

Evaluation of the most influent input variables on quantities of interest in a fire simulation

Titre: Evaluation des variables d'entrée les plus influentes sur des grandeurs d'intérêt dans une simulation d'incendie

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Abstract: This paper is concerned with a fire simulation model which aims at evaluating the effects from a fire source in a building on people evacuation. Three parameters are investigated to assess the safety of the building : the maximal upper layer temperature, the maximal lower layer temperature and the minimal layer height. The combination of these three parameters is supposed to let the evacuation paths practicable in case of a fire. The parameters are computed through a computational code in a measurement context. Consequently their knowledge is supposed to be based on a measurement uncertainty evaluation and this paper aims at evaluating the contribution of a set of input parameters to the variance of these three parameters. In measurement science, the use of Monte Carlo methods to evaluate measurement uncertainty is quite new and therefore, tools for sensitivity analysis associated to Monte Carlo methods are not well known. In this framework, we will briefly introduce the main stochastic techniques to cope with this problem and show some practical results concerning fire risk assessment of a tertiary building.

Résumé : Ce papier traite d'un modèle de simulation incendie dont le but est d'évaluer les effets sur l'évacuation du public d'un incendie dans un bâtiment. Trois paramètres caractérisent la sécurité dans le bâtiment : la température maximale en couche chaude, la température maximale en couche froide et la hauteur minimale de fumées. La combinaison de ces trois paramètres doit assurer que les chemins d'évacuation restent praticables en cas d'incendie. Les paramètres sont calculés à partir d'un code de calcul dans le contexte de la mesure. Par conséquent, leur connaissance s'accompagne d'une évaluation de l'incertitude de mesure et ce papier a pour but d'évaluer la contribution d'un ensemble de grandeurs influentes à la variance de ces trois paramètres. Dans la science de la mesure, l'utilisation des méthodes de Monte Carlo pour évaluer l'incertitude de mesure est assez récente et, par conséquent, les outils pour l'analyse de sensibilité associés aux méthodes de Monte Carlo sont peu connus. Dans ce cadre, nous introduisons brièvement les principales techniques stochastiques pour traiter ce problème et montrons quelques résultats pratiques concernant la détermination du risque incendie dans un bâtiment public.

Keywords: measurement uncertainty, fire engineering, sensitivity analysis, sobol' indices, local polynomial smoother

Mots-clés : incertitude de mesure, ingénierie du feu, analyse de sensibilité, indices de sobol', polynômes locaux

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1. Introduction

In measurement science, the accuracy of the results is very important, even more when safety of people is concerned. As a consequence, the evaluation of measurement uncertainty is an important tool for many reasons. In such a science, it is essential to estimate the reliability of a measurement because the true value of the measurand (the quantity to be measured) cannot be exactly known. Moreover, when a measurement process leads to a conformity assessment, the uncertainty has also to be taken into account. The evaluation of measurement uncertainty enables to avoid as much as possible bad decisions that may have a large impact in a field such as safety for example. Also, the metrologist aims perpetually at improving the measurement process, and reducing the measurement uncertainty. That's why a suitable sensitivity analysis, which must be able to point out the most influent input quantities, is required.

In this study, we are interested by a fire simulation model which aims at evaluating the effects from a fire source in a building on people evacuation. In the presence of a fire source in a public building, it is important that people have enough time to evacuate. The evaluation of the possibility to escape, as well as the Available Safe Egress Time (ASET) is linked to tenability criteria (temperature, free of smoke height, heat fluxes, toxicity, etc.) and therefore to the effect of fire on people [7]. In case the fire is strong enough relatively to the volume it occurs in, smoke and gases emitted by the fire goes up by buoyancy and accumulate under the ceiling. Stratification is observed and two zones may be ideally defined : the upper layer containing hot gases and smoke, and the lower layer, containing colder air, separated by an interface known as the neutral plane. In this case, the evaluation of the effects of fire on people may be done using simplified zone models, that strongly relies on this stratification hypothesis. Using Ordinary Differential Equations systems, such models allow to calculate output quantities related to the tenability criteria, such as the lower and upper layer temperatures and the height of the neutral plane, also know as the "layer height".

The software CFAST (v6.1) [17] allows to compute those parameters. This code needs, among others, input data characterizing the dimensions of the studied volume (area, height), its openings (height and width), inside and outside temperatures and properties of the fire source (Heat release rate, through heat release rate per unit area and surface area). Associated with a pre and post-treatment EXCEL workbook developed by LNE, this code may be used to evaluate thousands of fire scenarios, then allowing a statistical post analysis.

Today, there is no direct statement in fire regulation on how to deal with uncertainties using computational codes, nor formal requirement to do so. However, because computational codes are used in Fire Safety Engineering to determine whether people may survive or not, it is of much importance to assess the quality of their outputs. They therefore should be investigated against sensitivity and variance, in the same manner as measurements during real tests are, in order to have a better knowledge on how the output quantities are distributed.

