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## ACCESSIBLE EMBEDDINGS AND THE SOLUTION-SET CONDITION

by H. HU and M. MAKKAJ

Dedicated to the memory of Jan Reiterman

Résumé. Pour chaque catégorie localement présentable, on démontre qu'une sous-catégorie accessiblement plongée d'une catégorie localement présentable est accessible si et seulement si elle satisfait la condition de l'ensemble-solution.

### Introduction

The connection between accessibility and the solution set condition has been pointed out in several papers, including [7] and [9]. J. Adámek and J. Rosický have recently proved that each locally presentable category has the following property: a full subcategory closed under products and  $\kappa$ -filtered limits (for some infinite regular cardinal  $\kappa$ ) is accessible if the embedding satisfies the solution set condition, see [3]. It is well-known that a category is locally presentable iff it is accessible and complete, or equivalently cocomplete (cf. [7]), but no assumption is made of the existence of any particular limits for accessible categories. One is impelled to ask: does the above mentioned result hold for any full subcategory closed under  $\kappa$ -filtered colimits? In this paper we provide an affirmative answer to this question. We will show that, for any locally presentable category  $\mathbf{B}$ , a full subcategory  $\mathbf{A}$  of  $\mathbf{B}$  closed under  $\kappa$ -filtered colimits is accessible iff the embedding satisfies the solution set condition. The proof of this result is a modification of the proof of the above mentioned result of J. Adámek and J. Rosický in [3].

### 1 Generalities on Accessible Categories

Let  $\kappa$  be an infinite regular cardinal. Recall that a category  $\mathbf{A}$  is  $\kappa$ -filtered if for any graph  $\mathbf{G}$  of cardinality less than  $\kappa$ , any diagram  $D : \mathbf{G} \rightarrow \mathbf{A}$  has a cocone on it.  $\mathbf{A}$  has  $\kappa$ -filtered colimits, if  $\mathbf{A}$  has colimits of all diagrams whose domain is a  $\kappa$ -filtered category. Another concept is limit of  $\kappa$ -diagram, where a  $\kappa$ -diagram is a diagram whose domain category is of size less than  $\kappa$ .

An object  $A$  of a category  $\mathbf{A}$  is said to be  $\kappa$ -presentable if the representable functor  $\mathbf{A}(A, -)$  preserves  $\kappa$ -filtered colimits existing in  $\mathbf{A}$ . The full subcategory of  $\mathbf{A}$  whose objects are the  $\kappa$ -presentable ones is denoted by  $\mathbf{A}_\kappa$ . Note that for any category, a  $\kappa$ -colimit of a diagram in which the objects are  $\kappa$ -presentable is  $\kappa$ -presentable itself (see [5] and [7]).

**Definition 1.1** ([7]) *A category  $\mathbf{A}$  is  $\kappa$ -accessible if:*

- (i)  $\mathbf{A}$  has  $\kappa$ -filtered colimits;
  - (ii) *There is a small full subcategory  $\mathbf{C}$  of  $\mathbf{A}_\kappa$  such that every object of  $\mathbf{A}$  is a  $\kappa$ -filtered colimit of a diagram of objects in  $\mathbf{C}$ .*
- A category is accessible if it is  $\kappa$ -accessible for some infinite regular cardinal  $\kappa$ .*

Recall from [4] that a category is called locally  $\kappa$ -presentable if it is locally small, cocomplete, and has a small strong generator consisting of  $\kappa$ -presentable objects. A theorem (Theorem 6.1.4. in [7]) says that an accessible category is complete iff it is cocomplete. P. Gabriel and F. Ulmer have shown in [4] that a category  $\mathbf{B}$  is locally  $\kappa$ -presentable if it is equivalent to the category of the form  $L_\kappa(\mathbf{C}, \mathbf{Set})$ , the category of all functors preserving  $\kappa$ -limits; here  $\mathbf{C}$  is a small category with  $\kappa$ -limits, and  $\mathbf{Set}$  denotes the category of small sets.

Let  $\mathbf{B}$  be a  $\kappa$ -accessible category, and  $\mathbf{A}$  a full subcategory of  $\mathbf{B}$ .  $\mathbf{A}$  is said to be  $\kappa$ -accessibly embedded if  $\mathbf{A}$  is closed under  $\kappa$ -filtered colimits in  $\mathbf{B}$ .  $\mathbf{A}$  is accessibly embedded if it is  $\lambda$ -accessibly embedded for some regular cardinal  $\lambda$  with  $\lambda \geq \kappa$ .

