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FUNCTORS BETWEEN HOMOTOPY THEORIES

by Luciano STRAMACCIA¹

RÉSUMÉ. Dans cet article, on considère des catégories \mathcal{C} munies d'une notion d'homotopie, sous la forme d'une structure de I-catégorie au sens de Baues, engendrée par un foncteur cylindre I, et on étudie la préservation des propriétés d'homotopie relativement à un foncteur $S: \mathcal{C} \rightarrow \mathcal{A}$, en particulier lorsque S est un réflecteur. Le cas d'un proréflecteur est aussi examiné.

INTRODUCTION.

There are various ways to introduce a homotopy notion in a category \mathcal{C} , all related to the concept of model category of Quillen [9]. Most notably, those due to Brown [2] and, more recently, to Baues [1], seem to be very interesting and more manageable than the original one. However there exists, up to author's knowledge, a certain lack in the literature concerning subcategories and comparison of homotopy structures.

In this paper we are concerned with categories endowed with the structure of an I-category in the sense of [1], which is generated by a cylinder functor I on it [1,6]. We study the preservation of homotopy properties by means of a functor S from \mathcal{C} to \mathcal{A} . In particular, we are interested in the case where S is a reflector, which means that \mathcal{A} is a full subcategory of \mathcal{C} and S is left adjoint to the embedding functor $T: \mathcal{A} \rightarrow \mathcal{C}$. Also the case of a prorereflector P is considered.

1. PRELIMINARIES.

Let \mathcal{C} be a category and let Σ be a class of morphisms of \mathcal{C} which we call "weak equivalences". A new category $\mathcal{C}[\Sigma^{-1}]$ can be constructed by formally inverting weak equivalences. $\mathcal{C}[\Sigma^{-1}]$ has the same objects as \mathcal{C} and is defined by the following properties:

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(i) There is a functor $P_{\Sigma}: \mathbf{C} \rightarrow \mathbf{C}[\Sigma^{-1}]$ which is the identity on objects and which inverts all weak equivalences, that is, $P_{\Sigma}(s)$ is an isomorphism in $\mathbf{C}[\Sigma^{-1}]$, for every $s \in \Sigma$.

(ii) If $G: \mathbf{C} \rightarrow \mathbf{D}$ is a functor which inverts all weak equivalences, then there is a unique functor $G^*: \mathbf{C}[\Sigma^{-1}] \rightarrow \mathbf{D}$ such that $G^* \cdot P_{\Sigma} = G$.

$\mathbf{C}[\Sigma^{-1}]$ always exists, but its description is particularly nice whenever Σ admits a "calculus of left fractions" in \mathbf{C} [3].

Let \mathbf{A} be another category endowed with a notion of weak equivalence and let Λ be the class of such weak equivalences. A functor $F: \mathbf{C} \rightarrow \mathbf{A}$ can be extended to a functor $F^*: \mathbf{C}[\Sigma^{-1}] \rightarrow \mathbf{A}[\Lambda^{-1}]$ iff F preserves weak equivalences, that is $F(\Sigma) \subset \Lambda$. In such a case F^* is the unique functor with $F^* \cdot P_{\Lambda} = P_{\Sigma} \cdot F$. F^* acts on objects as F does.

PROPOSITION 1.1 (cf. [2], p. 426). *Let $T: \mathbf{A} \rightarrow \mathbf{C}$ and $S: \mathbf{C} \rightarrow \mathbf{A}$ be functors which preserve weak equivalences. If S is left adjoint to T , then S^* is left adjoint to T^* .*

DEFINITION 1.2. a) A cylinder functor for a category \mathbf{C} is a functor $I: \mathbf{C} \rightarrow \mathbf{C}$ together with natural transformations

$$e_0, e_1: 1_{\mathbf{C}} \rightarrow I \text{ and } \sigma: I \rightarrow 1_{\mathbf{C}}$$

such that $\sigma \cdot e_0 = \sigma \cdot e_1 = \text{identity}$.

Two morphisms $f, g \in \mathbf{C}(X, Y)$ are homotopic, written $f \simeq g$, whenever there is a "homotopy" $H: I(X) \rightarrow Y$ with $H \cdot e_0(X) = f$ and $H \cdot e_1(X) = g$. Shortly $H: f \simeq g$.

b) Once a cylinder functor is given for \mathbf{C} , one can define a morphism $t \in \mathbf{C}(X, Y)$ to be a weak equivalence when it has a homotopy inverse, that is there exists an

$$s \in \mathbf{C}(X, Y) \text{ such that } s \cdot t \simeq 1_X \text{ and } t \cdot s \simeq 1_Y.$$

Let Σ be the class of such weak equivalences in \mathbf{C} .