Consequently, the ISO Technical committee ISO/TC 92, Fire safety, Subcommittee SC 4, Fire safety engineering [9], has issued in 2008 the ISO16730 standard. This standard addresses the assessment, verification and validation of calculation methods for fire safety engineering in general, and its Annex A summarizes notions about measurement uncertainties. An example of such an assessment is currently under publication concerning CFAST [16] [10].

In the present study, the variables of interest are three parameters that may be lethal beyond a

given threshold, therefore subject to a strong regulation: the maximal upper layer temperature, through the radiating effect of smoke on people situated below, lower layer temperature and its direct effect on body temperature, and the minimal layer height indicating whether evacuees are under or in the smoke. All of them are represented in figure 1. These three parameters allow to evaluate if the evacuation paths remain practicable in case of a fire. The statistical analysis of these three output quantities deals with the standard deviation (which is also called standard uncertainty). It is determined using a Monte Carlo method and some sensitivity indices are investigated to point out the input quantities that are the most important to characterize these three safety criteria.

In this study, the computational time and the total number of input quantities, characterizing the dimensions of the room and openings and the properties of the fire determine what kind of sensitivity index is the most adapted. The Standardized Rank Regression Coefficients (SRRC) [1] and the variance-based sensitivity indices (estimated via Sobol's method [8], FAST technique [1] and local polynomial smoothers (LPS) [15]) are investigated.

In the following section, the different quantities involved in the fire simulation are detailed. Then, different kinds of sensitivity indices are presented and discussed in the context of measurement uncertainty. Finally, the practical case results are discussed.

2. Input data of the simulation model

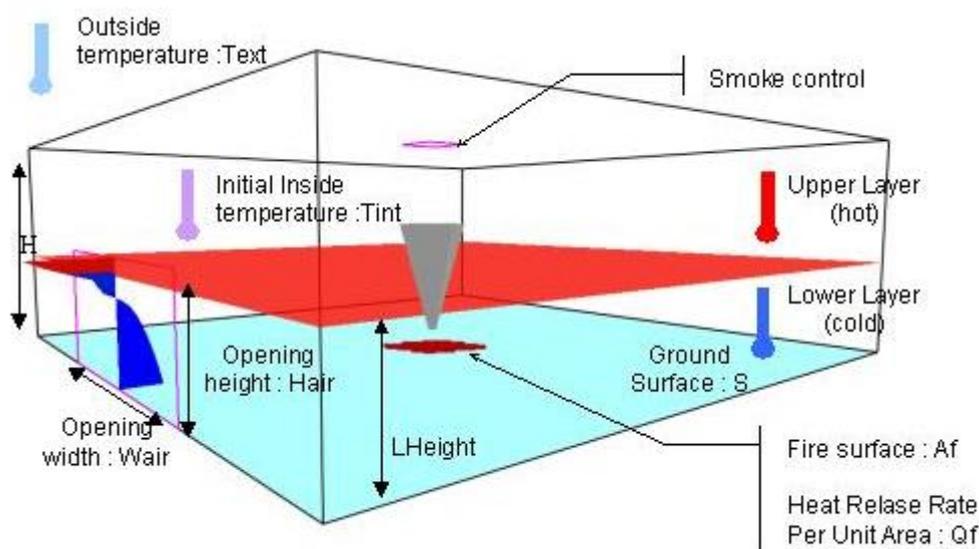


FIGURE 1. Illustration of the fire simulation

CFAST allows to take into consideration many input parameters. The set of parameters has been reduced to only eight parameters (see figure 1) that are suspected to be the most important regarding the output quantities.

Temperatures both inside and outside the building have been chosen to be representative of what is often found in France. Outside temperature (denoted as T_{ext}) could vary between -20°C and

+40 °C which are mean extreme values found in France according to meteorological survey. The chosen distribution law is normal with a standard deviation equal to the third of the semi-length of the interval of variation. Inside temperature (denoted as T_{int}) could vary between +15 °C and +27 °C according to extrema found in ISO7730 standard "Ergonomics of the thermal environment - Analytical determination and interpretation of thermal comfort using calculation of the PMV and PPD indices and local thermal comfort criteria". The chosen distribution law is normal with a standard deviation equal to the third of the semi-length of the interval of variation.

The studied volumes have a floor surface area (S) from 300 m² to 1000 m², representative of many single volumes found in public buildings. The chosen distribution law is uniform. As covered area of a building (rectangular vs square) was found of negligible influence in a preliminary study, all buildings have been modeled as squares.

The ceiling height (H) for these volumes has been chosen between 6 m and 15 m with a uniform distribution. For this application case, going under this 6 m limit would have required extra care and validation on a physics point of view, which was not in the scope of this statistical study. However, this will be considered in further studies, as there are of course volumes where ceiling height is under that limit.

