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1. The purpose of Algebraic Analysis will be to consider concrete phenomenons usually studied by Numerical Analysis and to attack them with the help of Homological Algebra extended in a convenient way. Phenomenons are antagonisms or <u>dialectics</u> and their study is the construction of the calculus describing their rules of modifications or <u>fluxions</u>. In this talk I have expressed how each mathematical theory (and each space) can be described in a precise way as a dialectic (it was the notion of figuration) and I have introduced the calculus of fluxions "along spaces" as an extension of the classical calculus of satellites and derived functors (so, finally, derived functors had been well christened). Of course, in the future it will be necessary to show how the classical differential calculus is embedded in this new kind of analysis.

2. A <u>figuration</u> is a datum $\begin{tikzcd} \underline{F} \arrow[r, bend left, "D"] \arrow[r, bend right, "P"'] \arrow[r, phantom, "\Downarrow u"] & \underline{S} \end{tikzcd}$ where $\underline{F}$ is the category of figures (or ideal datas, or arities, or "points"), $\underline{S}$ is the category of supports (or real world, or

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primitive domains, or "opens"), D is the bimodule of drawings, P is the bimodule of potential drawings and $u\colon D \longrightarrow P$ says that the drawings are potential drawings. Often we start with D and with $L\colon \underline{F} -\!\!\!+\!\!\!\longrightarrow \underline{F}$ a bimodule of composition laws and we take $P = D \boxtimes L$. Then an algebra is a (S,A) with $S \in \underline{S}_{\multimap}$ and A a co-section of

$$\ulcorner S \urcorner^{\circ} \boxtimes u : \ulcorner S \urcorner^{\circ} \boxtimes D \longrightarrow \ulcorner S \urcorner^{\circ} \boxtimes P$$

(where $\ulcorner S \urcorner : \mathbb{1} \longrightarrow \underline{S}$ and $\ulcorner S \urcorner \dashv \ulcorner S \urcorner^{\circ}$ as bimodules).
By analysis of contacts between figures and description of paradoxes it is possible to prove that figurative algebras are the same that models of (mixed) sketches. This imply that we have as examples: models of first order theories, partial and relational algebras, fuzzy algebras, elementary geometries, theories of equations, infinitesimal calculus, etc ...

3. A bimodule or a figuration where $u = Id_D$ (i. e. without laws) is seen as a space, following the fact that an ordinary topological space determines a discrete set $\underline{P}$ of points, an ordered set $\underline{O}$ of opens and a bimodule $E\colon \underline{P} -\!\!\!+\!\!\!\longrightarrow \underline{O}$ given by $E(x,U) = 1$, if $x \in U$, 0 otherwise.
A (bi)presentation of a bimodule (or a space) E is an exact square

$$\begin{tikzcd} \underline{K} \arrow[r, "T"] \arrow[d, "S"'] \arrow[dr, phantom, "\varphi \nearrow"] & \underline{P} \arrow[d, "V"] \\ \underline{O} \arrow[r, "U"'] & \underline{L} \end{tikzcd}$$

(i. e. $S \boxtimes T^{\circ} \overset{\tilde{\varphi}}{\simeq} U^{\circ} \boxtimes V$ with $U \dashv U^{\circ}$ and $T \dashv T^{\circ}$ as bimodules) such that $S \boxtimes T^{\circ} \cong E$.
If a presentation of E and a functor $H\colon \underline{O} \longrightarrow \underline{W}$ are given, then the germ of H along E (or at the points of E) is the extension of H.S along T . In fact it is the satellite $\mathbf{Sat}_{\longrightarrow E}\, H$ of H with respect to E , i. e. their exists an $h\colon H \boxtimes E \longrightarrow \mathrm{Sat}_{\longrightarrow E}\, H$ such that every $f\colon H \boxtimes E \longrightarrow F$, with F a functor, factors uniquely through h .
The computation of n^{th}- satellites by iterations is just to

play dominoes with the exact squares presenting the data.
If $\underline{A}$ is an abelian category then to take the satellite of $H: \underline{A} \longrightarrow \underline{B}$ along $EXT: \underline{A} \relbar\!\!\mapstochar\!\!\longrightarrow \underline{A}$ produces the (right) classical satellites. It is possible to prove syntactically by using convenient bipresentations of EXT and playing at dominoes the long exact sequence for $(EXT^n)_{0 \leq n}$ and the equality between satellites and derived functors.
It is also possible to construct a bimodule of exact squares (as elements) $\underline{CAT} \relbar\!\!\mapstochar\!\!\longrightarrow \underline{CAT}$ which produces an "achieved" cohomology theory for categories.