Journées Équations aux dérivées partielles

Mitsuru Ikawa

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Journées Équations aux dérivées partielles (1990), p. 1-9

<http://www.numdam.org/item?id=JEDP_1990____A6_0>

ON POLES OF SCATTERING MATRICES FOR SEVERAL CONVEX BODIES

MITSURU IKAWA

Department of Mathematics, Osaka University
Toyonaka, Osaka 560, Japan

1. Introduction. We shall consider scattering for the wave equation by obstacles. Let $\mathcal{O}$ be a bounded open set in $\mathbf{R}^3$ with smooth boundary Γ. We set

$$\Omega = \mathbf{R}^3 - \overline{\mathcal{O}},$$

and assume that Ω is connected. Consider the following acoustic problem:

$$(1.1)\qquad \begin{cases} \Box u = \dfrac{\partial^2 u}{\partial t^2} - \Delta u = 0 & \text{in } \Omega \times (-\infty, \infty), \\ u = 0 & \text{on } \Gamma \times (-\infty, \infty), \\ u(x,0) = f_1(x), \ \dfrac{\partial u}{\partial t}(x,0) = f_2(x). \end{cases}$$

We denote by $\mathcal{S}(z)$ the scattering matrix for this problem. The scattering matrix $\mathcal{S}(z)$ is an $\mathcal{L}(L^2(S^2))$−valued function analytic in $\{z; \operatorname{Im} z \le 0\}$ and meromorphic in the whole complex plane $\mathbf{C}$. It is known that the correspondance from obstacles to scattering matrices

$$\mathcal{O} \to \mathcal{S}(z)$$

is one to one. Thus, we may say all the informations of obstacles are contained in scattering matrices. One of the most interesting and important problems of scattering theory is to find concrete relationships between geometry of obstacles and analytic properties of scattering matrices.

Our actual problems are around the following question:

How the distribution of poles of scattering matrices relates to the geometry of obstacles?

Concerning this question, the following conjecture is fundamental:

MODIFIED LAX-PHILLIPS CONJECTURE. *When $\mathcal{O}$ is trapping, there is a positive constant α such that the scattering matrix $\mathcal{S}(z)$ has an infinite number of poles in $\{z; 0 < \operatorname{Im} z \le \alpha\}$.*

Hereafter, we say that MLPC(abbreviation of the modified Lax-Phillips conjecture) is valid for obstacle $\mathcal{O}$, when there is $\alpha > 0$ such that the scattering matrix $\mathcal{S}(z)$ corresponding to $\mathcal{O}$ has an infinite number of poles in $\{z; \operatorname{Im} z \le \alpha\}$.

Remark that, if $\mathcal{O}$ is nontrapping, the scattering matrix $\mathcal{S}(z)$ has only a finite number of poles in $\{z; \operatorname{Im} z \le \alpha\}$ for all $\alpha > 0$. Thus, if the above conjecture is true, the existence of such α becomes a characterization of trapping obstacles by means of the distribution of poles of scattering matrices. But, we may say that the above conjecture remains essentially open. Namely, at the present time there are only a few examples for which is proved its validity.

To my best knowledge, the examples for which is proved the validity of MLPC are obstacles consisting of two convex bodies. Here we would like to mention about the difference of geometry of the domains outside of two strictly convex bodies and of more than two.

For an obstacle $\mathcal{O}$ consisting of two strictly convex bodies, the number of primitive periodic rays in Ω is only one. On the other hand, for $\mathcal{O}$ consisting of more than two, there are generally an infinite number of primitive periodic rays in Ω. The infiniteness of the primitive periodic rays makes the problem difficult, that is, this fact makes us impossible to use the methods in Ikawa[4] and Gérard[3] that work well for two strictly convex bodies. In order to get informations about poles it is necessary to controle the complexity comming from the infiniteness of primitive periodic rays, but we cannot do it for general obstacles consisting of several strictly convex bodies. Here, we apply the methods of ergodic theory in [2, 12, 13] to controle the complexity of the geometry of periodic rays, but we can do it only for obstacles consisting of several small balls.