Passive smoke control by roof vents was considered in this study, as the French regulation stipulates it: the surface area of the vents was equal to 0.5 % of the total floor surface.

For simplicity purposes, the openings of the building have been considered as a single opening, with height (H_{air}) varying between 2 m and 6 m (assuming a uniform distribution), and a width (W_{air}) dependent of the dimensions of the building, from 20 % to 80 % of its width (a triangular distribution has been chosen). According to the regulation, the surface of this opening has to be high enough (more than 1.5 times the smoke vents surface) to ensure a proper ventilation and the efficiency of the smoke control system. Considering the chosen ranges, this is always the case in the present study.

The fire has also been considered according to the French regulation. In the smoke control chapter of this regulation, the fire to be taken into account to assess smoke control systems is conventionally considered as a surface, noted A_f [6], associated with a heat release rate per unit area, noted Q_f . This surface may be 9 m², 18 m² or 36 m², depending on the main purpose of the building. In the present study, surfaces (A_f) from 1 m² to 36 m² have been considered, using an uniform law. For smoke control engineering studies, usage is to consider a heat release rate per unit area (Q_f) that lies between 300 kW/m² and 500 kW/m². These bounds have been kept for this study. The chosen associated law is an uniform one.

By multiplying the surface by the heat release rate per unit area, this leads to fires with heat release rates from 300 kW to 18 MW. As the extent of the ranges that are used for surface and heat release rate per unit area are greatly different, one should expect the surface of the fire to be much more influent: in this model, effects from the fire are linked to its global heat release rate. Thus, as the surface is the most influent parameter in the calculation of the global heat release rate by the fire, it may be the most influent parameter on the output data.

3. Measurement uncertainty and sensitivity coefficients

In metrology, the expression "standard uncertainty" is defined as the standard deviation of the measurand. Two reference guides are available to evaluate measurement uncertainty: the Guide to

the evaluation of Uncertainty of Measurement [4], which deals with a Taylor quadratic approximation, and its supplement 1 [5], which deals with Monte Carlo Methods. Both methods rely on the same modeling of the measurement process. The measurement uncertainty framework (see figure 2) is derived from the common framework developed by the European Safety, Reliability and Data Association (ESReDA) [12, 13, 14].

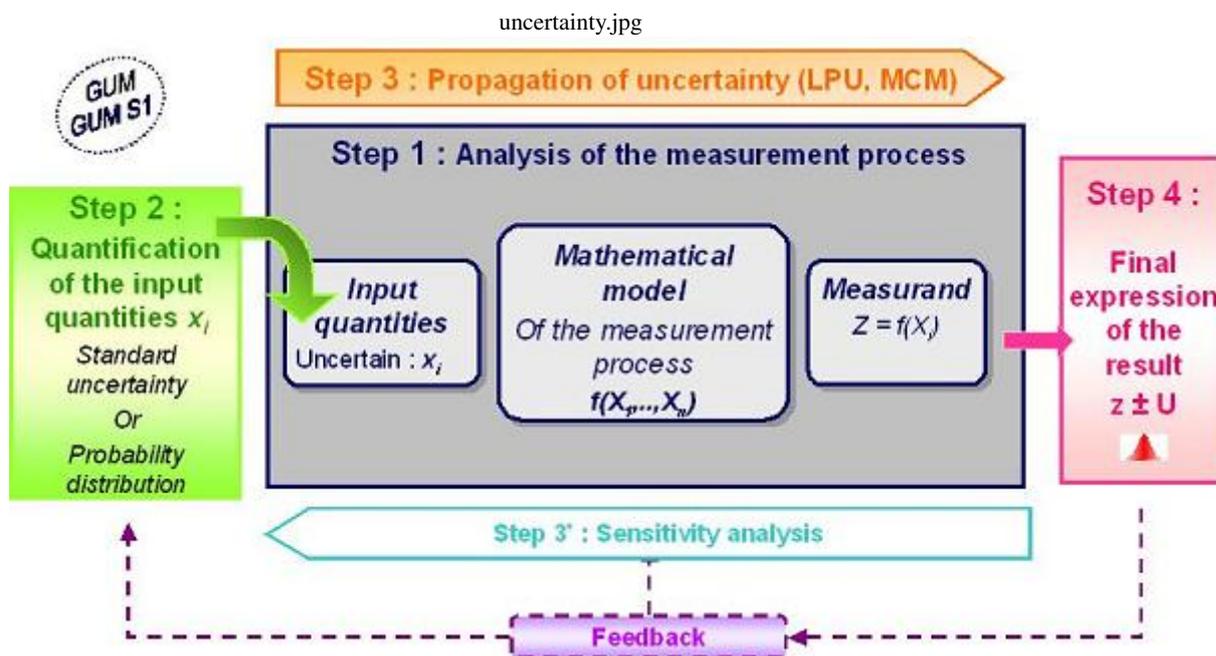


FIGURE 2. Evaluation of measurement uncertainty framework

The variable of interest, called measurand and denoted as $Z \in \mathbf{R}$, is obtained as the output of a computational code, interpreted as a function f of the set of input quantities $(X_1, \dots, X_n) \in \mathbf{R}^n$:

$$Z = f(X_1, \dots, X_n). \quad (1)$$

3.1. The Taylor quadratic approximation

The reference method that is being used for nearly twenty years relies on a Taylor series approximation of the measurement function f . First order Taylor approximation is used most of the time but second order ones may be used when the non linearity of the measurement function is significant. In metrology, the uncertainties associated to the measurements are often quite low. In such a case, the first order Taylor approximation is a satisfying approximation of the measurement function and the variance of the measurand is taken as the variance of the Taylor approximation.

$$V(Z) = \sum_{i=1}^n \left(\frac{\partial f}{\partial X_i} \right)^2 V(X_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n \left(\frac{\partial f}{\partial X_i} \right) \left(\frac{\partial f}{\partial X_j} \right) \text{Cov}(X_i; X_j) \quad (2)$$

where $V(\cdot)$ and $\text{Cov}(\cdot)$ denote the variance and covariance operators, and $\left(\frac{\partial f}{\partial X_i}\right)$ is evaluated for $X = E(X)$.

The partial derivatives, computed at the best estimate $E(X)$ of the input vector (X_1, \dots, X_n) are taken as the sensitivity coefficients for each input quantity, but they evaluate the sensitivity only around the best estimate point. Therefore, these indices are called local sensitivity indices. In opposite, global sensitivity indices [3] can be estimated by Monte Carlo methods. These indices allow to quantify the contribution of each input quantity to the overall variability of the measurand.

3.2. Monte Carlo Methods

Whereas a first order Taylor approximation may be inaccurate in the presence of non linearity or interactions in the measurement function, such an hypothesis is not required for the valid application of Monte Carlo methods. They have recently been introduced in the metrological community, in order to deal more easily with non linearizable or complex measurement models such as computational codes. A probability distribution is assigned to each input quantity, regarding the available information about the quantity. This leads to an empirical distribution for the measurand from which many statistical parameters may be estimated such as the mean, which is considered as the best estimate of the measurand, the standard deviation, which is taken as the standard uncertainty and the empirical percentiles that define an interval of possible values for the measurand.

Regarding sensitivity analysis, the supplement 1 to the GUM [5], which provides guidance to use Monte Carlo Methods to evaluate measurement uncertainty, proposes only one method to evaluate the sensitivity : the One-At-a-Time indices. It consists in analyzing the effect of varying one model input quantity at a time while keeping all the others fixed. It's a suitable sensitivity index in cases where no interactions are involved, but the computational time of these indices, which is equal to $n.M$ where n is the total number of input quantities and M is the total number of Monte Carlo simulations, may be very high. Thus, another sensitivity index is required to deal with more complicated measurement models.

Many goals may lead to conduct a sensitivity analysis [1]. As far as measurement uncertainty is concerned, the main objective is to prioritize the input quantities according to their respective contribution to the variance of the output quantity. The input quantities, for which a decrease of the variance implies the strongest decrease of the variance of the output quantity, have to be determined. This study leads to a general discussion about sensitivity analysis associated with the evaluation of measurement uncertainty. A simple sensitivity index can be performed if the relationships are monotonic: the Standardized Rank Regression Coefficient (SRRC) [1]. Sobol' sensitivity indices [8] estimate higher order terms, but with the crude Monte Carlo estimation technique, many evaluations of the model are needed in order to get good confidence, which is very important in measurement science. FAST (Fourier Amplitude Sensitivity Test) [1] method may be then used to compute Sobol' indices with a smaller simulation number. If total sensitivity indices are greater than first order indices, then second order indices may be computed using local polynomial smoothers [15].

In the following sections, the basis of the sensitivity methods are exposed as well as the sensitivity results of a practical case about the assesment of the propagation of energy and effluents from a fire source in a room.

4. Sensitivity analysis

4.1. Standardized Rank Regression Coefficient

A simple idea to estimate the sensitivity of a measurement model may be to estimate the regression model of the output quantity over the input quantities. Indeed, the regression coefficients can be used to provide a suitable measure of sensitivity if the measurement model is linear. The corresponding sensitivity index is called Standardized Regression Coefficient (SRC) [3] and is obtained as :

$$SRC(X_i; Z) = \beta_i \cdot \sqrt{\frac{V(X_i)}{V(Z)}} \quad (3)$$

where β_i denotes the regression coefficient associated to the input quantity X_i in the regression model.

