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## A FAST METHOD OF DEADLOCK AVOIDANCE (\*)

by Piotr BAK <sup>(1)</sup>

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*Abstract.* — *The paper presents a fast deadlock avoidance method for the single resource type system. Proposed method is similar to the one described in Habermann. It is based on the process rank modification during resource granting or releasing.*

*The auxiliary vector construction is simpler than in Habermann. The utilization of this method in multiple resource type system is also presented.*

*Résumé.* — *On présente une méthode rapide pour éviter l'interblocage dans un seul type de ressources. Cette méthode ressemble à celle présentée par Habermann. Elle est basée sur l'idée de modifier le rang d'un processus quand les ressources sont allouées ou sont rendues. La méthode de construction des vecteurs auxiliaires est plus simple que celle de Habermann. On propose aussi l'utilisation de cette méthode pour les systèmes à types de ressources multiples.*

### 1. INTRODUCTION

System deadlock avoidance methods are based on constant preservation of so-called "system safe state" [4, 5, 2]. After every demand of resources the special-test is used to decide if the resources may be granted. The banker's algorithm is used but its execution time is proportional to the product of number of processes and number of resource types. The substantial computational complexity of this algorithm often limits its utilisation in practice. In [5] one of the first attempts for its simplification is presented, the execution time being in this case proportional to the number of resources only. The method presented in [5] is valid only for single-resource-type system.

The utilization of this results from [5] for multiple-resource-type system is presented in [1]. In this paper the deadlock prevention method [3] for different types of resources has been used.

In a safety test proposed in [5] the "state vector" must be hold. However its actualisation is complicated. The paper [5] dosen't present the complete proof

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of the proposed condition of system safe state which causes its modifications to be difficult.

In this paper a new method of deadlock avoidance for the single-resource-type system as well as utilization of this method for multiple-resource-type system is presented.

## 2. ASSUMPTIONS AND DEFINITIONS

In the paper we will consider, a system which consists of the finite set of sequential processes  $\mathcal{P} = \{P_1, P_2, \dots, P_n\}$ , and the finite set of types of resources  $\mathcal{R} = \{R_1, R_2, \dots, R_m\}$ .

The processes dynamically demand resources, and all necessary conditions of deadlock occurrence are fulfilled [3].

The resource allocation state is defined similarly as in [4] by means of matrices  $\mathbf{B}$ ,  $\mathbf{C}$ ,  $\mathbf{D}$  and vector  $\mathbf{a}$ .

Vector  $\mathbf{a} = [a_i]$  ( $i = 1, 2, \dots, m$ ) describes the number of resources of each type available in the system ( $a_i$  - number of  $R_i$  type resources).

Matrix:

$$\mathbf{B} = [\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_n] = [b_{ij}] \quad (i = 1, 2, \dots, m, j = 1, 2, \dots, n),$$

describes claims for the system resources ( $b_{ij}$  - maximum number of resources of type  $R_i$  possibly allocated to process  $P_j$ ).

Matrix:

$$\mathbf{C} = [\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_n] = [c_{ij}] \quad (i = 1, 2, \dots, m, j = 1, 2, \dots, n),$$

describes the number of resources allocated to processes at a given moment of time ( $c_{ij}$  - number of resources of type  $R_i$  allocated to  $P_j$ ).

Matrix:

$$\mathbf{D} = [\mathbf{d}_1, \mathbf{d}_2, \dots, \mathbf{d}_n] = [d_{ij}] \quad (i = 1, 2, \dots, m, j = 1, 2, \dots, n),$$

describes current demands of the processes ( $d_{ij}$  - number of resources of types  $R_i$  demanded by process  $P_j$ ).

All demands of process  $P_j$  for resource type  $R_i$  are satisfied if  $d_{ij} = c_{ij}$ . The process demands  $(d_{ij} - c_{ij})$  resources of type  $R_i$  if  $d_{ij} > c_{ij}$ , and whenever if  $d_{ij} < c_{ij}$  process  $P_j$  declares the realise of  $(c_{ij} - d_{ij})$  resources of type  $R_i$ .

