CAHIERS DE
TOPOLOGIE ET GÉOMÉTRIE DIFFÉRENTIELLE
CATÉGORIQUES

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Cahiers de topologie et géométrie différentielle catégoriques, tome 34, nº 1 (1993), p. 2-12

<http://www.numdam.org/item?id=CTGDC_1993__34_1_2_0>

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VOL. XXXIV-1 (1993)

A GRAPH STRUCTURE OVER THE CATEGORY OF SETS AND PARTIAL FUNCTIONS

by Yoshihiro MIZOGUCHI

Résumé. En 1984 Raoult a proposé une formulation des réécritures des graphes utilisant des sommes fibrées dans la catégorie des graphes et des fonctions partielles. Cette note généralise sa méthode et propose une structure algébrique de graphe pour introduire un cadre plus général pour les réécritures de graphes et donner une démonstration simple du théorème d'existence de sommes fibrées à l'aide du calcul relationnel.

1 Introduction

There are many researches about graph grammars and graph rewritings using the category theory. The structure of a directed graph is a function from the set E of edges to the product set $V \times V$ of the source vertices set and destination vertices set. Ehrig[4] characterized the graph grammar and rewriting rules using two pushout squares and pushout complements in the category of graphs. As the category of graphs is considered as a functor category over the category of sets and functions, it becomes a topos and has various useful properties. The existence theorem of pushout complements in a topos including the category of graph was generally proved by Kawahara[7].

Löwe and Ehrig[3, 11] also formulated graph rewritings using a single pushout in the category of graphs based on sets and partial functions.

Raoult and Kennaway's approach[13, 9] was different from Ehrig's formalization of graph rewritings. The graph structure in [13] is a function from a vertex set V to the set V^* of finite strings of vertices and Raoult defined graph rewritings by a single pushout square using partial functions.

This paper generalizes Raoult's method. For an endofunctor on the category **Pfn** of sets and partial functions we consider a graph structure as a function $V \to TV$ from a vertex set V to TV, where T is an endofunctor on **Pfn**. That is, we treat the coalgebras over **Pfn**. In the case $TV = V^*$ the structure is the same as Raoult's one. We prove an existence theorem of pushouts in our general settings avoiding many kind of conditional checks and case divisions by using the relational calculus. The relational calculus, a theory of binary relations, has been originally applied to the area of topology and homological algebra[1, 2, 5]. Recently it has been used in the area of computer science for representing the notion of nondeterminism in automata theory[6], the theory of assertion semantics of programs[8] and characterization of pushouts in the theory of graph grammars[7].

Our main result on the existence theorem of pushouts produces a modification of Raoult's result [13, Proposition 5] which lacked a condition. A counter example to his result is given.

2 Preliminary

In this section, we recall some relational notations and properties of the category **Pfn** of sets and partial functions.

Let A, B and C be sets. When α is a subset of $A \times B$, we call α a *relation* from A to B and denote it by $\alpha : A \rightharpoondown B$. For relations $\alpha : A \rightharpoondown B$ and $\beta : B \rightharpoondown C$, we define a *composite relation* $\alpha\beta : A \rightharpoondown C$ by $\alpha\beta = \{(a,c) \in A \times C | (a,b) \in \alpha, (b,c) \in \beta \text{ for some } b \in B\}$. For relation $\alpha : A \rightharpoondown B$, we define the *inverse relation* $\alpha^\sharp : B \rightharpoondown A$ by $\alpha^\sharp = \{(b,a) \in B \times A | (a,b) \in \alpha\}$. We identify a function $f : A \to B$ with a relation $\{(a, f(a)) \in A \times B | a \in A\}$ (the graph of f). The unique function from a set X to one point set $1 = \{*\}$ is denoted by $\Omega_X : X \to 1$. We define a subset $\mathrm{dom}(\alpha)$ of A for a relation $\alpha : A \rightharpoondown B$ by $\mathrm{dom}(\alpha) = \{a \in A | (a,*) \in \alpha\Omega_A\}$ and a relation $\mathrm{d}(\alpha) : A \rightharpoondown A$ by $\mathrm{d}(f) = \{(a,a) \in A \times A | a \in \mathrm{dom}(\alpha)\}$. For two relations $\alpha, \beta : A \rightharpoondown B$, $\alpha \cup \beta$ and $\alpha \cap \beta$ denote the set union and intersection, respectively.