Now we shall state the main theorem. Let P_j, $j = 1, 2, \cdots, L$, be points in $\mathbf{R}^3$. We set for $\varepsilon > 0$

$$\mathcal{O}_\varepsilon = \cup_{j=1}^{L} \mathcal{O}_{j,\varepsilon}, \quad \mathcal{O}_{j,\varepsilon} = \{x; |x - P_j| < \varepsilon\}.$$

Now we have

THEOREM 1. *Suppose that*

(A.1) *any triple of P_j's does not lie on a straight line.*

Then, there is $\varepsilon_0 > 0$ such that, for any $0 < \varepsilon < \varepsilon_0$, the modified Lax and Phillips conjecture is valid for $\mathcal{O}_\varepsilon$.

We considered in [8] the same problem and showed that MLPC for several small balls requiring some additional conditions, which restrict the configuration of the centers of balls. Grace of the result in [9] we can remove the additional conditions.

The plan of the proof of Theorem 1 is as follows: With the aid of a general theorem for several strictly convex bodies, we reduce the validity of MLPC to the verification of the existence of singularities of a function determined by the geometry of the periodic rays in Ω. It is also known that the function has a close relation wih a zeta function of the dynamical system in Ω. Thus it suffices to check the existence of poles for the zeta function of the dynamical system. But it seems us also difficult to check the existence of poles of zeta functions in general. If we restrict $\mathcal{O}$ to the ones consisting of small balls, we can get a singularity of the zeta function. Indeed, when the bodies are small the dynamical system in Ω can be approximated by that of a graph, whose zeta function is much easier to treat.

2. A general theorem for several strictly convex bodies and reduction of the problem.

First we present a theorem in [6] without proof.

Let $\mathcal{O}_j$, $j = 1, 2, \cdots, L$, be bounded open sets with smooth boundary Γ_j satisfying

(H.1) every $\mathcal{O}_j$ is strictly convex,

(H.2) for every $j_1, j_2, j_3 \in \{1, 2, \cdots, L\}^3$ such that $j_l \neq j_{l'}$ if $l \neq l'$,
(convex hull of $\overline{\mathcal{O}_{j_1}}$ and $\overline{\mathcal{O}_{j_2}}$) $\cap \overline{\mathcal{O}_{j_3}} = \phi$.

We set

$$\mathcal{O} = \cup_{j=1}^{L} \mathcal{O}_j, \quad \Omega = \mathbf{R}^3 - \overline{\mathcal{O}} \text{ and } \Gamma = \partial\Omega. \tag{2.1}$$

Denote by γ an oriented periodic ray in Ω, and we shall use the following notations:

d_γ : the length of γ,
T_γ : the primitive period of γ,
i_γ : the number of the reflecting points of γ,
P_γ : the Poincaré map of γ.

We define a function $F_D(s)$ $(s \in \mathbf{C})$ by

$$F_D(s) = \sum_{\gamma} (-1)^{i_\gamma} T_\gamma |I - P_\gamma|^{-1/2} e^{-sd_\gamma} \tag{2.2}$$

where the summation is taken over all the oriented periodic rays in Ω and $|I - P_\gamma|$ denotes the determinant of $I - P_\gamma$.

Concerning the periodic rays in Ω we have

$$\#\{\gamma; \text{periodic ray in } \Omega \text{ such that } d_\gamma < r\} < e^{a_0 r} \tag{2.3}$$

and

$$|I - P_\gamma| \geq e^{2a_1 d_\gamma}, \tag{2.4}$$

where a_0 and a_1 are positive constants depending on $\mathcal{O}$. The estimates (2.3) and (2.4) imply that the right hand side of (2.2) converges absolutely in $\{s \in \mathbf{C}; \operatorname{Re} s > a_0 - a_1\}$. Thus $F_D(s)$ is well defined in $\{s \in \mathbf{C}; \operatorname{Re} s > a_0 - a_1\}$, and holomorphic in this domain.

Now we have

THEOREM 2.1. *Let $\mathcal{O}$ be an obstacle given by (2.1) satisfying (H.1) and (H.2). If $F_D(s)$ cannot be prolonged analytically to an entire function, then MLPC is valid for $\mathcal{O}$.*

The proof is based on the trace formula due to Bardos, Guillot and Ralston[1]. The essential part of the proof is given in [8, Section 2].

Now we consider the relationship between the function $F_D(s)$ and the zeta function of symbolic flows. We introduce some notations of symbolic flows.