We will introduce an auxiliary matrix:

$$\mathbf{Y} = \mathbf{B} - \mathbf{C} = [\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_n] = [y_{ij}] \quad (i = 1, 2, \dots, m, j = 1, 2, \dots, n),$$

which defines the number of resources that the processes can additionally request in order to complete. Let  $y_{ij}$  represent the "rank" of process  $P_j$  in relation to the type  $R_i$  resources [5].

In the paper we will consider only the system realizable state [4, 5].

The presentation of the safety test is based on the following definitions:

DEFINITION 2.1: We say that system is in its local safe state with regard to resource type  $R_i$  when it is possible to compose the processes realization sequence:

$$(2.1) \quad P_{f_i(1)} P_{f_i(2)} \dots P_{f_i(n)}$$

such that:

$$(2.2) \quad \forall_{k \in [1, n]}, \quad b_{if_i(k)} \leq r_i + \sum_{l \leq k} c_{if_i(l)}$$

where:  $b_{if_i(k)}$ ,  $c_{if_i(k)}$  denote maximal and allocated number of resources of type  $R_i$  to the process which occupies  $k$ -th place in the sequence, respectively;  $r_i$  describes how many resources of type  $R_i$  remain free at a given moment of time;  $f_i(k) = z$  denotes that the process  $P_z \in \mathcal{P}$  occupies the  $k$ -th place in the sequence.

DEFINITION 2.2: We say that the system is in its safe state when it is possible to compose the processes realization sequence:

$$(2.3) \quad P_{f(1)} P_{f(2)} \dots P_{f(n)}$$

$$(2.4) \quad \forall_{k \in [1, n]}, \quad \mathbf{b}_{f(k)} \leq \mathbf{r} + \sum_{l \leq k} \mathbf{c}_{f(l)}$$

where:  $\mathbf{b}_{f(k)}$ ,  $\mathbf{c}_{f(k)}$ -vector of maximal demands and vector of allocated number of resources to the process which occupies  $k$ -th place in the sequence, respectively:

$$\mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \end{bmatrix}$$

— describes how many resources of each remain free at the given moment of time;

–  $f(k)=z$  denotes that the process  $P_z \in \mathcal{P}$  occupies the  $k$ -th places in the sequence. It is easy to show that the Def. 2.2 is equivalent to safe state definition presented in [4].

The set of processes  $\mathcal{P}$  is divided into two disjoint subsets:

$$\mathcal{P} = \mathcal{P}_1 \cup \mathcal{P}_2 \quad (\mathcal{P}_1 \cap \mathcal{P}_2) = 0.$$

Subset  $\mathcal{P}_2$  is a set of passive processes for which:

$$\forall_{P_j \in \mathcal{P}_2}, \quad \forall_{i \in [1, m]}, \quad c_{ij} = 0$$

and  $\mathcal{P}_1$  is a set of active and suspended processes, for which:

$$\forall_{P_j \in \mathcal{P}_1}, \quad \exists_{i \in [1, m]}, \quad c_{ij} \neq 0.$$

Let  $u$  describe cardinality of subset  $\mathcal{P}_1$ .

### 3. A SAFETY TEST

We have noticed that the banker's algorithm may be used as the safety test. Because of its substantial complexity the simpler methods are being looked for.

To present the proposed safety test we will prove the following theorem:

**THEOREM 3.1:** *The realizable system state is a local safe state for the  $R_i \subset \mathcal{R}$  type of resources if and only if in this state there exists the sequence of all active and suspended processes:*

$$(3.1) \quad P_{f_i(1)} P_{f_i(2)} \dots P_{f_i(u)}$$

which satisfies the following conditions:

$$(3.2) \quad 1. \quad y_{if_i(1)} \leq y_{if_i(2)} \dots \leq y_{if_i(u)}$$

$$(3.3) \quad 2. \quad \forall_{j \in [1, u]}, \quad b_{if_i(j)} - c_{if_i(j)} \leq r_i + \sum_{k=1}^{j-1} c_{if_i(k)}$$

