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A LOWER BOUND FOR REVERSIBLE AUTOMATA

PIERRE-CYRILLE HÉAM¹

Abstract. A reversible automaton is a finite automaton in which each letter induces a partial one-to-one map from the set of states into itself. We solve the following problem proposed by Pin. Given an alphabet A , does there exist a sequence of languages K_n on A which can be accepted by a reversible automaton, and such that the number of states of the minimal automaton of K_n is in $O(n)$, while the minimal number of states of a reversible automaton accepting K_n is in $O(\rho^n)$ for some $\rho > 1$? We give such an example with $\rho = \left(\frac{9}{8}\right)^{\frac{1}{12}}$.

Résumé. Un automate réversible est un automate fini dans lequel chaque lettre réalise une fonction injective de l'ensemble des états dans lui-même. On résout dans cet article le problème suivant posé par Pin : étant donné un alphabet A , existe-t-il une suite de langages K_n sur A qui peuvent être reconnus par un automate réversible, et tels que le nombre d'états de l'automate minimal de K_n soit en $O(n)$ alors que le nombre minimal d'états d'un automate réversible reconnaissant K_n soit en $O(\rho^n)$ avec $\rho > 1$? On donne un tel exemple avec $\rho = \left(\frac{9}{8}\right)^{\frac{1}{12}}$.

AMS Subject Classification. 68Q45, 68Q70.

INTRODUCTION

In this paper we answer an open question on reversible automata proposed by Pin in [10]. Reversible automata form a natural class of automata with links to artificial intelligence [1], and to biprefix codes [4]. Moreover reversible automata are used to study inverse monoids, inverse automata [12, 13] and certain topological problems [7, 10, 12, 13]. We prove that computing a reversible automaton which

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recognizes a language given by its minimal automaton can not be done, in some cases, faster than in exponential time (if the alphabet contains at least two letters).

We denote by $|K|$ the cardinality of a set K .

We assume that the reader is familiar with the basic definitions on words and formal languages. For more information we refer the reader to [2]. Let us denote by $|u|$ the length of a word and by u^{\sim} its mirror.

Let us recall that a finite automaton is a 5-tuple $\mathcal{A} = (Q, A, E, I, F)$ where Q is a finite set of states, A is the alphabet, $E \subseteq Q \times A \times Q$ is the set of edges (or transitions), $I \subseteq Q$ is the set of initial states and $F \subseteq Q$ is the set of final states. A path in \mathcal{A} is a finite sequence of consecutive edges:

$$p = (q_0, a_0, q_1), (q_1, a_1, q_2), \dots, (q_{n-1}, a_{n-1}, q_n).$$

The label of the path p is the word $a_1 a_2 \dots a_n$, its origin is q_0 and its end is q_n . A word is accepted by \mathcal{A} if it is the label of a path in \mathcal{A} having its origin in I and its end in F . The set of words accepted by \mathcal{A} is denoted by $L(\mathcal{A})$. For every state q and language K , we denote by $q.K$ the subset of Q of all the states which are the end of a path having its origin in q and its label in K . An automaton is said to be *trim* if for each state q there exists a path from an initial state to q and a path from q to a final state. An automaton is *deterministic* if it has a unique initial state and does not contain any pair of edges of the form (q, a, q_1) and (q, a, q_2) with $q_1 \neq q_2$. An important result of automata theory states that for an automaton \mathcal{A} there exists exactly one (up to isomorphism) deterministic automaton with a minimal number of states which accepts the same language as \mathcal{A} . It is called the *minimal automaton* of $L(\mathcal{A})$. A *reversible* automaton is a finite automaton in which each letter induces a partial one-to-one map from the set of states into itself. It also is an automaton which does not contain any pair of edges of the form (q, a, q_1) and (q, a, q_2) with $q_1 \neq q_2$ or (q_1, a, q) and (q_2, a, q) with $q_1 \neq q_2$. It may happen that a reversible automaton is not deterministic because it may have several initial states. Reversible automata were studied by Pin [10] and by Silva [11]. For an automaton $\mathcal{A} = (Q, A, E, I, F)$, let us introduce the following languages, for p and q in Q :

$$L_{p,q}(\mathcal{A}) = \{u \in A^* \mid p.u = q\}.$$

If $I = \{i\}$ we write $L_p(\mathcal{A})$ for $L_{i,p}(\mathcal{A})$, and if $F = \{f\}$ we write $R_q(\mathcal{A})$ for $L_{q,f}(\mathcal{A})$. If there is no ambiguity we just write $L_{p,q}$, L_p and R_q .