If the relationship is monotonic but non linear, the linear regression has no longer meaning. Nevertheless, in this case, the regression model that can be estimated over the rank values of the quantities involved is linear. Consequently the Standardized Rank Regression Coefficient (SRRC) can be obtained as :

$$SRRC(X_i; Z) = \beta_i^R \cdot \sqrt{\frac{V(R_{X_i})}{V(R_Z)}} \quad (4)$$

where R_{X_i} and R_Z are respectively the ranks of the input quantity X_i and the output quantity Z and β_i^R denotes the regression coefficient associated to the rank vector R_{X_i} in the regression model.

Nevertheless, SRRC has to be considered with care. Indeed, such a transformation of the original data implies that the sensitivity which is evaluated in this case is the sensitivity about the model implying the ranked vectors, which is more linear than the original model. As a consequence, first-order effects may be overestimated and the interactions wrongly ignored.

4.2. Sobol' indices

In order to estimate the sensitivity of an input quantity without any assumption about the measurement function f , the decomposition of the variance can be considered:

$$V(Z) = V[E(Z|X_i)] + E[V(Z|X_i)]. \quad (5)$$

The importance of X_i on the variance of Z lies in the variance of the conditional expectation (VCE) so that it can be measured by the first-order sensitivity index:

$$S_i = \frac{V[E(Z|X_i)]}{V(Z)}. \quad (6)$$

The second-order sensitivity index can be defined to take into account interactions between two input quantities:

$$S_{ij} = \frac{V[E(Z|X_i, X_j)] - V[E(Z|X_i)] - V[E(Z|X_j)]}{V(Z)}. \quad (7)$$

Many methods are available to estimate those sensitivity indices, specially the method of Sobol'. Sobol' proposed to estimate the first-order sensitivity indices by the quantity [8]:

$$\hat{V}[E(Z|X_i)] = \frac{1}{M-1} \sum_{j=1}^M f(x_1^{(j)}, x_2^{(j)}, \dots, x_n^{(j)}) \cdot f(\tilde{x}_1^{(j)}, \tilde{x}_2^{(j)}, \dots, \tilde{x}_{(i-1)}^{(j)}, x_i^{(j)}, \tilde{x}_{(i+1)}^{(j)}, \dots, \tilde{x}_n^{(j)}) - \hat{f}_0^2 \quad (8)$$

where $(x_i^{(j)})_{j=1..M, i=1..n}$ and $(\tilde{x}_i^{(j)})_{j=1..M, i=1..n}$ are two independent samples of the input quantities, and $\hat{f}_0 = \frac{1}{M} \sum_{j=1}^M f(x_1^{(j)}, x_2^{(j)}, \dots, x_n^{(j)})$ is the estimate of the mean of the output quantity. The total number of evaluations of the measurement function to estimate all the first and second-order sensitivity indices is of the order of Mn^2 . Moreover, in measurements science, the regulation authorities state that the results should be presented with one or two significant decimal digits. In order to obtain such a numerical precision, it is highly recommended to use $M = 10^6$ Monte Carlo simulations. Considering the computational time cost of one run with CFAST (several seconds), the total required computational time for the estimation of the Sobol's indices would be too high. Two other methods, both based on a decomposition of the variance, are investigated: the FAST method and the local polynomial smoothers.

4.3. Fourier Amplitude Sensitivity Test

The Fourier Amplitude Sensitivity Test (FAST) has been developed by Cukier et al. [11], who shows that a decomposition of the variance of Y may be obtained using a Fourier transformation of f . The main idea is to switch from a n -dimensional problem $Y = f(X_1, \dots, X_n)$ to a one-dimensional problem $Z = h(s)$ using transformation functions G_i for $i = 1, \dots, n$ so that:

$$x_i = G_i(\sin(\omega_i s)) \quad (9)$$

where ω_i is supposed to be a set of chosen integer angular frequencies, s is a random variable uniformly distributed over the interval $[-\pi; \pi]$ and G_i the suitable transformation function for the random variable x_i . The transformation function must be chosen in order to assign the suitable probability distribution to the input quantity X_i .