*Proof:* First we assume that the system state is a local safe state with regard to  $R_i$  type of resources. Thus, there exists at least one local safe sequence of processes for resource type  $R_i$ , for example:

$$(3.4) \quad P_{g_i(1)} P_{g_i(2)} \dots P_{g_i(k)} \dots P_{g_i(n-1)} P_{g_i(n)}$$

Let:

$$P_{g_i(k)} \in \mathcal{P}_2 \quad \text{so} \quad c_{ig_i(k)} = 0.$$

From the Def. 2.1 it results that the process  $P_{g_i(k)}$  can be moved at the end of the above sequence without changing the locations of other processes. In the same way we move all passive processes at the end of (3.4), so finally we have:

$$(3.5) \quad P_{h_i(1)} P_{h_i(2)} \dots P_{h_i(l)} P_{h_i(l+1)} \dots P_{h_i(u)} P_{h_i(u+1)} \dots P_{h_i(n)}$$

which satisfies:

$$(3.6) \quad \forall_{j \in [u+1, n]}, \quad c_{ih_i(j)} = 0.$$

In order to examine that the state is local safe state for resource type  $R_i$  we need to examine only the subset of active and suspended processes. Now we assume, that the sequence of processes:

$$(3.7) \quad P_{h_i(1)} P_{h_i(2)} \dots P_{h_i(l-1)} P_{h_i(l)} P_{h_i(l+1)} P_{h_i(l+2)} \dots P_{h_i(u)}$$

satisfies:

$$(3.8) \quad \exists_{l \in [1, u]}, \quad y_{ih_i(l+1)} \geq y_{ih_i(l)}.$$

Writing inequalities (2.2) for the elements  $l$  and  $(l+1)$  we have:

$$(3.9) \quad b_{ih_i(l)} - c_{ih_i(l)} \leq r_i + \sum_{k=1}^{l-1} c_{ih_i(k)},$$

$$(3.10) \quad b_{ih_i(l+1)} - c_{ih_i(l+1)} \leq r_i + \sum_{k=1}^l c_{ih_i(k)}.$$

Because condition (3.8) is satisfied, the inequalities (3.9) and (3.10) can be rewritten as below:

$$(3.11) \quad b_{ih_i(l+1)} - c_{ih_i(l+1)} \leq r_i + \sum_{k=1}^{l-1} c_{ih_i(k)},$$

$$(3.12) \quad b_{ih_i(l)} - c_{ih_i(l)} \leq r_i + \sum_{k=1}^{l-1} c_{ih_i(k)} + c_{ih_i(l+1)}.$$

Therefore processes  $P_{h_i(l)}$  and  $P_{h_i(l+1)}$  in the sequence (3.7) can be swapped with the locations of the other processes left unchanged. It is further possible

to swap all other processes in the sequence (3.7) for which the conditions (3.2) and (3.3) are satisfied. Finally it is possible to obtain the sequence with properties (3.2) and (3.3). We can extend this sequence for all system's processes by placing the passive processes at the end of sequence (3.7), so that the system state is safe, from Def. 2.1, with regard to the  $R_i$  resource type.

**THEOREM 3.2:** *The realizable state is local safe state for the  $R_i \in \mathcal{R}$  resource type if the inequality:*

$$(3.13) \quad \mathbf{v}_i \leq \mathbf{q}_i,$$

is satisfied.

In (3.13) we have denoted:

$$(3.14) \quad \mathbf{v}_i^T = [v_{i1}, v_{i2}, \dots, v_{ij}, \dots, v_{ia_{\max}}],$$

$$(3.15) \quad v_{ij} = \sum_{k=j}^{a_i} w_{ik},$$

$$(3.16) \quad \mathbf{w}_i = [w_{i1}, w_{i2}, \dots, w_{ij}, \dots, w_{ia_i}],$$

$w_{ij}$  = the number of  $R_i$  type resources allocated to the process  $P_k \in \mathcal{P}$  with the rank  $y_{ik} = j - 1$ :

$$(3.17) \quad \mathbf{q}_i^T = [q_{i1}, q_{i2}, \dots, q_{ij}, \dots, q_{ia_{\max}}],$$

$$q_{ij} = \begin{cases} a_i - j + 1 & \text{for } j \leq a_i, \\ 0 & \text{for } j > a_i, \end{cases}$$

$$a_{\max} = \max_{a_i} \{ a_1, a_2, \dots, a_i, \dots, a_m \}.$$

*Proof:* From Def. 2.1 we have that the system state is safe, for  $R_i$  type of resource, if there exists the realisation sequence of processes (2.1) satisfying (2.2).