For a language L which is recognized by a reversible automaton we introduce two invariants: $m(L)$, the number of states of its minimal automaton and $c(L)$ the minimal number of states of a reversible automaton accepting L . We denote by $r(n)$ and $R(n)$ the following functions:

$$r(n) = \min\{c(L) \mid m(L) = n\} \text{ and } R(n) = \max\{c(L) \mid m(L) \leq n\}.$$

It is proved in [10] that $r(n) = O\left(\frac{\ln n}{\ln \ln n}\right)^2$. Let us remark that $R(n)$ is an increasing function of n . We prove here that $R(n) = \Omega(\rho^n)$ for some $\rho > 1$.

1. MAIN THEOREM

Theorem 1.1. *The relation $R(n) = \Omega(\rho^n)$ holds with $\rho = \left(\frac{3\sqrt{2}}{4}\right)^{\frac{1}{6}} \simeq 1.001$ if the alphabet contains at least two letters.*

For the proof, let us consider the language $L = \{aa, ab, ba\}$ on the alphabet $A = \{a, b\}$. Before we proceed, let us note the following technical result.

Lemma 1.2. *Let $\mathcal{A} = (Q, A, E, i, F)$ be a reversible trim automaton accepting a non-empty subset of $L = \{aa, ab, ba\}$. For each state $s \in Q$, let L_s be the set of words which can be read from state i to s . Then one of the following holds:*

- (a) $|F| = 3$ and $|L_s| = 1$ for each state $s \in F$;
- (b) $|F| = 2$, $F = \{s_1, s_2\}$, $|L_{s_1}| = 2$ and $|L_{s_2}| = 1$;
- (c) $|F| = 2$ and $|L_s| = 1$ for each state $s \in F$;
- (d) $|F| = 1$, $F = \{s\}$ and $|L_s| = 2$;
- (e) $|F| = 1$, $F = \{s\}$ and $|L_s| = 1$.

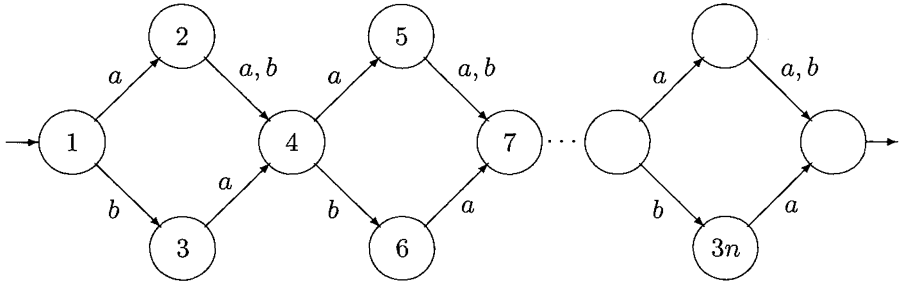
Proof. First we observe that in a reversible automaton on 2 letters, each state is the origin (resp. the end) of at most 2 transitions. Since $|L| = 3$ and \mathcal{A} is trim, F has at most 3 elements, and if $|F| = 3$, then $|L_s| = 1$ for each final state s , that is (a) holds.

If $|F| = 2$, say $F = \{s_1, s_2\}$, then $|L_{s_1}| + |L_{s_2}| \leq 3$. Since \mathcal{A} is trim $|L_s| > 0$ for every state, so (b) or (c) holds.

Finally, let us assume that $F = \{s\}$ is a singleton. We need to verify that $|L_s| \neq 3$, that is, \mathcal{A} does not accept all of L . If there is no a -labeled transition out of i , then $L_s = L(\mathcal{A}) = \{ba\}$ and (e) holds. If there is no b -labeled transition out of i , then $L_s \subseteq \{ab, aa\}$ and (d) or (e) holds. We now consider the case where both an a -labeled transition and a b -labeled transition start in state i . If $i.a = i.b$, then there is no b -labeled transition from $i.a$ to the final state s since bb cannot be accepted, and hence $L_s = \{aa, ba\}$ and (d) holds. On the other hand, suppose that $i.a \neq i.b$. Since \mathcal{A} is trim, there is an a -labeled transition from state $i.b$ to s . But \mathcal{A} is reversible, so there is no a -labeled transition from state $i.a$ to s , and the word aa is not accepted. In particular, $L_s = \{ab, ba\}$ and (d) holds. □

For every integer n , L^n is a finite language which can be recognized by a reversible automaton (it is easy to check that every finite language can be recognized by a reversible automaton). We prove that $c(L^{2n}) \geq \rho^n$, while $m(L^{2n}) = O(n)$.