The cylinder functor I is said to be generating for \mathbf{C} (compare [6]) whenever (\mathbf{C}, I, \dagger) is an I -category in the sense of Baues [1], with respect to the classes Σ of weak equivalences above and the class Γ of cofibrations, defined by the usual homotopy extension property. Let us denote by \dagger the initial object of \mathbf{C} .

Whenever I is generating, the class Σ of weak equivalences admits a calculus of left fractions in \mathbf{C} and the category

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$\mathbf{C}[\Sigma^{-1}] = \text{Ho}\mathbf{C}$ is called the *homotopy category* of \mathbf{C} with respect to I . For every pair of objects X, Y of \mathbf{C} , $\text{Ho}\mathbf{C}(X, Y) = [X, Y]$ is the set of homotopy classes of morphisms $X \rightarrow Y$ in \mathbf{C} .

c) Let $J = (J, d_0, d_1, \delta)$ and $I = (I, e_0, e_1, \sigma)$ be cylinder functors for the categories \mathbf{A} and \mathbf{C} , respectively. We say that $F: \mathbf{C} \rightarrow \mathbf{A}$ respects the cylinder functors whenever the following hold:

- (i) $F \cdot I = J \cdot F$,
- (ii) a) $F \cdot e_i = d_i \cdot F$, $i = 0, 1$; b) $F \cdot \sigma = \delta \cdot F$.

2. FUNCTORS PRESERVING CYLINDERS.

It is easily seen that a functor $F: \mathbf{C} \rightarrow \mathbf{A}$ which respects the cylinder functors preserves homotopies: in particular F preserves weak equivalences and induces a uniquely determined functor $\text{Ho}F: \text{Ho}\mathbf{C} \rightarrow \text{Ho}\mathbf{A}$ between the homotopy categories.

The converse is not true in general: the simplest example is perhaps a constant functor $\text{TOP} \rightarrow \text{TOP}$ which induces a constant functor between the homotopy categories, but does not preserve homotopies. We wish to study this situation in detail, in the case where \mathbf{A} is a reflective subcategory of \mathbf{C} with inclusion T such that $T \cdot I = I \cdot T$ and reflector S which preserves weak equivalences.

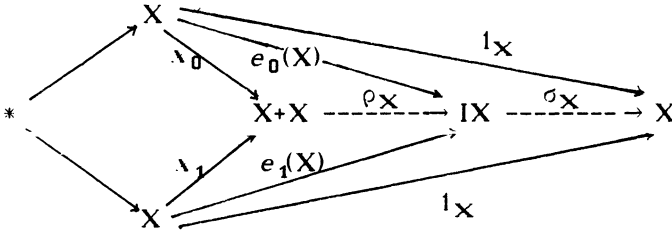
Let us denote by $\alpha: 1 \rightarrow T \cdot S$ the unit of this adjunction: then by the universal property of the reflection there exists a unique morphism t_X which renders the following diagram commutative:

$$\begin{array}{ccc}
 IX & \xrightarrow{\alpha_{IX}} & S(IX) \\
 \downarrow \alpha_X & & \downarrow t_X \\
 & & IS(X)
 \end{array}$$

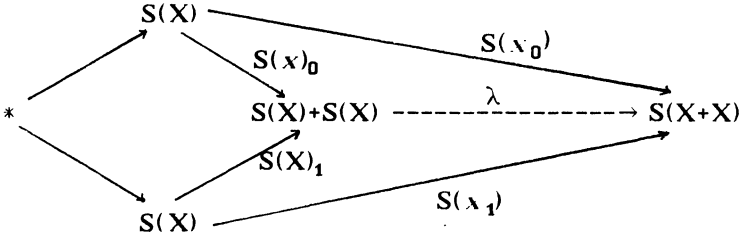
Assume now that I is a generating cylinder functor for \mathbf{C} . For every object $X \in \mathbf{C}$, there exists the pushout

$$\begin{array}{ccc}
 & \xrightarrow{\quad} & X \\
 \downarrow & & \downarrow \lambda_0 \\
 X & \xrightarrow{\lambda_1} & X + X
 \end{array}$$

As for notations, let us also consider the following commutative diagrams



from which it follows that $\sigma_X \cdot \rho_X = (1_X, 1_X)$ is the folding map for X .