Let $A = (A(i,j))_{i,j=1,2,\cdots,L}$ be a zero-one $L \times L$ matrix. We set

$$\Sigma_A = \{\xi = (\cdots, \xi_{-1}, \xi_0, \xi_1, \cdots);\ \xi_j \in \{1,2,\cdots,L\} \text{ and } A(\xi_j, \xi_{j+1}) = 1 \text{ for all } j\}.$$

Denote by σ_A the shift transformation defined by

$$(\sigma_A \xi)_j = \xi_{j+1}.$$

For $r \in C(\Sigma_A)$ we define $\mathrm{var}_n\, r$ and $\|r\|_\infty$ by

$$\mathrm{var}_n\, r = \sup\{|r(\xi) - r(\psi)|;\ \xi,\ \psi \in \Sigma_A \text{ and } \xi_j = \psi_j \text{ for } -n \le j \le n\},$$
$$\|u\|_\infty = \sup\{|r(\xi)|; \xi \in \Sigma_A\}.$$

We set for $0 < \theta < 1$

$$\|u\|_\theta = \sup_{n \ge 1}\ \mathrm{var}_n\, r/\theta^n, \quad |||\, r\, |||_\theta = \max\{\|r\|_\infty, \|u\|_\theta\},$$
$$\mathcal{F}_\theta(\Sigma_A) = \{r \in C(\Sigma_A);\ |||\, r\, |||_\theta < \infty\}.$$

Let $r(\xi, s)$ be a $\mathcal{F}_\theta(\Sigma_A)$–valued holomorphic function of s defined in a domain of $\mathbf{C}$, and define $Z(s)$ by

$$Z(s) = \exp\left(\sum_{n=1}^{\infty} \frac{1}{n} \sum_{\sigma_A^n \xi = \xi} \exp S_n r(\xi, s)\right)$$

where

$$S_n r(\xi, s) = r(\xi, s) + r(\sigma_A \xi, s) + \cdots + r(\sigma_A^{n-1}\xi, s).$$

Note that $Z(s)$ is nothing but the zeta function $\zeta(r(\cdot, s))$ in the sense of Parry[16, Section 3], and we call $Z(s)$ the zeta function of a symbolic flow (Σ_A, σ_A) associated to $r(\cdot, s)$.

Now consider relationships between Σ_A and bounded broken rays in the outside of $\mathcal{O}_j$'s satisfying (H.1) and (H.2). We take the matrix $A = (A(i,j))_{i,j=1.\cdots,L}$ as

$$A(i,j) = \begin{cases} 1, & \text{if } i \ne j \\ 0, & \text{if } i = j. \end{cases} \tag{2.5}$$

As was shown in [5], if we denote by $\{\cdots, l_{-1}, l_0, l_1, \cdots\}$ the reflection order of a broken ray in Ω which repeats reflections on the boundary Γ infinitely many times in the both directions, $\{\cdots, l_{-1}, l_0, l_1, \cdots\}$ belongs to Σ_A. Conversely, for each element of $\xi \in \Sigma_A$ there exists a unique broken ray with the reflection order ξ. Note that a periodic ray in Ω corresponds to a periodic element $\xi \in \Sigma_A$, that is, $\sigma_A^n \xi = \xi$ for some n. We set

$$f(\xi) = |X_0 X_1|$$

where X_j denote the j-th reflection point of the broken ray corresponding to ξ.

Denote by $\lambda_1(\xi)$ and $\lambda_2(\xi)$ the eigenvalues of P_γ greater than 1, and by $\kappa_l(\xi)$, $l=1,2$, the principal curvatures at X_0 of the wave front of the phase function $\varphi_{\mathbf{i},0}^\infty$ defined in [5, Section 5], where $\mathbf{i}=(\xi_0,\cdots,\xi_{n-1})$. Then we have

$$(2.6)\qquad \lambda_1(\xi)\lambda_2(\xi)=\prod_{j=1}^{n}(1+f(\sigma_A^j\xi)\kappa_1(\sigma_A^j\xi))(1+f(\sigma_A^j\xi)\kappa_2(\sigma_A^j\xi)).$$

It is easy to check that

$$(2.7)\qquad \lambda_1(\xi)\lambda_2(\xi)\geq e^{cn}\quad (c>0).$$

Since the other eigenvalues of P_γ are $\lambda_1^{-1}$and λ_2^{-1}, it holds that

$$(2.8)\qquad |\,\lambda_1\lambda_2-|I-P_\gamma|\,|\leq C(\lambda_1+\lambda_2)\quad \text{for all}\ \ \gamma.$$