Recall from [3] that a full subcategory  $\mathbf{A}$  of  $\mathbf{B}$  is said to be cone-reflective if the inclusion functor  $\mathbf{A} \rightarrow \mathbf{B}$  satisfies the solution set condition, i.e., for each object  $B$  of  $\mathbf{B}$  there exists a small cone  $\langle r_i : B \rightarrow A_i \rangle_{i \in I}$  with  $A_i \in \mathbf{A}$  such that for any  $A \in \mathbf{A}$ , every morphism  $B \rightarrow A$  factors through some  $r_i$ . Let  $\mathbf{D}$  be a set of objects of  $\mathbf{A}$ . We say that  $\mathbf{D}$  weakly reflects  $B$  (in  $\mathbf{A}$ ) if for every  $A \in \mathbf{A}$  and  $f : B \rightarrow A$  there is  $D \in \mathbf{D}$  and a factorization

$$\begin{array}{ccc}
 B & \xrightarrow{m} & D \\
 \downarrow f & & \swarrow f' \\
 & & A
 \end{array}$$

where  $m$  and  $f'$  are some morphisms. Note that  $\mathbf{A}$  is cone-reflective in  $\mathbf{B}$  iff for every  $B \in \mathbf{B}$ , there is a small set  $\mathbf{D} \subset \mathbf{A}$  weakly reflecting  $B$ . If  $\mathbf{B}$  is accessible, then this is equivalent to saying that for every  $B \in \mathbf{B}$  there is  $\kappa$  such that  $\mathbf{D}_\kappa = \mathbf{A} \cap \mathbf{B}_\kappa$  weakly reflects  $B$ . Note that, of course, if  $\kappa < \kappa'$  and  $\mathbf{D}_\kappa$  weakly reflects  $B$ , so does  $\mathbf{D}_{\kappa'}$ .

The following lemma can be found in [7] (Lemma 1.1.2.).

**Lemma 1.2** *Suppose that  $\mathbf{J}$  is  $\kappa$ -filtered and the functor  $F : \mathbf{I} \rightarrow \mathbf{J}$  satisfies that for every  $J \in \mathbf{J}$ , there exists  $I$  in  $\mathbf{I}$  and a morphism  $J \rightarrow F(I)$ . If  $F$  is full and faithful, then  $\mathbf{I}$  is  $\kappa$ -filtered and  $F$  is final, i.e., for any diagram  $\Sigma : \mathbf{J} \rightarrow \mathbf{A}$ ,  $\text{colim}\Sigma$  exists if and only if  $\text{colim}\Sigma \circ F$  exists and the canonical morphism  $\text{colim}\Sigma(F) \rightarrow \text{colim}\Sigma$  is an isomorphism.*

**Proposition 1.3** *Let  $\mathbf{B}$  be a  $\kappa$ -accessible category, and  $\mathbf{A}$  a  $\kappa$ -accessibly embedded subcategory of  $\mathbf{B}$ . If every  $B \in \mathbf{B}_\kappa$  is weakly reflected in  $\mathbf{A}$  by  $\mathbf{D} = \mathbf{A} \cap \mathbf{B}_\kappa$ , then  $\mathbf{A}$  is  $\kappa$ -accessible.*

**Proof:**  $\mathbf{A}$  has  $\kappa$ -filtered colimits, by assumption. For any  $A \in \mathbf{A}$ , we have a canonical diagram  $G : \mathbf{B}_\kappa/A \rightarrow \mathbf{B}$ , and  $A = \text{colim}G$ , by Proposition 2.1.5. in [7]. This colimit is  $\kappa$ -filtered. Let  $\mathbf{D} = \mathbf{A} \cap \mathbf{B}_\kappa$ . Since  $\mathbf{A}$  is closed under  $\kappa$ -filtered colimits in  $\mathbf{B}$ , all objects in  $\mathbf{D}$  are  $\kappa$ -presentable in  $\mathbf{A}$ . We have a full and faithful functor  $F : \mathbf{D}/A \rightarrow \mathbf{B}_\kappa/A$ . Let  $G' : \mathbf{D}/A \rightarrow \mathbf{B}$  be the canonical diagram. Given an object  $f : B \rightarrow A$  in  $\mathbf{B}_\kappa/A$ , by assumption, there is a factorization  $f = f' \circ m$ , with  $f' : D \rightarrow A$  in  $\mathbf{D}/A$ . By Lemma 1.2,  $A$  is the  $\kappa$ -filtered colimit  $\text{colim}G'$  in  $\mathbf{B}$ , and as a consequence, also in  $\mathbf{A}$ . Thus,  $\mathbf{A}$  is  $\kappa$ -accessible.