LEMMA 2.1. For every $X \in \mathcal{C}$ the following holds:

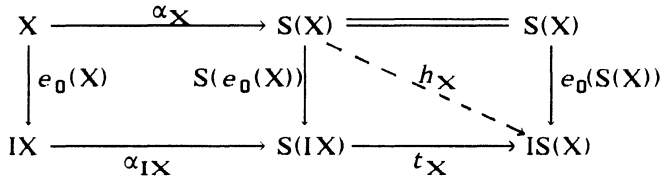
(i) $t_X \cdot S(e_0(X)) = e_0(S(X))$; in particular t_X is a weak equivalence.

(ii) $S(1_X \cdot 1_X) \cdot \lambda = (1_{S(X)}, 1_{S(X)})$.

(iii) $t_X \cdot S(\rho_X) \cdot \lambda = \rho_{S(X)}$.

(iv) $\sigma_{S(X)} \cdot t_X = S(\sigma_X)$.

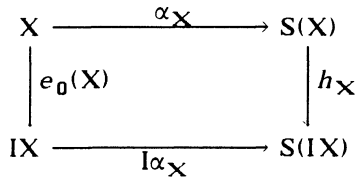
PROOF. Consider the diagram



where the left square is commutative. By the universal property of the reflection, there exists a unique morphism h_X such that

$$h_X \cdot \alpha_X = t_X \cdot \alpha_{IX} \cdot e_0(X) = \alpha_{IX} \cdot e_0(X).$$

Hence the following diagram is also commutative:



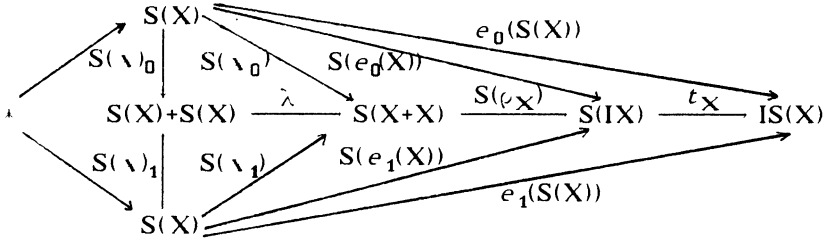
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and this forces $h_X = e_0(S(X))$. Finally

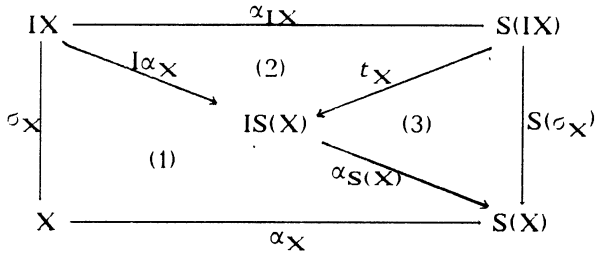
$$t_X \cdot \alpha_{IX} \cdot e_0(X) = e_0(S(X)) \cdot \alpha_X \text{ and } t_X \cdot S(e_0(X)) \cdot \alpha_X = e_0(S(X)) \cdot \alpha_X.$$

Hence $t_X \cdot S(e_0(X)) = e_0(S(X))$. t_X is a weak equivalence since $e_0(X)$ is, for every X , and S preserves weak equivalences.

Part (ii) follows from the second diagram, applying the functor S . Part (iii) follows from (i) considering the diagram:



(iv) Consider again a diagram:

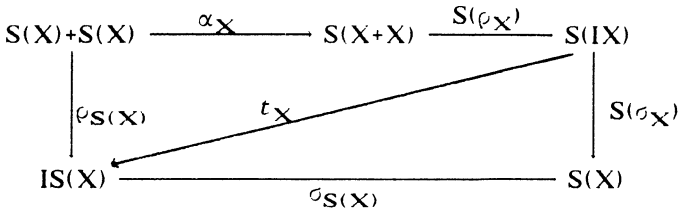


The outer square is commutative since α is a natural transformation. Square (1) commutes since σ is a natural transformation. Triangle (2) commutes by assumption. Let us prove that triangle (3) is also commutative:

$$(\sigma_{S(X)} \cdot t_X) \cdot \alpha_{IX} = \sigma_{S(X)} \cdot (t_X \cdot \alpha_{IX}) = \sigma_{S(X)} \cdot l\alpha_X = S(\sigma_X) \cdot \alpha_{IX}.$$

By the universal property of the reflection it follows that $\sigma_{S(X)} \cdot t_X = S(\sigma_X)$.