By using, the properties of Fourier series [11], an approximation of the variance of Z may be obtained by:

$$V(Z) = 2 \sum_{j=1}^{\infty} (A_j^2 + B_j^2) \quad (10)$$

where A_j and B_j are the Fourier coefficients and are defined as follows:

$$A_j = \frac{1}{2\pi} \int_{-\pi}^{\pi} h(s) \cos(js) ds \quad (11)$$

and

$$B_j = \frac{1}{2\pi} \int_{-\pi}^{\pi} h(s) \sin(js) ds. \quad (12)$$

First order sensitivity indices are computed by evaluating the Fourier coefficients A_j and B_j for the fundamental frequency ω_i , for $i = 1, \dots, n$, and its higher harmonics:

$$D_{\omega_i} = 2 \sum_{j=1}^{\infty} (A_{j\omega_i}^2 + B_{j\omega_i}^2). \quad (13)$$

As the Fourier amplitudes decrease as p increases, this quantity may be estimated by:

$$\hat{D}_{\omega_i} = 2 \sum_{j=1}^M (A_{j\omega_i}^2 + B_{j\omega_i}^2) \quad (14)$$

where M is the maximum harmonic considered. The first-order sensitivity index is then:

$$S_i = \frac{\hat{D}_{\omega_i}}{\hat{D}} \quad (15)$$

with $\hat{D} = 2 \sum_{j \in \mathbb{Z} - \{0\}} (A_j^2 + B_j^2)$ is the estimator of the total variance.

A method to compute total order sensitivity indices has been developed [2]. The idea is to evaluate the spectrum at the frequencies ω_{-i} :

$$\hat{D}_{\omega_{-i}} = 2 \sum_{j=1}^M (A_{j\omega_{-i}}^2 + B_{j\omega_{-i}}^2) \quad (16)$$

and the total sensitivity indices are given by:

$$TS_i = 1 - \frac{\hat{D}_{\omega_{-i}}}{\hat{D}}. \quad (17)$$

If the total order sensitivity indices are close to the first order sensitivity indices, it means that higher effects are negligible. Otherwise, second order sensitivity indices should be estimated. To this extend, a method based on local polynomial smoothers, developed by DaVeiga et al. [15] can be used.

4.4. Local polynomial smoothers

This method considers each single input quantity X_i relatively to the output quantity Z . Assuming X_i and Z are square integrable, a regression model is used to approximate the measurement function f and may be written as:

$$Z_j = m(X_j) + \sigma(X_j) \varepsilon_j \quad j = 1, \dots, N \quad (18)$$

where $m(x) = E(Z|X = x)$, $\sigma^2(x) = V(Z|X = x)$, $\varepsilon_1, \dots, \varepsilon_N$ are independent random variables such that $E(\varepsilon_i|X_i) = 0$ and $V(\varepsilon_i|X_i) = 1$ and N is the size of the data fitted.

Local polynomial fitting consists in approximating locally the regression function m by a p -th order polynomial

$$m(z) = \sum_{k=0}^p \beta_k (z-x)^k \quad (19)$$

for z in a neighborhood of x . This polynomial is then fitted to the observations by solving the least-squares problem

$$\min_{\beta} \sum_{j=1}^N \left(Z_j - \sum_{k=0}^p \beta_k (X_j - x)^k \right)^2 K_1 \left(\frac{X_j - x}{h_1} \right) \quad (20)$$

where K_1 is a kernel function and h_1 a smoothing parameter. The estimate $\hat{m}(x)$ is then

$$\hat{m}(x) = \hat{\beta}_0(x). \quad (21)$$

The first-order sensitivity index is then based on the variance of the estimate $\hat{m}(x)$.

Let $(\tilde{X}_j)_{j=1, \dots, N'}$ be another sample with the same probability distribution as X_j , the variance of the conditional expectation can be estimated by

$$\hat{T}_1 = \frac{1}{N' - 1} \sum_{j=1}^{N'} (\hat{m}(\tilde{X}_j) - \hat{m})^2 \quad (22)$$

where $\hat{m} = \frac{1}{N'} - \sum_{j=1}^{N'} \hat{m}(\tilde{X}_j)$.

The estimator of the first-order sensitivity index [15] is then obtained as:

$$\hat{S}_1 = \frac{\hat{T}_1}{\hat{\sigma}_Z^2} \quad (23)$$

where $\hat{\sigma}_Z^2$ is the estimator of the total variance.

Second-order indices can also be performed when considering a regression model with two dependent variables X_i and X_j . The VCE to X_i and X_j can be estimated

$$\hat{T}_{ij} = \frac{1}{N' - 1} \sum_{k=1}^{N'} (\hat{m}(\tilde{X}_{ki}, \tilde{X}_{kj}) - \hat{m})^2. \quad (24)$$

Such an estimation takes into account the interaction effect between X_i and X_j , as well as the individual effects of the two input quantities. In order to estimate the interaction effect alone, the individual effects have to be characterized and subtracted to this quantity. Then, the second-order effect is:

$$\hat{S}_{ij} = \frac{\hat{T}_{ij} - \hat{T}_i - \hat{T}_j}{\hat{\sigma}_Z^2}. \quad (25)$$

5. Results

We apply now the sensitivity analysis methods previously described to the fire simulation model CFAST.

