these lattices, at least three of the μ 's are equal; we relabel these μ_0, μ_1, μ_2 . The corresponding extended lattice Γ_Σ with Σ the permutation group on $\{0, 1, 2\}$ coincides with the group $\text{A}\Gamma(\varphi)$ by § 4. We list below the p and t -parameters of the corresponding Γ_Σ , labelling each Picard lattice by its position on the list of DM (14.3).

Clearly $p = \left(\frac{1}{2} - \mu_0\right)^{-1}$. By § 4,

$$t = k_{03}^{-1} - k_{04}^{-1} = (1 - \mu_0 - \mu_3) - (1 - \mu_0 - \mu_4) = \mu_4 - \mu_3.$$

We order the indices so that $\mu_3 \leq \mu_4$. As a result $k_{03} > 0$ and $k_{03} < |k_{04}|$.

(Of the five Picard lattices not on the list, two are non-arithmetic.)

DM#	D	$D\mu_0$	$D\mu_3$	$D\mu_4$	p	t	k_{03}	k_{04}	$\Gamma_2 = \text{A}\Gamma$ or Γ
1	3	1	1	2	6	$\frac{1}{3}$	3	∞	$\text{A}\Gamma$
2	4	2	1	1	∞	0	4	4	Γ
3	4	1	2	3	4	$\frac{1}{4}$	4	∞	Γ
4	5	2	2	2	10	0	5	5	Γ
5	6	2	3	3	6	0	6	6	$\text{A}\Gamma$
6	6	3	1	2	∞	$\frac{1}{6}$	3	6	$\text{A}\Gamma$
8	6	2	1	5	6	$\frac{2}{3}$	2	— 6	Γ
9	8	3	3	4	8	$\frac{1}{8}$	4	8	Γ
10	8	2	5	5	4	0	8	8	Γ
11	8	3	1	6	8	$\frac{5}{8}$	2	— 8	Γ
12	9	4	2	4	18	$\frac{4}{18}$	3	9	$\text{A}\Gamma$

DM#	D	D_{μ_0}	D_{μ_3}	D_{μ_4}	p	t	k_{03}	k_{04}	$\Gamma_2 = A\Gamma$ or Γ
13	10	4	1	7	10	$\frac{6}{10}$	2	— 10	Γ
14	12	5	4	5	12	$\frac{1}{12}$	4	6	Γ
16	12	5	3	6	12	$\frac{3}{12}$	3	12	$A\Gamma$
17	12	4	5	7	6	$\frac{2}{12}$	4	12	Γ
21	12	5	1	8	12	$\frac{7}{12}$	2	— 12	Γ
22	12	3	7	8	4	$\frac{1}{12}$	6	12	$A\Gamma$
23	12	3	5	10	4	$\frac{5}{12}$	3	— 12	$A\Gamma$
24	15	6	4	8	10	$\frac{10}{30}$	3	15	$A\Gamma$
25	18	8	1	11	18	$\frac{10}{18}$	2	— 18	Γ
26	20	5	11	14	4	$\frac{3}{20}$	5	20	Γ
27	24	9	7	14	8	$\frac{7}{24}$	3	24	$A\Gamma$

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