Let us observe that the previous lemma implies that the following diagram is commutative:



In ([1], §5a, p. 112) a diagram similar to the above is constructed in order to prove via a "weak lifting" $L=(h \cdot j)$ of it, that the correspondence

$$[IX, Y] \xrightarrow{S} [S(IX), S(Y)] \xrightarrow{L^* = h^* (j^*)^{-1}} [IS(X), S(Y)]$$

given by $(L^* \cdot S)(H) = L^*(S(H))$, carries (homotopy classes of) homotopies to (homotopy classes of) homotopies. A condition on a general functor S , for L^* to be a bijection is that S be compatible with pushouts of the form $X+X$ (see [1]). In case S is a reflector, as we do assume, the work above allows us to obtain the following

THEOREM 2.2. $L^* = (t_X^*)^{-1}$.

In other words L^* is a bijection, which may be restated by saying that the reflector S "respects the cylinder functor up to homotopy".

Let us observe that the phrase " S respects the cylinder functor" above is not correct since \mathbf{A} has not its own cylinder functor as well. To be precise we put the following definitions.

DEFINITION 2.3. a) A full subcategory \mathbf{A} of \mathbf{C} is called a *homotopy subcategory* (*h-subcategory*, for short) whenever $I(A) \in \mathbf{A}$, for every $A \in \mathbf{A}$.

In other words, \mathbf{A} is a h-subcategory of \mathbf{C} when the restriction of I to \mathbf{A} is a cylinder functor for \mathbf{A} itself.

Let us denote by $\text{Ho}\mathbf{A}$ the category obtained by formally inverting the weak equivalences of \mathbf{C} that are contained in \mathbf{A} . $\text{Ho}\mathbf{A}$ is the full subcategory of $\text{Ho}\mathbf{C}$ having the same objects as \mathbf{A} .

b) Let now I and J be cylinder functors for \mathbf{C} and \mathbf{A} , respectively, let again $S: \mathbf{C} \rightarrow \mathbf{A}$ be left adjoint to $T: \mathbf{A} \rightarrow \mathbf{C}$ and assume that T respects the cylinder functors.

We say that the functor $S: \mathbf{C} \rightarrow \mathbf{A}$ *respects homotopies* whenever the correspondence

$$\mathbf{A}(JS(X), A) \rightarrow \mathbf{A}(SI(X), A), \text{ given by } K \mapsto K \cdot t_X,$$

is onto, for every $A \in \mathbf{A}$. Then S respects homotopies iff t_X is a section, as one easily verifies.

PROPOSITION 2.4. Let \mathbf{A} be an epireflective h-subcategory of \mathbf{C}

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with reflector $S: \mathbf{C} \rightarrow \mathbf{A}$ which respects homotopies. If I preserves epimorphisms then S respects the cylinder functor and $\text{HoS}: \text{Ho}\mathbf{A} \rightarrow \text{Ho}\mathbf{C}$ is a reflector.

We note that, since the epimorphisms in $\mathbf{C} = \text{TOP}$ are the onto continuous maps, then, whenever S is the epireflector of a full subcategory \mathbf{A} in TOP , which contains the unit interval I , the following statements are equivalent (cf. [10] Th. 1.2):

- i) $S(X \times I) = S(X) \times I$, for every space X .
- ii) S respects homotopies.
- iii) S takes homotopic maps to homotopic maps.

$\text{HoS}: \text{HoTOP} \rightarrow \text{Ho}\mathbf{A}$ is a reflector whenever these hold.

It is shown in [10] that every quotient reflective subcategory \mathbf{A} of TOP such that $I \in \mathbf{A}$ satisfies the conditions above. This can be obtained from the following more general result, using Theorem 3.5 of Schwarz [9].

PROPOSITION 2.5. *Let \mathbf{C} be a monotopological category with a cylinder functor $- \times I$ where I is an exponential object of \mathbf{C} . Let \mathbf{A} be a quotient reflective subcategory of \mathbf{C} such that $I \in \mathbf{A}$. Then the reflector S respects the cylinder functor and HoS is still a reflector.*