First observe that, for every positive integer n , $|L^n| = 3^n$, $L^n = (L^n)^\sim$ and every word of L^n has length $2n$. Furthermore it is easy to check that the minimal automaton recognizing L^n is the following one:



Thus the minimal automaton of L^n has $m(L^n) = 3n + 1$ states. Moreover, if \mathcal{A} is a trim automaton which recognizes a sublanguage of L^n , then for all pairs of states p and q of \mathcal{A} , all the words of $L_{p,q}$ have the same length.

Let $\mathcal{A} = (Q, A, E, \{i\}, \{f\})$ be a reversible automaton with a unique initial state and a unique final state recognizing a sublanguage of L^{2n} . Let $M(\mathcal{A}) = i.A^{2n}$ be the set of states of \mathcal{A} we can reach from i by reading a word of length $2n$ (half of the length of the words of L^{2n}).

Proposition 1.3. *With the above notation, the following relation holds*

$$|L(\mathcal{A})| \leq \frac{1}{2} \left(\sum_{q \in M(\mathcal{A})} |L_q|^2 + |R_q^\sim|^2 \right). \tag{1}$$

Proof. By definition we have:

$$L(\mathcal{A}) = \bigcup_{q \in M(\mathcal{A})} L_q R_q.$$

But \mathcal{A} is deterministic and hence, if $p \neq q$ then $L_p \cap L_q = \emptyset$. Consequently, if $p, q \in M(\mathcal{A})$ then $L_p R_p \cap L_q R_q = \emptyset$. Indeed the prefix of length $2n$ of a word of $L_p R_p \cap L_q R_q$ would have to be in $L_p \cap L_q$. Thus we get:

$$|L(\mathcal{A})| = \sum_{q \in M(\mathcal{A})} |L_q| |R_q|.$$

Since R_q is finite we have $|R_q| = |R_q^\sim|$. But for every pair of real numbers (x, y) we have $xy \leq \frac{1}{2}(x^2 + y^2)$. From this we deduce (1). □

For the next proposition and the next three lemmas we consider a reversible automaton $\mathcal{A} = (Q, A, E, \{i\}, F)$ recognizing a sublanguage of L^n (where n is a

strictly positive integer), and having a unique initial state i . For every state q of Q let us denote by n_q the cardinal of L_q .

Proposition 1.4. *The following equality holds:*

$$\sum_{q \in F} n_q^2 \leq 8^n. \tag{2}$$

To prove Proposition 1.4 we proceed by induction on n and we need some technical lemmas. For the proof we can assume without loss of generality that \mathcal{A} is trim.

Lemma 1.5. *If $n = 1$, Proposition 1.4 is true.*

Proof. Applying Lemma 1.2 to \mathcal{A} , we get:

In case (a), $\sum_{q \in F} n_q^2 = |L_{i,s_1}|^2 + |L_{i,s_2}|^2 + |L_{i,s_3}|^2 = 3.$

In case (b), $\sum_{q \in F} n_q^2 = |L_{i,s_1}|^2 + |L_{i,s_2}|^2 = 4 + 1 = 5.$

In case (c), $\sum_{q \in F} n_q^2 = |L_{i,s_1}|^2 + |L_{i,s_2}|^2 = 1 + 1 = 2.$

In case (d), $\sum_{q \in F} n_q^2 = |L_{i,s_1}|^2 = 4.$

In case (e), $\sum_{q \in F} n_q^2 = |L_{i,s_1}|^2 = 1.$

This completes the proof. □

We denote by S the set of states which we can reach by reading a word of length $2n - 2$ (every word of L^n has length $2n$).