We will divide the set  $\mathcal{P}_1 \subset \mathcal{P}$ , with regard to the resource type  $R_i$ , into subsets:

$$\Delta_1, \Delta_2, \dots, \Delta_k, \dots, \Delta_l, \dots, \Delta_{a_i}$$

satisfying:

$$\forall_{P_j \in \Delta_k}, \quad y_{ij} = k - 1,$$

i. e.: each process  $P_j \in \Delta_k$  needs at most  $(k-1)$  resources of type  $R_i$  to complete.

Let us assume that the above subsets are ordered in the following way:

$$(3.20) \quad \forall_{P_p \in \Delta_k}, \quad \forall_{P_j \in \Delta_l}, \quad y_{ip} \leq y_{ij}$$

The verification if the system state is local safe state is then possible (by means of the Theorem 3.1) by subsequent verification of the following subsets:

$$(3.21) \quad 1) \quad \exists_{P_j \in \Delta_1}, \quad b_{ij} - c_{ij} \leq r_i,$$

$$(3.22) \quad k) \quad \exists_{P_j \in \Delta_k}, \quad b_{ij} - c_{ij} \leq r_i + \sum_{P_p \in \Delta^{k-1}} c_{ip},$$

$$(3.23) \quad l) \quad \exists_{P_j \in \Delta_l}, \quad b_{ij} - c_{ij} \leq r_i + \sum_{P_p \in \Delta^{l-1}} c_{ip},$$

where:

$$\Delta^{k-1} = \Delta_1 \cup \Delta_2 \cup \dots \cup \Delta_{k-1},$$

$$\Delta_{l-1} = \Delta_1 \cup \Delta_2 \cup \dots \cup \Delta_{l-1}.$$

We find that the system state is not local safe state, with regard to the resource number  $R_i$ , when at least one inequality is not satisfied.

Let all processes in the subsets:

$$\Delta_1, \Delta_2, \dots, \Delta_{l-1},$$

satisfy the above inequalities. Now, we are to test subset  $\Delta_l$ , so:

$$(3.24) \quad \forall_{P_j \in \Delta_l}, \quad b_{ij} - c_{ij} \leq r_i + \sum_{P_p \in \Delta^{l-1}} c_{ip}.$$

We may write the right part of the inequality (3.24) in the form:

$$(3.25) \quad r_i + \sum_{P_p \in \Delta^{l-1}} c_{ip} = a_i - \sum_{P_p \in \Delta^{a_i}} c_{ip} + \sum_{P_p \in \Delta^{l-1}} c_{ip} = a_i - \sum_{P_p \in \Delta_l^{a_i}} c_{ip},$$

where:

$$\Delta_l^{a_i} = \Delta_l \cup \Delta_{l+1} \cup \dots \cup \Delta_{a_i}.$$

From (3.19) we have that the number of resources needed by process  $P_j \in \Delta_l$  to complete itself equals  $(l-1)$ . Hence according to (3.24) and (3.25):

$$(3.26) \quad a_i - \sum_{P_p \in \Delta_l^{a_i}} c_{ip} \geq l-1$$



and:

$$(3.27) \quad \sum_{P_p \in \Delta_l^{a_i}} c_{ip} \leq a_i - l + 1.$$

Notice, that if subset  $\Delta_l$  is empty and if all previous subsets satisfy conditions (2.2) then the condition:

$$(3.28) \quad \sum_{P_p \in \Delta_{l-1}^{a_i}} c_{ip} \leq a_i - l.$$

is also satisfied.