3. THE HOMOTOPY STRUCTURE.

Let us recall from [4.5] that the cylinder functor I induces on \mathbf{C} a *semicubical homotopy system* $Q_I: \mathbf{C} \cdot \mathbf{C} \rightarrow \mathbf{K}$. For every $X, Y \in \mathbf{C}$, $Q_I(X, Y)$ is the *semicubical complex* having $\mathbf{C}(I^n(X), Y)$ as the set of n -cubes, where $I^0(X) = X$ and, for every $n \geq 1$, $I^n(X) = I(I^{n-1}(X))$. Face and degeneracy operators are defined, respectively, by the following:

$$\varepsilon_n^i = \mathbf{C}(I^{i-1}(e_c(I^{n-1}(X)), 1_Y)) \text{ and } \xi_n^j = \mathbf{C}(I^{j-1}(d_c(I^{n+1-j}(X)), 1_Y))$$

$\varepsilon = 0, 1$. The *edge* of a $\varphi \in \mathbf{C}(I^n X, Y)$ is defined to be

$$D\varphi = (0_n^1 \varphi, 1_n^1 \varphi, \dots, 0_n^n \varphi, 1_n^n \varphi).$$

For every pair $X, Y \in \mathbf{C}$, we can construct the *fundamental groupoid* $\Pi_I(X, Y)$. Its objects are the 0-cubes of $Q_I(X, Y)$, while a morphism $f - g$ in $\Pi_I(X, Y)$ is an equivalence class $[\alpha]$ of 1-cubes with $D\alpha = (f, g)$, with respect to the following relation:

if $\alpha, \beta \in \mathbf{C}(IX, Y)$, then $\alpha \approx \beta$ whenever a $\varphi \in \mathbf{C}(I^2X, Y)$ exists, in such a way that

$$D\varphi = (\alpha, \beta, \xi_0^1 0_1^1 \alpha, \xi_0^1 1_1^1 \alpha).$$

The fundamental groupoid may also be considered as a functor $\Pi_1: \mathbf{C} \rightarrow \mathbf{C}\text{-Grd}$.

If $f: X \rightarrow Y$ is a homotopy equivalence in \mathbf{C} , there are induced natural transformations

$$f_*: \Pi_1(X, Z) \rightarrow \Pi_1(Y, Z), \quad f^*: \Pi_1(Z, X) \rightarrow \Pi_1(Z, Y)$$

which are natural equivalences of groupoids, for every $Z \in \mathbf{C}$.

Moreover, any functor $F: \mathbf{C} \rightarrow \mathbf{A}$ which respects the cylinder functors (I for \mathbf{C} and J for \mathbf{A}) induces a natural transformation $\Pi_1(X, Y) \rightarrow \Pi_J(F(X), F(Y))$. In fact, F preserves homotopies, hence it takes n -cubes to n -cubes, and also it preserves the equivalence of 1-cubes, as one verifies with a short calculation.

THEOREM 3.1. *Let $S: \mathbf{C} \rightarrow \mathbf{A}$ be left adjoint to $T: \mathbf{A} \rightarrow \mathbf{C}$ and assume that they respect the cylinder functors. Then*

- (i) *S preserves weak equivalences and cofibrations;*
- (ii) *For every $A \in \mathbf{A}$ and $X \in \mathbf{C}$, there is an isomorphism of groupoids $\Pi_1(X, T(A)) \approx \Pi_J(S(X), A)$.*

PROOF. For (i) we have only to show that S preserves cofibrations. Let $i: Y \rightarrow X$ be a cofibration in \mathbf{C} and consider a morphism $b: S(X) \rightarrow B$ and a homotopy $\psi: JS(Y) \rightarrow B$ in \mathbf{A} , such that $\psi \cdot d_0(S(Y)) = b \cdot S(i)$. Consider the commutative diagram.

$$\begin{array}{ccccc}
 Y & & \xrightarrow{e_0(Y)} & & IY \\
 & \searrow \alpha_Y & & & \downarrow I(\alpha_Y) \\
 & & TS(Y) & \xrightarrow{e_0(TS(Y))} & TS(IY) = T(JS(Y)) = ITS(Y) \\
 & & \downarrow TS(i) & & \downarrow T(\psi) \\
 X & \xrightarrow{\alpha_X} & TS(X) & \xrightarrow{T(b)} & T(B)
 \end{array}$$

There exists a homotopy $\Phi: I(X) \rightarrow T(B)$ in \mathbf{C} , such that

$$\Phi \cdot e_0(X) = T(b) \cdot \alpha_X \quad \text{and} \quad \Phi \cdot I(i) = T(\psi) \cdot I(\alpha_Y).$$

Then $\beta_B \cdot S(\Phi): SI(X) = IS(X) \rightarrow B$ is a homotopy in \mathbf{A} such that

$$\beta_B \cdot S(\Phi) \cdot d_0(S(X)) = \beta_B \cdot S(\Phi) \cdot S(e_0(X)) = \beta_B \cdot S(\Phi \cdot e_0(X))$$

