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PARITY COMPLEXES: CORRIGENDA

by Ross STREET

The purpose of this note is to correct some typographical errors and a mathematical error in my paper "Parity complexes" [St2]. While the errors are not so serious as to affect the paper's *raison d'être,* they could undermine the confidence of diligent readers.

There is an unfortunate typographical mistake in the Definition at the beginning of Section 4 (page 330). The fourth line of the Definition should read:

$$\mu(x)_{n-1} = \mu(x)_n^{\mp}, \qquad \pi(x)_{n-1} = \pi(x)_n^{\pm} \qquad \text{for } p \geq n > 0.$$

The author is very grateful to Dominic Verity for pointing out the mathematical error that Lemma 3.2 (a) is false as stated. The statement is false even for the simplex example[1]. Therefore Lemma 3.2 (a) should be deleted; it concerns a segment property which is not used at all in the proof of parts (b), (c) of Lemma 3.2. The error in the "proof" of Lemma 3.2(a) occurs in the assertion, in the last three lines of page 325, that w lies in $Z^{\pm}$; this comes from an erroneous formula for $Z^{\pm}$ at the top of that page: see the detailed line-by-line corrections below. The only application of Lemma 3.2 (a) in the whole paper is in the proof of Theorem 4.1 at the start of the excision of extremals algorithm (page 330, line 7 of the Algorithm). There the segment property is only required for sets of the form $\mu(u)$. This can be repaired by adding the further hypothesis for the whole of Section 4 that each of the sets $\mu(x)$ should be *tight* in the following sense.

A subset R of C is called *tight* when $u \triangleleft v$ and $v \in R$ imply $u^- \cap R^{\pm} = \emptyset$. For example, x^+ is tight by Proposition 1.2. Then we have the following strengthening of Proposition 1.3.

Proposition 1.4 *If* R *is tight,* S *is well formed, and* $R \subset S \subset C$ *then* R *is a segment of* S.

Proof Suppose $w < u \triangleleft_S v$ with $w, v \in R$, so that there is $y \in w^+ \cap u^-$. Since R is tight, $u^- \cap R^{\pm} = \emptyset$. So $y \notin R^{\pm}$. Now $y \in w^+ \subset R^+$. So $y \in R^-$. So $y \in z^-$ where $z \in R$. So $y \in u^- \cap z^-$. But $u, z \in S$ well formed. So $u = z \in R$. qed

It remains to see how this tightness assumption carries over to products and joins of parity complexes. Recall that, for a relevant element x (see the

[1] So that Lemma 3.13 of [St1] is also false as stated. Of course, the correction given herein will repair the simplex case too.

Definition on page 331), the following equations hold:

$$\mu(x)_{n-1} = \pi(x)_n^{\mp}, \quad \pi(x)_{n-1} = \mu(x)_n^{\pm} \quad \text{for } 0 < n < p-1.$$

These last equations are interesting in their own right and will be called *the globularity condition.*

Proposition 5.2 *If the globularity condition holds and the relation* $\blacktriangleleft$ *is antisymmetric then each* $\mu(x)$ *is tight.*

Proof Suppose $u \lhd v$, $v \in \mu(x)_n$ and $y \in u^- \cap \pi(x)_{n-1}$. Then $y < u \lhd v \blacktriangleleft x \blacktriangleleft y$, contrary to antisymmetry.$_{\text{qed}}$

The next two Propositions follow from the formulas for μ and π in a product (see the Remark in the middle of page 334) and the corresponding formulas in a join.

Proposition 5.5 *The globularity condition is inherited by products.*

Proposition 5.8 *The globularity condition is inherited by joins.*

Much light is thrown on the material of [St2] Section 6 by the recent work of Verity [V] which has, as its principal concern, a proof of Conjecture 5.3 of [St1]. In particular, he makes explicit the canonical co-ω-structure of [St2] Proposition 6.2.

The reader should compare parity complexes with Steiner's directed complexes [Sn] which are designed to cover the same basic examples from a more local approach. The various descriptions of the free n-categories in approaches of Michael Johnson, John Power, Richard Steiner and the author will surely be of value to future workers.

Line-by-line Changes to [St2]

Page 321 Replace Proposition 1.3 and its Proof by the definition of *tight*, and Proposition 1.4 and its Proof (see above).

Page 324 Delete statement (a) of Lemma 3.2. In the fifth last line of the page, delete "(a) *and*". In the second last line, delete "and x^+ is well formed (Axiom 2)".

Page 325 Line 2 should be replaced by

$$Z^{\pm} = (X^{\pm} \cap \neg x^+) \cup (X^+ \cap x^-) \subset (M_n \cup x^-) \cap \neg x^+ = N_n .$$

In line 3, delete "$Z^{\pm}$ is a segment of N_n,". Delete the last 12 lines of the page.

Page 326 Delete the first 4 lines of the page. On lines 20, 23, 26, delete the clauses containing the words "is a segment of".

Page 330 As mentioned above, in line 13, interchange the superscripts $^{\mp}$ and $^{\pm}$. After the Definition, add the blanket hypothesis for Section 4 that *each of the sets* $\mu(x)$ *is tight.* In line 7 of the Algorithm, replace "By Lemma 3.2 (a)" by "By our blanket hypothesis for this Section and Proposition 1.4,".

Page 331 The Definition should include naming the *globularity conditions* as described above.

Page 334 Proposition 5.2 should be inserted just before the last two lines of the page.

Page 335 Proposition 5.5 (above) should be included after the Proof of Theorem 5.4.

Page 336 Proposition 5.8 (above) should be included just before Section 6.

References

[Sn] R. Steiner, The algebra of directed complexes, *Applied Categorical Structures* 1 (1993) 247-284.

[St1] Ross Street, The algebra of oriented simplexes, *J. Pure Appl. Algebra* 49 (1987) 283-335.

[St2] Ross Street, Parity complexes, *Cahiers de topologie et géométrie différentielle catégoriques* 32 (4) (1991) 315-343.

[V] Dominic Verity, Higher dimensional algebra via simplicial sets with structure (in preparation).

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