Lemma 1.6. *For every q in F we have*

$$n_q = \sum_{s \in S} n_s |L_{s,q}|. \tag{3}$$

Proof. Since every word of $L(\mathcal{A})$ has length $2n$ we have

$$L_q = \bigcup_{s \in S} L_s L_{s,q}. \tag{4}$$

But \mathcal{A} is deterministic, so as in the proof of Proposition 1.3 the union in (4) is disjoint. Since $n_q = |L_q|$ for every state q in Q , we can now deduce formula (1.6) from formula (4). □

We now prove that:

Lemma 1.7. *For all $s \in S$ we have*

$$\sum_{q \in F} \sum_{t \in S} |L_{s,q}| |L_{t,q}| \leq 8. \tag{5}$$

Proof. Let $s \in S$ and let F_s be the set of states in F , which we can reach from s . If $q \in F$, we denote by I_q the set of states of S from which q can be reached. The sum in formula (5) is in fact equal to

$$\sum_{q \in F_s} \sum_{t \in I_q} |L_{s,q}| |L_{t,q}|.$$

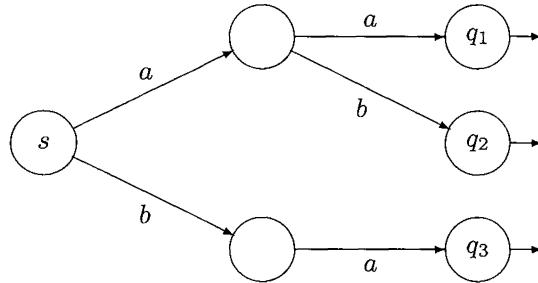
Since \mathcal{A} is trim, the words which can be read from a state in S to a state in F , are all in L . Moreover, as \mathcal{A} is reversible, if $q \in F$, then the languages $L_{t,q}$ ($t \in I_q$) are pairwise disjoint and hence

$$\sum_{t \in I_q} |L_{t,q}| \leq 3.$$

Consider the automaton obtained from \mathcal{A} by first making s the initial state and then trimming. This automaton satisfies the hypothesis of Lemma 1.2 and one of the following five cases arises.

- (a) $|F_s| = 3$ and $|L_{s,q}| = 1$ for each $q \in F_s$.

We have, say, the following configuration



Note that there is no state $t \in I_{q_1}$ such that $t.b = s.a$: if there was, we would have $t.bb = q_2$, so $bb \in L_{t,q_2} \subseteq L$, a contradiction. Since \mathcal{A} is trim, it follows that $ba \notin \bigcup_{t \in I_{q_1}} L_{t,q_1}$ and hence

$$\sum_{t \in I_{q_1}} |L_{t,q_1}| \leq 2.$$

We already observed that

$$\sum_{t \in I_{q_2}} |L_{t,q_2}| \leq 3 \text{ and } \sum_{t \in I_{q_3}} |L_{t,q_3}| \leq 3$$

so

$$\sum_{q \in F_s} \sum_{t \in I_q} |L_{s,q}| |L_{t,q}| = \sum_{q \in F_s} \sum_{t \in I_q} |L_{t,q}| \leq 2 + 3 + 3 = 8.$$

(b) $F_s = \{q_1, q_2\}$ with $|L_{s,q_1}| = 2$ and $|L_{s,q_2}| = 1$. Since \mathcal{A} is reversible we have either $I_{q_1} = \{s\}$ or $I_{q_1} = \{s, u\}$ with $|L_{u,q_1}| = 1$.

(b1) If $I_{q_1} = \{s\}$ we have

$$\sum_{t \in I_{q_1}} |L_{t,q_1}| |L_{s,q_1}| = 4$$

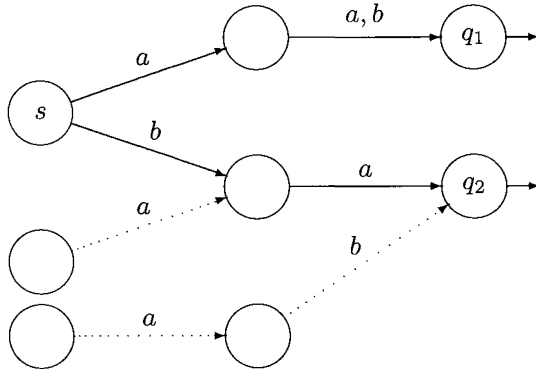
and

$$\sum_{t \in I_{q_2}} |L_{t,q_2}| |L_{s,q_2}| = \sum_{t \in I_{q_2}} |L_{t,q_2}| \leq 3$$

so

$$\sum_{q \in F_s} \sum_{t \in I_q} |L_{s,q}| |L_{t,q}| \leq 4 + 3 = 7.$$

This case can be illustrated in the following figure:



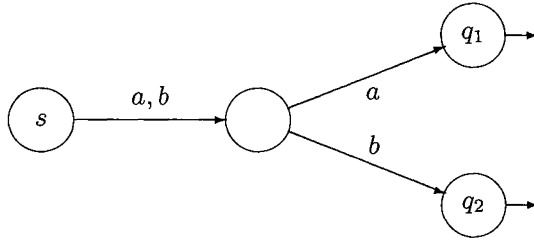
(b2) Assume that $I_{q_1} = \{s, u\}$. By reversibility, we have $L_{u,q_1} = L \setminus L_{s,q_1}$. Since $|L_{s,q_1}| = 2$, it follows that $|L_{u,q_1}| = 1$ and

$$\sum_{t \in I_{q_1}} |L_{s,q_1}| |L_{t,q_1}| = 2 (|L_{s,q_1}| + |L_{u,q_1}|) = 6.$$

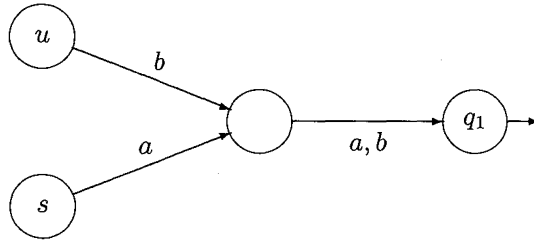
If $I_{q_2} = \{s\}$, we are finished:

$$\sum_{q \in F_s} \sum_{t \in I_q} |L_{s,q}| |L_{t,q}| = 6 + 1 = 7.$$

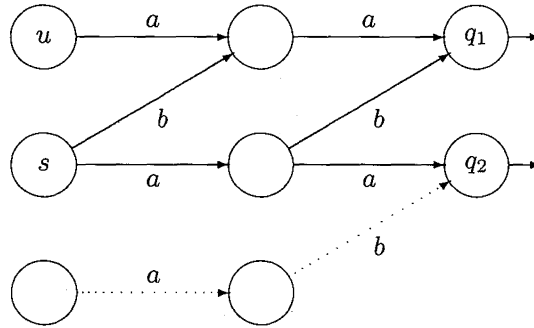
In order to go further, let us analyze this situation in more detail. If $L_{s,q_1} = \{aa, ba\}$, then by reversibility we have $s.b = s.a$. Moreover in that case $L_{s,q_2} = \{ab\}$, so that $(s.a).b = q_2$. This implies in turn that $s.bb = q_2$, $bb \in L_{s,q_2}$, a contradiction.



Now if $L_{s,q_1} = \{aa, ab\}$ then by determinism both the a -labeled and the b -labeled transitions into q_1 come from $s.a$. Moreover $L_{u,q_1} = \{ba\}$, so $u.b = s.a$ and we now have $u.bb = q_1$, $bb \in L_{u,q_1}$, again a contradiction.



Thus $aa \notin L_{s,q_1}$ and we have $L_{s,q_1} = \{ab, ba\}$, $L_{s,q_2} = L_{u,q_1} = \{aa\}$ and the following configuration



Suppose that $v \in I_{q_2}$, $v \neq s$. Then $aa \notin L_{v,q_2}$ by reversibility. Also $ba \notin L_{v,q_2}$: otherwise $v.b = s.a$ and $v.bb = q_1$. This implies $bb \in L_{v,q_1} \subseteq L$, a contradiction. Thus if $v \neq s$ lies in I_{q_2} , then $L_{v,q_2} = \{ab\}$. This shows that

$$\sum_{t \in I_{q_2}} |L_{s,q_2}||L_{t,q_2}| = \sum_{t \in I_{q_2}} |L_{t,q_2}| \leq 2$$

and hence

$$\sum_{q \in F_s} \sum_{t \in I_q} |L_{s,q}| |L_{t,q}| \leq 8.$$

(c) $|F_s| = 2$ and $|L_{s,q}| = 1$ for each $q \in F_s$. Since $\sum_{t \in I_q} |L_{t,q}| \leq 3$ for each $q \in F$, we have

$$\sum_{q \in F_s} \sum_{t \in I_q} |L_{s,q}| |L_{t,q}| = \sum_{q \in F_s} \sum_{t \in I_q} |L_{t,q}| \leq 2 \times 3 = 6.$$

(d-e) $F_s = \{q\}$ and $|L_{s,q}| = 2$ (resp. 1). Again using that $\sum_{t \in I_q} |L_{t,q}| \leq 3$, we find that

$$\sum_{q \in F_s} \sum_{t \in I_q} |L_{s,q}| |L_{t,q}| \leq 6 \text{ (resp. 3).}$$

□

Now we can prove Proposition 1.4.