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$$= \beta_B \cdot ST(b) \cdot S(\alpha_X) = b \cdot \beta_{S(X)} \cdot S(\alpha_X) = b.$$

Moreover

$$\begin{aligned} \beta_B \cdot S(\Phi) \cdot J(S(i)) &= \beta_B \cdot S(\Phi) \cdot S(I(i)) = \beta_B \cdot S(\Phi I(i)) \\ &= \beta_B \cdot ST(\psi) \cdot SI(\alpha_Y) = \beta_B \cdot S(T(\psi)) \cdot JS(\alpha_Y) = \psi \cdot \beta_{JS(Y)} \cdot JS(\alpha_Y) \\ &= \psi \cdot \beta_{SI(Y)} \cdot S(\alpha_{IY}) = \psi \cdot S(\text{identity}) = \psi. \end{aligned}$$

It follows that $S(i):S(Y) \rightarrow S(X)$ is a cofibration in \mathbf{A} . Part (ii) follows from the discussion above and Proposition 2.3.

4. A GENERALIZATION.

We wish to consider now the case of *proreflectors*. Such functors arise in Shape Theory [7] and are a weakened form of reflectors. Here one deals with the procategory $\text{Pro } \mathbf{C}$ of the given category \mathbf{C} , whose objects are the inverse systems of objects of \mathbf{C} of the form $\underline{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$. The cylinder functor I on \mathbf{C} extends naturally to $\text{Pro } \mathbf{C}$ by taking $I\underline{X} = (IX_\lambda, Ip_{\lambda\lambda'}, \Lambda)$.

Let \mathbf{A} be a full subcategory of \mathbf{C} , then a proreflector $P: \mathbf{C} \rightarrow \text{Pro } \mathbf{A}$ is a functor which assigns to every $X \in \mathbf{C}$ an inverse system $\underline{X} \in \text{Pro } \mathbf{A}$ and a morphism $X \rightarrow \underline{X}$ in $\text{Pro } \mathbf{C}$ which is initial with respect to every other morphism $X \rightarrow \underline{Y}$, $\underline{Y} \in \text{Pro } \mathbf{A}$.

We refer to [7, 8, 11] for the definition of a procategory and related concepts.

Let us recall [11] that \mathbf{A} is proreflective in \mathbf{C} by means of P iff $\text{Pro } \mathbf{A}$ is reflective in $\text{Pro } \mathbf{C}$ by means of the functor P^* given by the composition of the extension of P to the procategory, $\text{Pro } P: \text{Pro } \mathbf{C} \rightarrow \text{Pro } \text{Pro } \mathbf{A}$, with the inverse limit functor

$$\text{invlm}: \text{Pro } \text{Pro } \mathbf{A} \rightarrow \text{Pro } \mathbf{A}.$$

Recently Porter [8] has shown that the right homotopy category $\text{Ho } \text{Pro } \mathbf{C}$ of $\text{Pro } \mathbf{C}$ is that obtained by formally inverting the level homotopy equivalences in $\text{Pro } \mathbf{C}$. A level morphism in $\text{Pro } \mathbf{C}$ is a morphism between inverse systems indexed over the same directed set, which is actually a natural transformation.

THEOREM 4.1. *Let \mathbf{A} be a proreflective h -subcategory of \mathbf{C} , with proreflector $P: \mathbf{C} \rightarrow \text{Pro } \mathbf{A}$. If P respects the cylinder functors, then $\text{Ho } \text{Pro } \mathbf{A}$ is reflective in $\text{Ho } \text{Pro } \mathbf{C}$ by means of $\text{Ho } P^*$.*

PROOF. We have only to show that P^* takes level homotopy equivalences in $\text{Pro } \mathbf{C}$ to level homotopy equivalences in $\text{Pro } \mathbf{A}$. Let $\underline{f}: \underline{X} \rightarrow \underline{Y}$ be a level homotopy equivalence in $\text{Pro } \mathbf{C}$; then $\text{Pro } P(\underline{f})$ is an inverse system of homotopy equivalences in $\text{Pro } \mathbf{A}$. Finally $P(\underline{f})$ is a level homotopy equivalence in $\text{Pro } \mathbf{A}$. To see

this one can look at the explicit description of the functor P , as given in ([11], 2.6) and making use of the reindexing theorem ([7], 3.3. Ch. 1).

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