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MODELS FOR ACTIONS OF CERTAIN GROUPOIDS

by Peter GREENBERG

Résumé. Soit $B\Gamma^n$ l'espace classifiant pour les feuilletages C^∞ de codimension n, et soit M^n le monoïde discret des plongements C^∞ de $\mathbf{R}^n$ dans $\mathbf{R}^n$. G. Segal a montré qu'il existe une équivalence d'homotopie faible $BM^n \to B\Gamma^n$, et D. McDuff a obtenu des résulats analogues pour les espaces classifiants de feuilletages C^∞ avec une forme volume transverse, de codimension au moins 3. On généralise ici ces résultats.

1. INTRODUCTION.

Let $B\Gamma^n$ be the classifying space for C^∞ codimension n foliations, and let M^n be the discrete monoid of C^∞ embeddings from $\mathbf{R}^n$ to $\mathbf{R}^n$. G Segal showed [13] that there is a weak homotopy equivalence $BM^n \to B\Gamma^n$, and D. McDuff [10] obtained similar results for the classifying spaces for C^∞ foliations with transverse volume form, with codimension at least 3. This paper generalizes these results.

Much of this work was done at M.I.T. and is my doctoral thesis ; I'd like to thank my advisors Daniel Kan and Sol Jekel. Sol Jekel has obtained similar results with other methods [7].

1.1. Classifying spaces for Haefliger structures.

A pseudogroup G of transformations of a space X is a collection, closed under composition and inverse, of homeomorphisms between open subsets of X. There are various models for a classifying space BG ; such a classifying space is of interest in the homotopy theory of foliations whose transverse geometric structure is modelled on X. In this paper we start with a topological category Γ called a *groupoid of homeomorphisms* (2.1) of X ; the classifying space (1.7) $|\Gamma|$ of this category is a standard model for BG.

1.2. Monoids of immersions.

We associate to a groupoid of homeomorphisms Γ of a space X a discrete *monoid of immersions* M (2.3), which acts on X by maps $m : X \to X$ which are locally one-to-one. Let $|M\backslash X|$ denote the homotopy quotient of the action (sometimes denoted $EM \times_M X$) ; if X is contractible there is an (homotopy) equivalence $|M\backslash X| \to BM$.

1.3. Theorem. *If the images mX, $m \in M$ form a basis for the topology of X, then there is a weak (homotopy) equivalence* $|M\backslash X| \to |\Gamma|$.

For example, let Γ be the groupoid of all area preserving diffeomorphisms between open subsets of the plane ; $|\Gamma|$ is the classifying space for codimension 2 foliations with a transverse area form. Let M be the monoid of area preserving immersions of the plane. Then there is a weak equivalence $BM \to |\Gamma|$.

1.4. Modelling actions.

Let Y be a space with a map $p : Y \to X$, and suppose that the homeomorphisms between open subsets of X in the pseudogroup G lift naturally to homeomorphisms between the inverse image of the open sets in Y. Such extra data may correspond to some facet of the geometry of X. For example, consider the tangent bundle $p : T_* \mathbf{R}^n \to \mathbf{R}^n$. If $f : U \to V$ is a diffeomorphism between open subsets of $\mathbf{R}^n$, there is a natural induced isomorphism $f_* : T_* U \to T_* V$ of tangent bundles.

We define (2.2) an action of a groupoid of homeomorphisms Γ on a space Y over X. There is an appropriate definition of the homotopy quotient $|\Gamma \backslash Y|$, of the action ; in [5] we show that there is a spectral sequence abutting to $H_* |\Gamma \backslash Y|$, with

$$E^2_{pq} = H_p(\Gamma ; H_q Y) ,$$

where the latter expression is defined in analogy with group homology. In some cases the E^2 term has some geometrical significance [4].

If Γ is a groupoid of homeomorphisms of X, and $p : Y \to X$ is a map, and Γ acts on Y over X, then the monoid of immersions M acts on Y.

1.5. Theorem. *If the images mX, $m \in M$, form a basis for the topology of X, then there is a weak equivalence* $|M \backslash Y| \to |\Gamma \backslash Y|$.

It turns out that 1.3 is a corollary to 1.5.

1.6. Organization. In Section 2, definitions are given, along with restatements of the main results and some examples. Section 3 contains the main body of proof, with a major Lemma 3.4 proved in Sections 4 and 5.

1.7. Notation.

We assume familiarity with semisimplicial notations; references are [9] and [14].