## 2 Main Theorem and Some Remarks

In what follows,  $\kappa$ ,  $\lambda$  and subscripted variants of them always denote infinite cardinals.

The proof of the following theorem follows closely the lines of the corresponding proof in [3].

**Theorem 2.1** *Let  $\mathbf{B}$  be a locally presentable category, and  $\mathbf{A}$  an accessibly embedded subcategory of  $\mathbf{B}$ . If  $\mathbf{A}$  is cone-reflective, then it is accessible.*

**Proof:** We may assume that  $\mathbf{B}$  is a functor category  $(\mathbf{C}, \mathbf{Set})$ , for some small category  $\mathbf{C}$ . The reason is that every locally presentable category is a reflective subcategory of a functor category  $(\mathbf{C}, \mathbf{Set})$  with  $\mathbf{C}$  small, and the inclusion functor is accessibly embedded. If  $\lambda$  is a regular cardinal bigger than the cardinality of  $\mathbf{C}$  and  $\aleph_0$ , then a functor  $F \in \mathbf{B}$  is  $\lambda$ -presentable in  $\mathbf{B}$  iff the cardinality of  $\coprod_{C \in \mathbf{C}} F(C)$  is less than  $\lambda$ . It easily follows that if  $\mu = \sup_{i < \nu} \kappa_i$  with  $\kappa_i \leq \kappa_j$  for  $i < j < \nu$ , and  $B \in \mathbf{B}_{\mu+}$ , then we can write  $B$  as a colimit of a  $\nu$ -chain,  $B = \text{colim}_{i < \nu} B_i$ , with  $B_i \in \mathbf{B}_{\kappa_i}$ .

Let  $\kappa$  be a regular cardinal such that  $\mathbf{A}$  is closed under  $\kappa$ -filtered colimits in  $\mathbf{B}$ . Let us define  $\kappa_i$  for  $i < \kappa$  by transfinite induction. Let  $\kappa_0 = \kappa$ . Given  $0 < i < \kappa$ , having defined  $\kappa_j$  for all  $j < i$ , for  $i$  limit, let  $\kappa_i$  be a regular cardinal bigger than  $\kappa_j$

for all  $j < i$ , and for  $i = j + 1 < \kappa$ , let  $\kappa_{j+1}$  be a regular cardinal  $\geq \kappa_j$  such that all objects in  $\mathbf{B}_{\kappa_j}$  have a weak reflection in  $\mathbf{D}_{\kappa_{j+1}}$ ; since each  $\mathbf{B}_{\kappa'}$  is small, and since every  $B \in \mathbf{B}$  has a weak reflection in  $\mathbf{D}_{\kappa'}$ , for some  $\kappa'$ , such  $\kappa_{j+1}$  clearly exists.

Let  $\mu = \sup_{i < \kappa} \kappa_i$ , and  $\lambda = \mu^+$ . We claim that every  $B \in \mathbf{B}_\lambda$  has a weak reflection in  $\mathbf{D}_\lambda$ . Since  $\mathbf{B}$  is clearly  $\kappa'$ -accessible for all  $\kappa' \geq \aleph_0$ , in particular, for  $\lambda = \kappa'$ , and  $\lambda > \kappa$ , by Proposition 1.3, this will suffice for the proof of the theorem.

Let  $B \in \mathbf{B}_\lambda$ . As above, let us represent  $B$  as the colimit of a  $\kappa$ -chain  $(b_{i,j} : B_i \rightarrow B_j)_{i < j < \kappa}$ , with  $B_i \in \mathbf{B}_{\kappa_i}$ . Let  $\phi_i : B_i \rightarrow B$  be the colimit coprojection. Let  $A \in \mathbf{A}$  and  $f : B \rightarrow A$  be arbitrary; we want to find  $A^* \in \mathbf{D}_\lambda$  with a factorization

$$\begin{array}{ccc}
 B & \xrightarrow{f^*} & A^* \\
 \downarrow f & \searrow a^* & \\
 A & & 
 \end{array}$$

By transfinite induction on  $i < \kappa$ , we will define objects  $A_i \in \mathbf{D}_{\kappa_{i+1}}$  and morphisms  $a_{i,j} : A_i \rightarrow A_j$ ,  $f_i : B_i \rightarrow A_i$ ,  $\psi_i : A_i \rightarrow A$  such that the morphisms  $a_{i,j}$  form a chain, the morphisms  $\psi_i$  form a compatible cocone, and the following diagram

$$\begin{array}{ccccc}
 B_i & \xrightarrow{b_{i,j}} & B_j & \xrightarrow{\phi_j} & B \\
 \downarrow f_i & & \downarrow f_j & & \downarrow f \\
 A_i & \xrightarrow{a_{i,j}} & A_j & \xrightarrow{\psi_i} & A
 \end{array}$$

commutes.