From above, we have that condition (3.27) is also satisfied if:

$$(3.29) \quad \sum_{P_p \in \Delta_1} c_{ip} = 0.$$

If condition (3.28) is not satisfied we don't need to verify (3.27) for resource type  $R_i \in \mathcal{R}$ .

Condition (3.27) should be tested for all subsets  $\Delta_l$ , so from (3.14), (3.15), (3.16) we have:

$$(3.30) \quad v_i \leq q_i,$$

where:

$$(3.31) \quad v_{il} = \sum_{P_p \in \Delta_{l-1}^{a_i}} c_{ip},$$

which completes the proof.

EXAMPLE 3.1: We have the system in which:

$$\mathcal{P} = \{P_1, P_2\}, \quad \mathcal{R} = \{R_1, \dots, R_i, \dots, R_m\}$$

and  $a_i = 5$ . Let the elements of matrix  $\mathbf{B}$ , for resource type  $R_i$ , equal:

$$b_{i1} = 4, \quad b_{i2} = 3.$$

In the initial moment to the:

$$c_{i1} = 2, \quad c_{i2} = 2,$$

i. e.:

$$y_{i1} = 3 \quad \text{and} \quad y_{i2} = 1.$$

Vectors  $w_i$ ,  $v_i$  and  $q_i$  are as below:

$$w_i = [0 \quad 2 \quad 0 \quad 1 \quad 0],$$

$$v_i^T = [3 \quad 3 \quad 1 \quad 1 \quad 0],$$

$$q_i^T = [5 \quad 4 \quad 3 \quad 2 \quad 1].$$

Hence, the condition

$$v_i^T \leq q_i^T,$$

is satisfied.

The system state is then the local safe state for resource type  $R_i$ .

#### 4. ONE AND MANY TYPES OF RESOURCES

In the case of the single-resource-type system (for example  $\mathcal{R} = \{R_i\}$ ) the condition of the local safe state is the same as of the system safe state. According to this property we can apply the condition (3.13) from Theorem 3.2 for testing the system safe state.

This type of system is identical with the system in [5].

The results obtained here are more suitable for use because of the way the vector  $w_i$  is being constructed.

In the case of the multiple type resource system the existence of the local safe state sequence for any resource type is not sufficient for the existence of the system safe state [1].

The above conditions can be used if the deadlock prevention method is to be used for individual type of resources.

In order to accomplish it, the following theorem is proved in [1] (Theorem 2).

**THEOREM 3.3:** *The system state is safe if in this state there exist local safe sequences for every resource type  $R_i (i=1, 2, \dots, m)$  and the sequence of processes:*

$$(4.1) \quad P_{f(1)} P_{f(2)} \dots P_{f(n)},$$

satisfying:

$$(4.2) \quad y_{f(1)} \leq y_{f(2)} \leq \dots \leq y_{f(n)}$$

can be found.

It is easy to show that according to the Theorem 3.1 the process sequence (4.1) may consist only of the passive and active processes.

The examination of the system safe state using (3.30) for  $R_i$  ( $i=1, 2, \dots, m$ ) and Theorem 3.3, requires the additional constraints to be imposed on the manner processes receive and release the resources.

These constraints cause that large number of safe states is rejected in the system.

In [1] it is determined that the minimal number of rejected states (35%) is obtained when matrix  $\mathbf{B}$  is satisfying:

$$(4.3) \quad \sigma = 0,$$

where:

$$\sigma = \sum_{i=1}^m \sum_{j=1}^n |\bar{r}_j - b_{ij}|,$$

$$\bar{r}_j = \frac{1}{m} \sum_{i=1}^m b_{ij}.$$

The application of Theorem 3.2 for local safe state examination and prevention of deadlock occurring within different types of resources is expected to give good results.

## 5. CONCLUSIONS

The paper presents a fast deadlock avoidance method. Its computational complexity is proportional only to the number of resource types and number of resource unit of each type.

The results obtained here are similar to ones obtained in [5] with simpler construction of the auxiliary vector.

The utilization of this method for multiple type resources system is also proposed.

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