We consider three output quantities:

- Maximal Upper Layer Temperature (*ULTemp*), with a criterion at 206 °C. Beyond this point, the radiant flux from smoke and hot gases exceeds what body temperature regulation system can cope;

TABLE 1. Summary statistics for the three output quantities

Effect	Mean value	Standard error	2.5% percentile	97.5% percentile
ULTemp	156.0	67.4	37.6	288.9
LLTemp	29.5	10.9	16.8	56.1
LHeight	4.1	1.0	2.0	5.92

TABLE 2. First order sensitivity indices for the maximal upper layer temperature

$Y_1 = \text{Maximal Upper Layer Temperature (ULTemp)}$			
Effect	SRRC	FAST	Local Polynomial
Text	0.02	0.03	0.02
Tint	0.00	0.00	0.00
S	0.10	0.10	0.11
H	0.04	0.04	0.04
Hair	0.01	0.01	0.01
Wair	0.00	0.00	0.00
Af	0.80	0.74	0.75
Qf	0.04	0.04	0.04

- Maximal Lower Layer Temperature (*LLTemp*), with a criterion at 60 °C. Beyond this point, evacuation is greatly compromised, due to a failure of thermal regulation of the body;

- Minimal Layer Height (*LHeight*), with a criterion at 1.80 m, lowest accepted bound for the smoke in the French regulation.

150000 iterations of a Monte Carlo simulation have been performed to obtain the summary statistics (Table 1) and an empirical distribution of the three output quantities.

The total number of Monte Carlo simulations for the estimation of the SRRC and the local polynomial smoother is 150000. The sample size (1025 evaluations for each input quantity, hence a total of 8200 simulations) and the set of frequencies used for the computation of FAST indices are given by Saltelli et al. [2]. The local polynomial smoothers have been estimated over 30 samples (each containing 5000 simulation points) of the total set of 150000 values, which enables us to estimate average sensitivity indices associated to a standard deviation over the 30 samples. The maximal standard deviation of the local polynomial smoother indices is 0.02. The first order sensitivity indices are detailed in Table 2, Table 3 and Table 4 and the most important second order sensitivity indices, computed from the local polynomial smoothers are presented in Table 5.

The area of the fire source is the main input quantity to explain the variance of the maximal upper layer temperature. The higher the fire source area is, the higher the upper layer temperature is. The area of the room is the second influent input quantity. The smallest the room is, the higher the upper layer temperature is. This is in accordance with common sense: for a given size of room with its smoke vents and openings, raising the fire heat release rate means raising the energy input in the system. If the fire is big enough, only a part of this rise is dealt by the smoke vents and the openings: the energy output is also raised, but not as much as the energy input. Consequently the temperature of the hot layer is raised. Considering a given fire, raising the surface area of the building means also raising the surface area of the smoke vents. This means more heat is extracted from the system, therefore a lower temperature for the hot upper layer.

Regarding the lower layer temperature, the main effect comes from the outside temperature,

TABLE 3. First order sensitivity indices for the maximal lower layer temperature

$Y_2 = \text{Maximal Lower Layer Temperature (LLTemp)}$			
Effect	SRRC	FAST	Local Polynomial
Text	0.71	0.66	0.61
Tint	0.07	0.02	0.02
S	0.02	0.02	0.02
H	0.01	0.00	0.01
Hair	0.02	0.02	0.03
Wair	0.01	0.01	0.01
Af	0.16	0.11	0.15
Qf	0.01	0.01	0.01

and, to a lower extent, from the area of the fire source. In a well ventilated fire, smoke and hot gases are extracted from the building in its upper part, and fresh air enters the building feeding the lower part, with a direct influence on its temperature. Energy exchanges also exist between the hot upper layer and the cold lower layer, which means the fire size have an indirect influence on the lower layer temperature. The size of the opening of the building (H_{air} , W_{air}) was felt to have a fair influence on the lower layer temperature, at least on first intuition : a bigger opening could allow more external air to enter, thus influencing the temperature of the cold layer. However, this is not the case, and may be explained considering aeraulic fluxes: whatever the dimensions of the opening are, its area is largely in excess to ensure a proper ventilation. Changing the area of the opening in the chosen range consequently poorly impacts the quantity of fresh air entering the building, which, in this study where external wind is not taken into account, is driven by the fire needs. This can be checked using Pettersson's relation. It allows to calculate the maximum rate of heat release that can be obtained at steady state in a single volume (eg a room), considering the maximum flux of combustive that can enter through an opening to sustain the combustion. This relation is:

$$\dot{m}_{air} = 0.5 \times S_{air} \times \sqrt{H_{air}} \quad (26)$$

where \dot{m}_{air} (kg/s) is the entering air mass flux, S_{air} the surface (m^2) of the opening and H_{air} its height (m). the entering oxygen mass flux can be deduced knowing the mass ratio of oxygen in air is about 0.23:

$$\dot{m}_{O_2} = 0.23 \times \dot{m}_{air}. \quad (27)$$

Finally, the heat release rate can be deduced using the Thornton factor (13100 $kJ.kg^{-1}$: 13100 kJ are released when 1 kg of O_2 is consumed). This factor is valid at $\pm 5\%$ for most usual combustible materials, thus giving a good estimation of heat release rate. The smallest opening is about 7 m with a height of 2 m, which leads to an oxygen mass flux of about $0.22 kg.s^{-1}$ and a heat release rate of about 17.5 MW. This value is near with the maximum heat release rate considered in the study (18 MW): the smallest opening is big enough to let air enter from outside as the most intense fire requires it.