For  $i = 0$ , we let  $A_0 \in \mathbf{D}_{\kappa_1}$ ,  $f_0 : B_0 \rightarrow A_0$  and  $\psi_0 : A_0 \rightarrow A$  such that

$$\begin{array}{ccc}
 B_0 & \xrightarrow{\phi_0} & B \\
 \downarrow f_0 & & \downarrow f \\
 A_0 & \xrightarrow{\psi_0} & A
 \end{array}$$

commutes; these items are obtained from a suitable factorization of the morphism  $f \circ \phi_0 : B_0 \rightarrow A$ , possible by the choice of  $\kappa_1$  and  $B_0 \in \mathbf{D}_{\kappa_0}$ .

Fix  $k$ ,  $0 < k < \kappa$ , and assume that all items with indices  $< k$  have been defined. Let  $C = \text{colim}(a_{i,j} : A_i \rightarrow A_j)_{i < j < k}$  with coprojections  $a_i : A_i \rightarrow C$ , and  $B^* = \text{colim}(b_{i,j} : B_i \rightarrow B_j)_{i < j < k}$  with coprojections  $b_i^* : B_i \rightarrow B^*$ .

Since  $A_i \in \mathbf{B}_{\kappa_i} \subset \mathbf{B}_{\kappa_k}$ , and  $\mathbf{B}_{\kappa_k}$  is closed under  $< \kappa \leq \kappa_k$ -sized colimits,  $C \in \mathbf{B}_{\kappa_k}$ . Similarly,  $B_i \in \mathbf{B}_{\kappa_{i+1}} \subset \mathbf{B}_{\kappa_k}$ , and so  $B^* \in \mathbf{B}_{\kappa_k}$ .

By the universal property of  $B^*$ , we have  $b^* : B^* \rightarrow B_k$  such that

$$\begin{array}{ccc} B_i & \xrightarrow{b_i^*} & B^* \\ \downarrow b_{i,k} & \searrow b^* & \\ B_k & & \end{array}$$

commute, and  $c : B^* \rightarrow C$  such that

$$\begin{array}{ccc} B_i & \xrightarrow{b_i^*} & B^* \\ \downarrow f_i & & \downarrow c \\ A_i & \xrightarrow{a_i} & C \end{array}$$

commute, for all  $i < k$ .

By the universal property of  $C$ , we have  $a : C \rightarrow A$  such that

$$\begin{array}{ccc} A_i & \xrightarrow{a_i} & C \\ \downarrow \psi_i & \searrow a & \\ A & & \end{array}$$

commute for all  $i < k$ . We form the pushout of  $c$  and  $b^*$ :

$$\begin{array}{ccc}
 B^* & \xrightarrow{b^*} & B_k \\
 \downarrow c & & \downarrow h \\
 C & \xrightarrow{g} & D
 \end{array}$$

Since  $B^*, B_k, C \in \mathbf{B}_{\kappa_k}$ , we have  $D \in \mathbf{B}_{\kappa_k}$ . We prove that  $a \circ c = f \circ \phi_k \circ b^*$  by showing that

$$\begin{aligned}
 a \circ c \circ b_i^* &= a \circ a_i \circ f_i \\
 &= \psi_i \circ f_i \\
 &= f \circ \phi_i \\
 &= f \circ \phi_k \circ b_i^* \circ b_i^*
 \end{aligned}$$

for each projection  $b_i^*$ . By using the universal property of pushout  $D$ , we have a unique morphism  $l : D \rightarrow A$  such that  $a = l \circ g$  and  $f \circ \phi_k = l \circ h$ . Since  $D \in \mathbf{B}_{\kappa_k}$ , and every object in  $\mathbf{B}_{\kappa_k}$  is weakly reflected by  $\mathbf{D}_{\kappa_{k+1}}$ , there is  $A_k \in \mathbf{D}_{\kappa_{k+1}}$  with  $\psi_k : A_k \rightarrow A$  such that the diagram

$$\begin{array}{ccc}
 D & \xrightarrow{m} & A_k \\
 \downarrow l & \searrow \psi_k & \\
 A & & 
 \end{array}$$

commutes. We have defined the items  $A_k$  and  $\psi_k$ .