A *partial function* is a relation $f : A \rightharpoondown B$ satisfying $f^\sharp f \subset \mathrm{id}_B$ and denote it by $f : A \to B$, where $\mathrm{id}_B : B \to B$ is a identity function of B. A partial function $f : A \to B$ is a *(total) function* if it satisfies $\mathrm{id}_A \subset f f^\sharp$.

Lemma 2.1 *Let $\alpha, \alpha' : A \rightharpoondown B$, and $\beta, \beta' : B \rightharpoondown C$ be relations.*

(1) *If $\alpha \subset \alpha'$ and $\beta \subset \beta'$, then $\alpha\beta \subset \alpha'\beta'$.*

(2) *If $\alpha \subset \alpha'$, then $\alpha^\sharp \subset \alpha'^\sharp$.*

(3) *$\alpha(\beta \cup \beta') = \alpha\beta \cup \alpha\beta'$ and $(\alpha \cup \alpha')\beta = \alpha\beta \cup \alpha'\beta$.*

(4) *$\mathrm{d}(\alpha) \subset \mathrm{d}(\alpha')$ iff $\mathrm{dom}(\alpha) \subset \mathrm{dom}(\alpha')$ iff $\alpha\Omega_B \subset \alpha'\Omega_B$.*

Lemma 2.2 *Let $f : A \to B$, $g : A \to B$, $h : B \to C$ be partial functions.*

(1) *f is a total function if and only if $f\Omega_B = \Omega_A$.*

(2) *If $f \subset g$ and $f\Omega_B = g\Omega_B$ then $f = g$.*

(3) *$f\mathrm{d}(h) = \mathrm{d}(fh)f$.*

Proposition 2.3 (Law of Puppe-Calenko) *If $\alpha : A \rightharpoondown B$, $\beta : B \rightharpoondown C$ and $\gamma : A \rightharpoondown C$ are relations, then $\alpha\beta \cap \gamma \subset \alpha(\beta \cap \alpha^\sharp\gamma)$.*

Fact 2.4 *The category* **Pfn** *has pushouts.*
Let

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[d, "g"'] & B \arrow[d, "h"] \\ C \arrow[r, "k"'] & D \end{tikzcd}$$

be a pushout square in **Pfn**. *For any functions $x : B \to S$, $y : C \to S$ satisfying $fx = gy$, there exists a unique function $t : D \to S$ such that $ht = x$ and $ht = y$, where $t = h^\sharp x \cup k^\sharp y$.*

Lemma 2.5 *For the pushout square in Fact 2.4,*

(1) $h^\sharp h \cup k^\sharp k = id_D$,

(2) $(B - f(A)) \subset \mathrm{dom}(h)$ *and* $(C - g(A)) \subset \mathrm{dom}(k)$.

3 Graphs over Pfn

In this section, we introduce an abstract definition of a category which represents graphs and graph homomorphisms. Graph rewritings are defined by using a single pushout in the category. We prove a necessary and sufficient condition for existence of pushouts. Some concrete categories of graphs including Raoult's[13] definition are shown.

Let T : **Pfn** $\to$ **Pfn** be an endofunctor. A *graph* constructed by T is a pair (A, a) of a set A and a total function $a : A \to TA$. A *graph morphism* $f : (A, a) \to (B, b)$ is a partial function $f : A \to B$ satisfying $fb = \mathrm{d}(f)aTf$. The *graph category* $G(T)$ is the category of graphs and graph morphisms associated with T.

Lemma 3.1 *Let T : **Pfn** $\to$ **Pfn** be a functor, $a : A \to TA$, $b : B \to TB$ and $c : C \to TC$ total functions and $f : A \to B$ and $g : B \to C$ partial functions. If $fb' = \mathrm{d}(f)aTf$ and $gc = \mathrm{d}(g)bTg$ then $fgc = \mathrm{d}(fg)aT(fg)$. That is $G(T)$ is in fact a category.*

Proof. It follows from a simple relational computation:

$$\begin{aligned} fgc &= f\mathrm{d}(g)bTg \\ &= \mathrm{d}(fg)fbTg \qquad \text{(Lemma 2.2)} \\ &= \mathrm{d}(fg)\mathrm{d}(f)aTfTg \qquad (fb = \mathrm{d}(f)aTf) \\ &= \mathrm{d}(fg)aT(fg) \qquad (\mathrm{d}(fg)\mathrm{d}(f) = \mathrm{d}(fg)). \end{aligned}$$

■

Choosing a suitable functor T, we consider the several kinds of graph structures.