Define $g(\xi)$ for an periodic element ξ by

$$g(\xi)=-\frac{1}{2}\log(1+f(\xi)\kappa_1(\xi))(1+f(\xi)\kappa_2(\xi)).$$

Then $g(\xi)$ can be extended to a function in $\mathcal{F}_\theta(\Sigma_A)$. Define $\zeta(s)$ by

$$\zeta(s)=\exp\left(\sum_{n=1}^{\infty}\frac{1}{n}\sum_{\sigma_A^n\xi=\xi}\exp S_n(-sf(\xi)+g(\xi)+\pi i)\right).$$

The estimates (2.7) and (2.8) imply that both $F_D(s)$ and $\zeta(s)$ converge absolutely for Re s large. Denote by ν_0 the abscissa of convergence of $\zeta(s)$, that is,

$$\nu_0=\inf\{\nu;\zeta(s)\text{ converges absolutely for }\operatorname{Re}s>\nu\}.$$

Then it holds that for $\operatorname{Re}s>\nu_0$

$$\begin{aligned}-\frac{d}{ds}\log\zeta(s)&=\sum_{n=1}^{\infty}\frac{1}{n}\sum_{\sigma_A^n\xi=\xi}S_nf(\xi)\exp\left(S_n(-sf(\xi)+g(\xi)+\pi i)\right)\\&=\sum_{n=1}^{\infty}\sum_{\sigma_A^n\xi=\xi}\frac{S_nf(\xi)}{n}(-1)^n\exp\left(S_ng(\xi)\right)\exp\left(-sS_nf(\xi)\right).\end{aligned}$$

Obviousely we have

$$S_n(\xi)=d_\gamma,\quad n=i_\gamma,\quad (\lambda_1(\xi)\lambda_2(\xi))^{-1/2}=\exp S_ng(\xi).$$

Taking account of the number of elements $\xi\in\Sigma_A$ corresponding to γ, we have

$$\sum_{\xi\in(\gamma)}\frac{S_nf(\xi)}{n}=T_\gamma$$

where the summation is taken over all ξ corresponding to γ. By using the relations

$$S_n(\xi) = d_\gamma, \quad n = i_\gamma, \quad (\lambda_1(\xi)\lambda_2(\xi))^{-1/2} = \exp S_n g(\xi).$$

we have

$$\begin{aligned} &F_D(s) - (-\frac{d}{ds}\log \zeta(s)) \\ &= \sum_\gamma T_\gamma(-1)^n\{|I - P_\gamma|^{-1/2} - (\lambda_1\lambda_2)^{-1/2}\}\exp(-sd_\gamma). \end{aligned} \tag{2.9}$$

Since $\Big|\ |I - P_\gamma|_{-1/2} - (\lambda_1\lambda_2)^{-1/2}\ \Big| \leq C(\lambda_1\lambda_2)^{-1/2}(\lambda_1 + \lambda_2)^{-1}$ the left hand side of (2.9) absolutely converges in $\mathrm{Re}\, s \geq \nu_0 - c/2$. Therefore the singularities of $F_D(s)$ and $-\frac{d}{ds}\log \zeta(s)$ coincide in $\{s; \mathrm{Re}\, s \geq \nu_0 - c/2\}$. Namely, if we can show the existence of poles of $-\frac{d}{ds}\log \zeta(s)$ in $\{s; \mathrm{Re}\, s \geq \nu_0 - c/2\}$, we get the existence of poles of $F_D(s)$.

3. On the proof of Theorem 1.

In order to show the existence of singularities of the zeta function associated to $\mathcal{O}_\varepsilon$ for small ε, we have to consider singular perturbations of symbolic flows. We present a theorem on singular perturbation of symbolic flows, which is the main result of [9].

Assume that a zero-one $L \times L$ matrix A satisfies

$$A^N > 0 \qquad \text{for some positive integer } N, \tag{3.1}$$

that is, all the entries of the matrix A^N are positive. Let $B = [B(i,j)]_{i,j=1,2,\cdots,L}$ be another zero-one $L \times L$ matrix. For a pair $i, j \in \{1, 2, \cdots, L\}$, we denote $i \underset{B}{\longrightarrow} j$ when there is a sequence $i_1, i_2. \cdots, i_p$ such that $B(i_1, i) = 1$, $B(i_{q+1}, i_q) = 1$ for $q = 1, 2, \cdots, p-1$ and $B(j, i_p) = 1$. We assume on B the following:

There is $1 < K \leq L$ such that

$$B(i,j) = 0 \quad \text{for all } j \text{ if } i \geq K+1, \tag{3.2}$$

$$i \underset{B}{\longrightarrow} i \quad \text{for all } 1 \leq i \leq K, \tag{3.3}$$

$$i \underset{B}{\longrightarrow} j \quad \text{implies} \quad j \underset{B}{\longrightarrow} i \quad \text{if } i, j \leq K \tag{3.4}$$

and

$$B(i,j) = 1 \quad \text{implies} \quad A(i,j) = 1. \tag{3.5}$$

Let f_ε, h_ε are functions with parameter $\varepsilon \geq 0$ satisfying

$$f_\varepsilon,\ h_\varepsilon \in \mathcal{F}_\theta(\Sigma_A^+) \qquad \text{for all } 0 \leq \varepsilon \leq \varepsilon_1, \tag{3.5}$$

where ε_1 is a positive constant, and let $k \in \mathcal{F}_\theta(\Sigma_A^+)$ satisfy

$$k(\xi) = \begin{cases} k(\xi) = 0 & \text{if } B(\xi_1, \xi_2) = 1 \\ k(\xi) > 0 & \text{if } B(\xi_1, \xi_2) = 0. \end{cases} \tag{3.6}$$

Suppose that

$$|||f_\varepsilon - f_0|||_\theta, \ |||h_\varepsilon - h_0|||_\theta \to \infty \quad \text{as} \quad \varepsilon \to 0. \tag{3.7}$$

For $0 < \varepsilon \leq \varepsilon_1$, we define zeta function $Z(s;\varepsilon)$ by

$$Z(s;\varepsilon) = \exp\left(\sum_{n=1}^{\infty} \frac{1}{n} \sum_{\sigma_A^n \xi = \xi} \exp\left(S_n r(\xi, s; \varepsilon)\right)\right) \tag{3.8}$$

where

$$r(\xi, s; \varepsilon) = -s f_\varepsilon(\xi) + h_\varepsilon(\xi) + k(\xi) \log \varepsilon. \tag{3.9}$$

Concerning the existence of singularities of $Z(s;\varepsilon)$ we have

THEOREM 3.1. *Suppose that (3.1)∼(3.7) are satisfied, and that*

$$\theta < 2^{-L} \tag{3.10}$$

$$f_0(\xi) > 0 \quad \textit{for all } \xi \in \Sigma_A^+, \tag{3.11}$$

$$h_0(\xi) \quad \textit{if } B(\xi_1, \xi_2) = 1. \tag{3.12}$$

Then there exist $s_0 \in \mathbf{R}$, D a neighborhood of s_0 in $\mathbf{C}$ and $\varepsilon_0 > 0$ such that, for every $0 < \varepsilon \leq \varepsilon_0$, $Z(s;\varepsilon)$ is meromorphic in D andit has a pole s_ε in D with

$$s_\varepsilon \to s_0 \quad \textit{as} \quad \varepsilon \to 0.$$

Theorem 3.1 is the main theorem of [9], which is an improvement of [7]. As we mentioned in Introduction, the improvement by Theorem 3.1 on the existence of pole for zeta functions permits us Theorem 1.

Next, we shall explain how to apply Theorem 3.1 to $F_D(s)$ corresponding to $\mathcal{O}_\varepsilon$, which will be denoted by $F_{D,\varepsilon}(s)$.

Suppose that P_j, $j = 1, 2, \cdots, L$, satisfy the condition (A.1). We choose as matrix $A = [A(i,j)]_{i,j=1,2,\cdots,L}$ the one defined by (2.5), which satisfies (3.1) for $N = 2$.

Set

$$d_{\max} = \max_{i \neq j} |P_i P_j|$$

and

$$B(i,j) = \begin{cases} 1 & \text{if } |P_i P_j| = d_{\max}, \\ 0 & \text{if } |P_i P_j| < d_{\max}. \end{cases} \tag{3.13}$$

By changing the numbering of the points if necessary, we may suppose that

$$\begin{aligned} B(i,j) &= 0 \qquad \text{for all } j \quad \text{if} \quad i \geq K+1, \\ B(i,j) &= 1 \qquad \text{for some } j \quad \text{if} \quad i \leq K. \end{aligned}$$

holds for some $2 \leq K \leq L$. Obviously (3.13) shows the symmetry of matrix B, and this implies that the matrix B satisfies the condition (3.2)~(3.5).

Remark that (A.1) implies (H.2) for $\mathcal{O}_\varepsilon$ when ε is small.