Proof. We proceed by induction on n . The proposition is true for $n = 1$ by Lemma 1.5.

Assume now that it is true for n . Let $\mathcal{A} = (Q, A, E, \{i\}, F)$ be a reversible trim automaton with a unique initial state i recognizing a subset of L^{n+1} . According to Lemma 1.6

$$n_q^2 = \sum_{(s,t) \in S^2} n_s n_t |L_{s,q}| |L_{t,q}|. \tag{6}$$

Thus we have:

$$\begin{aligned} \sum_{q \in F} n_q^2 &= \sum_{q \in F} \sum_{(s,t) \in S^2} n_s n_t |L_{s,q}| |L_{t,q}| = \sum_{(s,t) \in S^2} \left(\sum_{q \in F} |L_{s,q}| |L_{t,q}| \right) n_s n_t \\ &\leq \frac{1}{2} \sum_{(s,t) \in S^2} \left(\sum_{q \in F} |L_{s,q}| |L_{t,q}| \right) n_s^2 + \frac{1}{2} \sum_{(s,t) \in S^2} \left(\sum_{q \in F} |L_{s,q}| |L_{t,q}| \right) n_t^2 \\ &\leq \sum_{(s,t) \in S^2} \left(\sum_{q \in F} |L_{s,q}| |L_{t,q}| \right) n_s^2. \end{aligned}$$

From this, by Lemma 1.7, we have

$$\sum_{q \in F} n_q^2 \leq 8 \sum_{s \in S} n_s^2.$$

Now we can apply the induction hypothesis to the automaton deduced from \mathcal{A} by taking S for the set of final states rather than F , and by making it trim. We obtain:

$$\sum_{s \in S} n_s^2 \leq 8^n.$$

This establishes the proposition. □

Now we prove the main theorem.

Proof. Let n be a positive integer and let $\mathcal{A} = (Q, A, E, I, F)$ be a reversible automaton recognizing L^{2n} . For each $i \in I$ and $f \in F$ let us denote by \mathcal{A}_{if} the automaton $(Q, A, E, \{i\}, \{f\})$.

It is clear that

$$L(\mathcal{A}) = \bigcup_{i \in I, f \in F} L(\mathcal{A}_{if}). \tag{7}$$

According to formula (1) we have

$$|L(\mathcal{A}_{if})| \leq \frac{1}{2} \left(\sum_{q \in M(\mathcal{A}_{if})} |L_q(\mathcal{A}_{if})|^2 + |R_q(\mathcal{A}_{if})^\sim|^2 \right). \tag{8}$$

Now the reversible automata $(Q, A, E, \{i\}, M(\mathcal{A}_{if}))$ recognizes a sublanguage of L^n . By formula (2), we have

$$\sum_{q \in M(\mathcal{A}_{if})} |L_q(\mathcal{A}_{if})|^2 \leq 8^n.$$

Moreover, let us denote by $E' = \{(p, a_i, q) \mid (q, a_i, p) \in E\}$. The reversible automata $(Q, A, E', \{f\}, M(\mathcal{A}_{if}))$ also recognizes a sublanguage of L^n , and by (2), we have

$$\sum_{q \in M(\mathcal{A}_{if})} |R_q(\mathcal{A}_{if})^\sim|^2 \leq 8^n.$$

Thus, $|L(\mathcal{A}_{if})| \leq 8^n$ by formula 8.

By formula (7), it comes $|L(\mathcal{A})| \leq |I||F|8^n$. But $|L(\mathcal{A})| = 9^n$, so

$$|I||F| \geq \left(\frac{9}{8}\right)^n.$$

It follows that I or F contain a least r^n elements with $r = \frac{3\sqrt{2}}{4}$. Since I and F are subsets of Q we have $|Q| \geq r^n$.

We have proved that $c(L^{2n}) \geq r^n$. But $m(L^{2n}) = 6n + 1$. Hence $R(6n + 1) = \Omega(r^n)$ and, since $R(n)$ is an increasing function of n , this proves the theorem with $\rho = r^{\frac{1}{6}}$. □

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