Example 3.2 (Kleene functor) We define the Kleene functor $*$: **Pfn** $\to$ **Pfn** as follows. Let A, B be sets and $f : A \to B$ a partial function. $*A = A^*$ is the set of finite strings over A. We define $*f(= f^*) : A^* \to B^*$ as follows:

$$\begin{aligned} f^*(w) &= f(a_1)f(a_2)\cdots f(a_n) \qquad (w = a_1a_2\cdots a_n \in (\mathrm{dom}(f))^*), \text{ and} \\ f^*(\varepsilon) &= \varepsilon. \end{aligned}$$

An object of $G(*)$ can be seen as a kind of directed graph and a morphism of $G(*)$ is a node-mapping which preserves out-edges but not in-edges. The category $G(*)$ is equivalent to what is considered by Raoult[13].

Example 3.3 (Powerset functor) We define the power set functor P : **Pfn** $\to$ **Pfn** as follows. Let A, B be sets and $f : A \to B$ a partial function. $P(A)$ is the set of all subsets of A and $Pf : P(A) \to P(B)$ is defined by $Pf(X) = f(X)$, for all $X \subset A$. An object of $G(P)$ is a kind of directed graph in which the out-edges of a node are not orderd and there cannot be multiple edges between the same nodes.

Example 3.4 A set N^A of functions from A to the set $N = \{0, 1, \ldots\}$ of natural numbers is defined by $N^A = \{f : A \to N | \Sigma_{x \in A} f(a)$ is finite.$\}$. We define the functor W : **Set** $\to$ **Set** as follows. Let A, B be sets and $f : A \to B$ a partial function. $W(A) = N^A$ and $Wf : N^A \to N^B$ is defined as $Wf(\alpha)(y) = \Sigma\{\alpha(x) | f(x) = y, x \in A\}$, $(\alpha \in N^A, y \in B)$. An objects of $G(W)$ are a type of edge-weighted directed graph.

We can treat labeled graph structure choosing a functor like next two examples.

Example 3.5 (**L-labeled Kleene functor**) We fix a set L of labels for edges. We define a functor $(L \times -)^* : \mathbf{Pfn} \to \mathbf{Pfn}$ as follows. For a set A, $(L \times A)^*$ is the set of finite strings of pairs of a label and an element of A. Other definition of the functor is similar to Example 3.2. An object of $G((L \times -)^*)$ is similar to the closed term hypergraph of Kennaway [10].

Example 3.6 (**L-labeled powerset functor**) We similarly define a functor $P(L \times -) : \mathbf{Pfn} \to \mathbf{Pfn}$ like Example 3.3 and Example 3.5.

Theorem 3.7 *Let $f : (A,a) \to (B,b)$ and $g : (A,a) \to (C,c)$ be morphisms in $G(T)$. If the square*

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[d, "g"'] \arrow[dr, phantom, "(1)"] & B \arrow[d, "h"] \\ C \arrow[r, "k"'] & D \end{tikzcd}$$

is a pushout in $\mathbf{Pfn}$, *then there exists a unique partial function*

$$d = (h^\sharp bTh) \cup (k^\sharp cTk)$$

such that $hd = \mathrm{d}(h)bTh$, $kd = \mathrm{d}(k)cTk$. When that is so, the square (2)

$$\begin{tikzcd} (A,a) \arrow[r, "f"] \arrow[d, "g"'] \arrow[dr, phantom, "(2)"] & (B,b) \arrow[d, "h"] \\ (C,c) \arrow[r, "k"'] & (D,\delta) \end{tikzcd}$$

is a pushout in $G(T)$ if and only if $\delta = (h^\sharp bTh) \cup (k^\sharp cTk)$ and δ is a total function.