We denote $f(\xi)$, $g(\xi)$ and $\zeta(s)$ associated to $\mathcal{O}_\varepsilon$ by $f_\varepsilon(\xi)$, $g_\varepsilon(\xi)$ and $\zeta_\varepsilon(s)$ respectively. Note that the θ of $\mathcal{F}_\theta(\Sigma_A)$ to which f_ε, g_ε belong decreases to zero as ε tends to zero. Therefore, if we consider only small ε, we may suppose that (3.10) is satisfied. It is easy to see that, by setting $f_0(\xi) = |P_{\xi_0}P_{\xi_1}|$,

$$|\log \varepsilon| \; |||f_\varepsilon - f_0|||_\theta \to 0 \qquad \text{as} \quad \varepsilon \to 0. \tag{3.14}$$

Of course, f_0 satisfies the condition (3.11). From the relationship between the curvatures of the wave fronts of incident and reflected waves we have

$$\kappa_1(\xi) = \frac{2}{\varepsilon}(\cos\frac{\Theta(\xi)}{2})^{-1} + O(1), \quad \kappa_2(\xi) = \frac{2}{\varepsilon} + O(1)$$

where $\Theta(\xi) = \angle P_{\xi_{-1}}P_{\xi_0}P_{\xi_1}$. Thus we have immediately

$$|||g_\varepsilon(\xi) - \big(\log\varepsilon + \frac{1}{2}\log(\frac{1}{4}\cos\frac{\Theta(\xi)}{2})\big)|||_\theta \to 0 \qquad \text{as} \quad \varepsilon \to 0.$$

Then, by setting $\tilde{g}_\varepsilon(\xi) = g_\varepsilon(\xi) - \log\varepsilon$ and $\tilde{g}_0(\xi) = \frac{1}{2}\log(\frac{1}{4}\cos\frac{\Theta(\xi)}{2})$ we have

$$|||\tilde{g}_\varepsilon - \tilde{g}_0|||_\theta \to 0 \qquad \text{as} \quad \varepsilon \to 0. \tag{3.15}$$

Define $k(\xi)$ by

$$k(\xi) = 1 - f_0(\xi)/d_{\max}.$$

By putting $s' = s - (\log\varepsilon + \sqrt{-1}\,\pi)/d_{\max}$ we have

$$-sf_\varepsilon + g_\varepsilon + \sqrt{-1}\,\pi = -s'f_\varepsilon + h_\varepsilon + k\log\varepsilon,$$

where

$$h_\varepsilon = \tilde{g}_\varepsilon + \sqrt{-1}\,\pi\,k + (\log\varepsilon + \sqrt{-1}\,\pi)\frac{(f_0 - f_\varepsilon)}{d_{\max}}.$$

Evidently it follows from (3.14) that

$$h_0 = \tilde{g}_0 + \sqrt{-1}\,\pi\,k,$$

hence we have

$$h_0(\xi) = \tilde{g}_0(\xi) \quad \text{for } \xi \text{ satisfying } B(\xi_0, \xi_1) = 1.$$

Thus, h_0 satisfies (3.12). Then, $h_\varepsilon, h_\varepsilon, k$ satisfy the conditions required in Theorem 3.1. Let $Z(s;\varepsilon)$ be the zeta function defined by (3.8) with these $h_\varepsilon, h_\varepsilon, k$. Note that we have the relation

$$\zeta_\varepsilon(s) = Z(s - (\log \varepsilon + \sqrt{-1}\,\pi)/d_{\max}; \varepsilon).$$

On the other hand, Theorem 3.1 says that there exists $\varepsilon_0 > 0$, $s_0 \in \mathbf{R}$ and D such that $Z(s;\varepsilon)$ has a pole in D, which implies that $\zeta_\varepsilon(s)$ is meromorphic in $D_\varepsilon = \{s = z + (\log \varepsilon + \sqrt{-1}\,\pi)/d_{\max}; z \in D\}$ and has a pole near $s_0 + (\log \varepsilon + \sqrt{-1}\,\pi)/d_{\max}$. It is evident that this pole of $\zeta_\varepsilon(s)$ stays in the domain where the singularities of $\zeta_\varepsilon(s)$ and $F_{D,\varepsilon}(s)$ coincide. Moreover we see easily that $\zeta_\varepsilon(s)$ is holomorphic in a neighborhood of $[s_0 + \log \varepsilon/d_{\max}, \infty)$. Thus the existence of singularities of $F_{D,\varepsilon}(s)$ is proved.

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