Next, we define  $f_k = m \circ h : B_k \rightarrow A_k$  and  $a_{i,k} = m \circ g \circ a_i : A_i \rightarrow A_k$ . Note that the diagrams

$$\begin{array}{ccccc}
 B_i & \xrightarrow{b_i^*} & B^* & \xrightarrow{b^*} & B_k \\
 \downarrow f_i & & \downarrow c & & \downarrow h \\
 A_i & \xrightarrow{a_i} & C & \xrightarrow{g} & D
 \end{array}$$

commute for all  $i < k$ ; and  $b_{i,k} = b^* \circ b_i^*$ . Then the diagrams

$$\begin{array}{ccc}
 B_i & \xrightarrow{b_{i,k}} & B_k \\
 \downarrow f_i & & \downarrow f_k \\
 A_i & \xrightarrow{a_{i,k}} & A_k
 \end{array}$$

commute for all  $i < k$ , and the diagram

$$\begin{array}{ccc}
 B_k & \xrightarrow{\phi_k} & B \\
 \downarrow f_k & & \downarrow f \\
 A_k & \xrightarrow{\psi_k} & A
 \end{array}$$

commutes. It is clear that  $\psi_i = a_{i,k} \circ \psi_k$  and  $a_{i,k} = a_{i,j} \circ a_{j,k}$  hold for all  $i < j < k$ . This completes the construction.

Put  $A^* = \text{colim}(a_{i,j} : A_i \rightarrow A_j)_{i < j < \kappa}$  with coprojections  $p_i : A_i \rightarrow A^*$ . Since  $\mathbf{A}$  is closed under  $\kappa$ -filtered colimits in  $\mathbf{B}$ ,  $A^* \in \mathbf{A}$ . Also, since  $A_i \in \mathbf{B}_{\kappa_{i+1}} \subset \mathbf{B}_\lambda$  and  $\kappa < \lambda$ , we have that  $A^* \in \mathbf{B}_\lambda$ ; that is,  $A^* \in \mathbf{D}_\lambda$ . By the construction above, we have  $f^* : B \rightarrow A^*$  such that the diagrams

$$\begin{array}{ccc}
 B_i & \xrightarrow{\phi_i} & B \\
 f_i \downarrow & & \downarrow f^* \\
 A_i & \xrightarrow{p_i} & A^*
 \end{array}$$

commute for all  $i < \kappa$ ; also, we have  $a^* : A^* \rightarrow A$  such that the diagrams

$$\begin{array}{ccc}
 A_i & \xrightarrow{p_i} & A^* \\
 \psi_i \downarrow & \searrow a^* & \\
 A & & 
 \end{array}$$

commute for all  $i < \kappa$ ; hence we have that

$$f \circ \phi_i = \psi_i \circ f_i = a^* \circ p_i \circ f_i = a^* \circ f^* \circ \phi_i$$

for all  $i < \kappa$ . Since  $\langle \phi_i \rangle_{i < \kappa}$  is a colimit cocone, we conclude that the diagram

$$\begin{array}{ccc}
 B & \xrightarrow{f^*} & A^* \\
 f \downarrow & \searrow a^* & \\
 A & & 
 \end{array}$$

commutes. This completes the proof.

**Definition 2.2** For any locally presentable category  $\mathbf{B}$ , an accessibly embedded subcategory  $\mathbf{A}$  of  $\mathbf{B}$  is accessible iff  $\mathbf{A}$  is cone-reflective.

**Proof:** By Theorem 2.1 and Proposition 2.1.8. in [7].

**Remark 2.3** *Let  $\mathbf{A}$  be an accessibly embedded subcategory of  $\mathbf{B}$  closed under limits. J. Adámek and J. Rosický have shown in [2] that  $\mathbf{A}$  is a reflective subcategory of  $\mathbf{B}$ . Thus  $\mathbf{A}$  is locally presentable.*

**Remark 2.4** *Recall from [1] that Vopěnka's principle is the following statement: the category  $\mathbf{Gra}$  of graphs does not have a large discrete full subcategory. It has been shown in [8] that Vopěnka's principle is equivalent to the following statement: every accessibly embedded subcategory of a locally presentable category is accessible. Thus, assuming Vopěnka's principle, every accessible embedding of a locally presentable category satisfies the solution set condition.*

**Remark 2.5** *Let  $\mathbf{A}$  be an accessible full subcategory of an accessible category  $\mathbf{B}$ . Suppose that the inclusion functor from  $\mathbf{A}$  to  $\mathbf{B}$  satisfies the solution-set condition. J. Rosický and W. Tholen have recently proved that the inclusion functor is accessible (see Theorem 3.10 in [9]). Also, they have proved in [9] that Vopěnka's principle is equivalent to the the following statement: a functor between accessible categories is accessible if and only if it satisfies the solution-set condition.*

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