Proof. We first show $f\mathrm{d}(h)bTh = g\mathrm{d}(k)cTk$.

$$\begin{aligned} f\mathrm{d}(h)bTh &= \mathrm{d}(fh)fbTh \\ &= \mathrm{d}(fh)\mathrm{d}(f)aTfTh \\ &= \mathrm{d}(fh)aTfTh \\ &= \mathrm{d}(gk)aTgTk \\ &= \mathrm{d}(gk)\mathrm{d}(g)aTgTk \\ &= \mathrm{d}(gk)gcTk \\ &= g\mathrm{d}(k)cPk \end{aligned}$$

Since the square (1) is a pushout in **Pfn**, there exist a unique partial function $d : D \to TD$ such that $hd = \mathrm{d}(h)bTh$ and $hd = \mathrm{d}(k)cTk$, where $d = (h^\sharp bTh) \cup (k^\sharp cTk)$, by Fact 2.4. Assume the square (2) is a pushout. Graph morphisms h and k satisfys $h\delta = \mathrm{d}(h)bTh$ and $h\delta = \mathrm{d}(k)cTk$. Since $h^\sharp \mathrm{d}(h) = h^\sharp$, $k^\sharp \mathrm{d}(k) = k^\sharp$ and $(h^\sharp h \cup k^\sharp k) = \mathrm{id}_D$, we have $\delta = (h^\sharp h \cup k^\sharp k)d = (h^\sharp bTh) \cup (k^\sharp cTk)$. Conversely, assume $\delta = (h^\sharp bTh) \cup (k^\sharp cTk)$ is a total function. Let (S, s) be an object in $G(T)$, and $x : B \to S$, $y : C \to S$ morphisms in $G(T)$ satisfying $fx = gy$. Since the square (1) is a pushout in **Pfn**, there exist a unique partial function $t : D \to S$ such that $ht = x$ and $ht = y$, where $t = h^\sharp x \cup k^\sharp y$, by Fact 2.4. We only need to show that t is a graph morphism. Since $h^\sharp x \Omega_S = h^\sharp \mathrm{d}(h)\mathrm{d}(x)\Omega_B = h^\sharp \mathrm{d}(x) h \Omega_D$ and $k^\sharp y \Omega_S = k^\sharp \mathrm{d}(y) k \Omega_D$, we have $\mathrm{d}(t) = h^\sharp \mathrm{d}(x) h \cup k^\sharp \mathrm{d}(y) k$. Since

$$
\begin{aligned}
h^\sharp \mathrm{d}(x) h\delta Tt &= h^\sharp \mathrm{d}(x) h\delta Tt \\
&= h^\sharp \mathrm{d}(x)\mathrm{d}(h) bThTt \\
&= h^\sharp \mathrm{d}(x)\mathrm{d}(h) bTx \\
&= h^\sharp \mathrm{d}(h)\mathrm{d}(x) bTx \\
&= h^\sharp \mathrm{d}(h) xs \\
&= h^\sharp xs
\end{aligned}
$$

and $k^\sharp \mathrm{d}(y) k\delta Tt = k^\sharp ys$, we obtain $\mathrm{d}(t)\delta Tt = h^\sharp xs \cup k^\sharp ys = ts$. That is t is a graph morphism. So the square (2) is a pushout in $G(T)$. ■

Corollary 3.8 *Let $f : (A, a) \to (B, b)$ and $g : (A, a) \to (C, c)$ be morphisms in $G(T)$. If the square*

$$
\begin{array}{ccc}
A & \xrightarrow{f} & B \\
{\scriptstyle g}\downarrow & (1) & \downarrow{\scriptstyle h} \\
C & \xrightarrow[k]{} & D
\end{array}
$$

is a pushout in **Pfn**, *then the following conditions are equivalent:*

(1) $d = (h^\sharp bTh) \cup (k^\sharp cTk)$ *is a total function.*

(2) $b(\mathrm{dom}(h)) \subset \mathrm{dom}(Th)$ *and* $c(\mathrm{dom}(k)) \subset \mathrm{dom}(Tk)$

(3) $\mathrm{dom}(h) \subset \mathrm{dom}(bTh)$ *and* $\mathrm{dom}(k) \subset \mathrm{dom}(cTk)$

Proof. If $b^\sharp h\Omega_D \subset Th\Omega_{TD}$ then $h\Omega_D \subset bb^\sharp h\Omega_D \subset bTh\Omega_{TD}$. If $h\Omega_D \subset bTh\Omega_{TD}$ then $b^\sharp h\Omega_D \subset b^\sharp bTh\Omega_{TD} \subset Th\Omega_{TD}$. We have $c^\sharp k\Omega_D \subset Tk\Omega_{TD}$

if and only if $k\Omega_D \subset cTk\Omega_{TD}$. So we have shown that (2) and (3) are equivalent.