Nevertheless, on a pragmatic point of view, even if the width of the opening does not here affect the dynamic of the fire nor is an influent parameter on the lower layer temperature, one should not forget that wide openings globally ensure a better evacuation for people.

Three input quantities have to be carefully considered regarding the minimal layer height: the area of the room, the height of the air opening and the area of the fire source. For a given fire

TABLE 4. *First order sensitivity indices for the minimal layer height*

$Y_3 = \text{Minimal Layer Height (LHeight)}$			
Effect	SRRC	FAST	Local Polynomial
Text	0.08	0.10	0.07
Tint	0.00	0.00	0.00
S	0.33	0.27	0.28
H	0.09	0.07	0.08
Hair	0.23	0.18	0.19
Wair	0.00	0.01	0.01
Af	0.25	0.19	0.20
Qf	0.01	0.01	0.01

TABLE 5. *Most important second order sensitivity indices, based on local polynomial smoothers*

Second order sensitivity indices			
Interaction	ULTemp	LLTemp	LHeight
S*Af	0.03	0	0
Text*Af	0.01	0.04	0
S*H	0	0	0.02
S*Hair	0.01	0	0.03

source, smoke will be more prone to reach head level in small volumes with small openings than in bigger ones with larger openings, where more smoke is naturally extracted.

For each quantities of interest, it appears that the total indices and not significantly different to the first-order indices. Consequently, the second order indices are not significant given the input parameters of the study.

6. Discussion

In this study, three simple criteria have been considered to assess fire safety and the ability for people to escape in case of a fire. Four input parameters have been found of major influence on the output quantities related to these criteria. One is fire related (surface area, on which highly depends the global heat release rate of the fire), one is meteorologically induced (outside temperature), while the other ones are building related (size of the studied volume, size of the opening). As no action can be taken against outside temperature, this study points towards two ways of action to raise fire safety:

- ensure fire is controlled as much as possible and not allowed to reach an unacceptable level, through choosing materials with a better reaction to fire, or active suppression means as examples;
- design the building to better evacuate smoke and with volumes high enough to prevent smoke to come at head level too soon.

However, fire safety issues should be considered at a higher and more global level than this study case allowed to. Thermal and toxic effects of fire on people are known to interact while they have been considered separately here, and no consideration about dose effects have been taken into account. Considering the number of parameters to be taken into account, a holistic approach is highly recommended, where the tools used in this study are only a part of the global reflection that considers the problem as a whole.

Nevertheless, the statistical analysis performed in the present study is an important first step: it shows that useful help can be obtained through the use of statistical tools, at a reasonable computational cost, to point out influent parameters. With improvement in the method to take into account higher complexity, this kind of analysis could answer to legitimate "what if ?" questions. This might lead to decision making tools helping in the design process of a building or its smoke control system, and finally raise fire safety at a higher level.

7. Conclusion

The fire simulation exposed in this paper is surrounded by a normative context [9] in the field of measurement science, accompanied by an investigation of the measurement uncertainty taken as the variance of the output quantity. Thus, this study leads to the identification of the most important input quantities regarding the variance of the three quantities of interest that allow to evaluate if the evacuation paths remain practicable in case of a fire. Particularly, the surface of the fire appears to be important to explain the variance of the maximal upper layer temperature, the outside temperature appears to be the most important input quantity to explain the variance of the minimal lower layer temperature and both the surface of the room, the height of the openings and the surface of the fire appear to be important to explain the variance of the minimal layer height. If the sensitivity indices used for the purpose of this study are well known in the scientific community, they are quite new in the context of measurement uncertainty. In measurement science, people are used to perform a local sensitivity index based on the partial derivatives, associated to a propagation of measurement uncertainty using the Taylor quadratic approximation. The first goal of this study is then to introduce these methods in order to improve the knowledge on the measurement processes in many other fields than fire engineering [1].

Moreover, the regulation authorities state that the evacuation paths have to remain practicable. For these three quantities, it is then possible to define a threshold that ensures that the requirement is fulfilled. It seems then interesting to estimate the probability of failure of each parameter, associated to a sensitivity analysis that would point out the input quantities that are the most important when the parameters of interest are close to their threshold. For example, FORM-SORM methods [14] may then be used to this aim, which will be the subject of our future work.

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