Next we show the equivalence of (3) and (1). Assume $h\Omega_D \subset bTh\Omega_{TD}$ and $k\Omega_D \subset cTk\Omega_{TD}$. Since $h^\sharp h \cup k^\sharp k = \mathrm{id}_D$ by Lemma 2.5, we have $\Omega_D \subset (h^\sharp h\Omega_D) \cup (k^\sharp k\Omega_D) \subset (h^\sharp bTh\Omega_{TD}) \cup (k^\sharp cTk\Omega_{TD}) \subset d\Omega_{TD}$. This means d is a total function. Conversely, assume d is a total function. Since $hd = \mathrm{d}(h)bTh$ means $\mathrm{d}(h) = \mathrm{d}(h) \cap \mathrm{d}(bTh)$, we have $\mathrm{d}(h) \subset \mathrm{d}(bTh)$. Similaly $\mathrm{d}(k) \subset \mathrm{d}(cTk)$. ■

We note that if $T = P$, $T = W$ and $T = P(L \times -)$, then $Tf : TA \to TB$ is a total function for any partial function $f : A \to B$. This property is very convenient for existence of pushouts.

Corollary 3.9 *The categories $G(P)$, $G(W)$ and $G(P(L\times -))$ have pushouts.*

4 Observations

In this section, we provide the proof of Raoult's Proposition 5 in a view point of our framework.

Let $f : A \to B$ and $g : A \to C$ be partial functions. We define a relation $\Gamma_{(f,g)} : A \to 1$ by $\Gamma_{(f,g)} = \cup\{\alpha : A \to 1 | ff^\sharp\alpha = \alpha \text{ and } gg^\sharp\alpha = \alpha\}$, that is $\Gamma_{(f,g)}$ is the maximum relation satisfying $ff^\sharp\Gamma_{(f,g)} = \Gamma_{(f,g)}$ and $gg^\sharp\Gamma_{(f,g)} = \Gamma_{(f,g)}$.

Lemma 4.1 *Let*

$$\begin{array}{ccc} (A,a) & \xrightarrow{f} & (B,b) \\ {\scriptstyle g}\downarrow & & \downarrow{\scriptstyle h} \\ (C,c) & \xrightarrow[k]{} & (D,d) \end{array}$$

be a pushout in $G(T)$. Then $\Gamma_{(f,g)} = fh\Omega_D (= gk\Omega_D)$.

Proof. It is obvious $ff^\sharp fh\Omega_D = fh\Omega_D$ and $gg^\sharp fh\Omega_D = fh\Omega_D$. Asuume a relation $\alpha : A \to 1$ satisfies $ff^\sharp\alpha = \alpha$ and $gg^\sharp\alpha = \alpha$. Since $f^\sharp\alpha : B \to 1$ and $g^\sharp\alpha : C \to 1$ are partial functions and D is a pushout, there exist a unique function $\beta : D \to 1$ such that $h\beta = f^\sharp\alpha$ and $k\beta = g^\sharp\alpha$ hold. We obtain $fh\Omega_D \supset ff^\sharp\alpha = \alpha$. So $fh\Omega_D$ is the maximum relation. ■

We note that $\Gamma_{(f,g)} = fh\Omega_D$ means $f^{-1}(\mathrm{dom}(h)) = \{a \in A | (a,1) \in \Gamma_{(f,g)}\}$. By Lemma 2.5, $\mathrm{dom}(h) = (B - f(A)) \cup f(A')$ where $A' = \{a \in A | (a,1) \in \Gamma_{(f,g)}\}$.

Lemma 4.2 *Under the situation of Theorem 3.7, consider the functor* $T = *$. *Then following five conditions are equivalent:*

(1) $b(f(A')) \subset \mathrm{dom}(Th)(= (\mathrm{dom}(h))^*)$,

(2) $c(g(A')) \subset \mathrm{dom}(Tk)(= (\mathrm{dom}(k))^*)$,

(3) $Tf(a(A')) \subset (\mathrm{dom}(h))^*$,

(4) $Tg(a(A')) \subset (\mathrm{dom}(k))^*$, *and*

(5) $a(A') \subset (A')^*$,

where $A' = \{a \in A | (a, 1) \in \Gamma_{(f,g)}\}$.

Proof. Since $\mathrm{d}(f)aTf = fb$ and $\mathrm{d}(f)f = f$, we have $b^\sharp f^\sharp fh\Omega_B = (Tf)^\sharp a^\sharp fh\Omega_B$. This means $b(f(A')) = Tf(a(A'))$. So (1) and (3) are equivalent. (3) and (5) are equivalent by $(Tf)^{-1}(\mathrm{dom}(h)^*) = (A')^*$. Similarly, (2),(4) and (5) are equivalent, because of $\Gamma_{(f,g)} = gk\Omega_D$. ■

Using the last lemma, condition (2) in Corollary 3.8 is replaced to the form in the next proposition. The next proposition which was originally proved by Raoult [13, Proposition 5] is a special case of Theorem 3.7.

Proposition 4.3 *Let the square*

$$\begin{array}{ccc} A & \xrightarrow{f} & B \\ {\scriptstyle g}\downarrow & (1) & \downarrow{\scriptstyle h} \\ C & \xrightarrow[k]{} & D \end{array}$$

is a pushout in **Pfn**. *A commutative diagram*

$$\begin{array}{ccc} (A,a) & \xrightarrow{f} & (B,b) \\ {\scriptstyle g}\downarrow & & \downarrow{\scriptstyle h} \\ (C,c) & \xrightarrow[k]{} & (D,d) \end{array}$$

in $G(*)$ *is a pushout in* $G(*)$ *if and only if the following conditions (1), (2) and (3) hold:*

(1) $b(B - f(A)) \subset (\mathrm{dom}(h))^*$,

(2) $c(C - g(A)) \subset (\mathrm{dom}(k))^*$, *and*

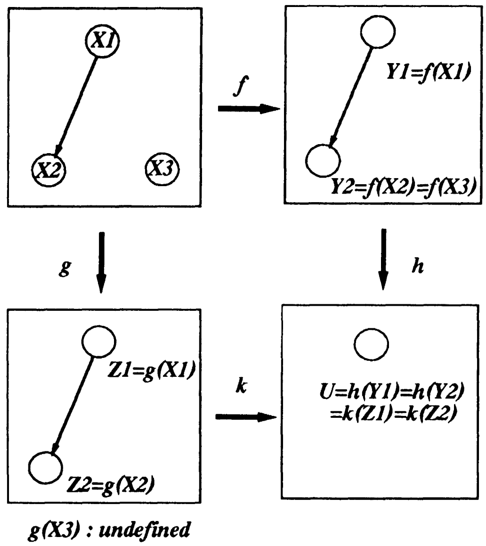

Figure 1:

(3) $a(A') \subset (A')^*$,

where $A' = \{a \in A | (a,1) \in \Gamma_{(f,g)}\}$.

The next example is a counter example of Raoult's Proposition 5[13] which lack a condition (3) of Proposition 4.3.

Example 4.4 Let $A = \{x_1 \to x_2, x_3\}$, $B = \{y_1 \to y_2\}$ and $C = \{z_1 \to z_2\}$ be graphs. Define graph morphisms $f : A \to B$ and $g : A \to C$ by $f(x_1) = y_1$, $f(x_2) = f(x_3) = y_2$, $g(x_1) = z_1$ and $g(x_2) = z_2$. The value of $g(x_3)$ is undefined(cf. Figure 1). It is easy to check $A' = \{x_1\}$. and the condition in Proposition 4.3(3) does not hold. Consider the pushout

$$\begin{array}{ccc} A & \xrightarrow{f} & B \\ {\scriptstyle g}\downarrow & & \downarrow{\scriptstyle h} \\ C & \xrightarrow[k]{} & D \end{array}$$

in the category **Pfn**. Since D is a one point set, h and k are not graph morphisms.

Acknowledgments

The author would like to thank Professor Yasuo Kawahara for the valuable suggestions and encouragement during the course of this study. The author wishes to thank a referee for his constructive comments to improve this manuscript.

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