Recall [2] that a *topological category* is a small category in which the sets of objects and morphisms are given topologies, such that the structure maps of the category are continuous. The *nerve* of a (topological) category C is a simplicial (space) set C_* ; a functor $F : C \to D$ induces a simplicial map $F_* : C_* \to D_*$. If C is a (topological) category, we denote by D, R : $C_n \to C_0$ the domain and range maps

$$D = d_1 \dots d_n , \qquad R = d_{n-1} \dots d_1 \, d_0 .$$

We employ the geometric realization functor

$$|\cdot| : \text{simplicial spaces} \to \text{spaces}$$

defined by Segal ([12], App. A ; Segal calls the functor $\|\cdot\|$). If A_* is a simplicial space, $|A_*|$ is defined as

$$|A_*| = \coprod A_n \times \Delta^n / \sim$$

where Δ^n is the standard n-simplex, and we set

$$(Fa_n \,;\, t_k) \sim (a_n \,,\, F^* t_k)$$

for every composite of face maps $F : A_n \to A_k$; here $F^* : \Delta^k \to \Delta^n$ is the inclusion induced by F. Segal's realization is useful to us because of the proposition :

1.8. Proposition. *(Segal, 12 , App. A.1.ii) If $f_* : A_* \to B_*$ is a map of simplicial spaces such that every $f_n : A_n \to B_n$ is a weak equivalence, then $|f_*| : |A_*| \to |B_*|$ is a weak equivalence.*

2. MAIN RESULTS.

2.1. Definition. A *groupoid of homeomorphisms* Γ of a space X is a topological category Γ with space of objects X, such that (denoting by Γ_1 the space of morphisms of Γ) :

i) every morphism in Γ_1 has an inverse.

ii) the domain and range maps $D, R : \Gamma_1 \to X$ are locally homeomorphisms.

let Γ be a groupoid of homeomorphisms of a space X. If $U \subset X$ is open, and $s : U \to \Gamma_1$ is a section of the domain map, then $Rs : U \to X$ is locally a homeomorphism. The sections such that Rs is a homeomorphism form a pseudogroup of transformations of X in the sense of Ehresmann [1], or Haefliger [6].

2.2. Definition [2]. Let Γ be a groupoid of homeomorphisms of X. Let $p : Y \to X$ be a continuous map. An *action of* Γ *on* Y *over* X is a groupoid of homeomorphisms $\Gamma\backslash Y$ of Y, and a functor $p : \Gamma\backslash Y \to \Gamma$, which on objects, is the map $p : Y \to X$, such that :

i) the diagram

$$\begin{array}{ccc} (\Gamma\backslash Y)_1 & \xrightarrow{D} & Y \\ {\scriptstyle p}\downarrow & & \downarrow{\scriptstyle p} \\ \Gamma_1 & \xrightarrow{D} & X \end{array}$$

is a pullback, so that we can write elements of $(\Gamma\backslash Y)_1$ as pairs (g, y) such that $Dg = py$;

ii) $pR(g, y) = Rg$;

iii) if $f, g \in \Gamma_1$ with $Df = Rg$, and $y \in Y$ with $Dg = py$, then

$$(f \circ g, y) = (f, R(g, y)) \circ (g, y).$$

The range map $R : (\Gamma\backslash Y)_1 \to Y$ of $\Gamma\backslash Y$ is called the *range map* for the action. Of course, Γ itself is an action of Γ on X.

2.3. Definition. Let Γ be a groupoid of homeomorphisms of a space X. Define the *monoid of immersions* M of X to be the set

$$M = \{m : X \to \Gamma_1 \mid Dm = \mathrm{id}\},$$

with the composition

$$m \circ n(x) = m(Rnx) \circ n(x),$$

where the composition in the latter expression is in Γ_1. (Note that if $m \in M$, $m(x) \in \Gamma_1$.)

2.4. Definition. Let Γ be a groupoid of homeomorphisms of a space X, $p : Y \to X$ a map, and $\Gamma\backslash Y$ an action of Γ on Y over X. To every $m \in M$ define a section $m : Y \to (\Gamma\backslash Y)_1$ of the domain map by

$$m(y) = (m(py), y).$$

Let $M\backslash Y$ denote the topological category with objects Y, morphisms $M \times Y$, and domain and range maps $D, R : M \times Y \to Y$ given by

$$D(m, y) = y, \quad R(m, y) = R\,m(y).$$

We define a functor $i_Y : M\backslash Y \to \Gamma\backslash Y$ to be the identity on objects, with $i_{Y,1}(m, y) = m(y) \in (\Gamma\backslash Y)_1$. In particular, there is a functor i: $M\backslash X \to \Gamma$, realizing $M\backslash X$ as the category of global sections of the domain map [3].

We can now restate Theorems 1.3 and 1.5.

2.5. Theorem. *Let Γ be a groupoid of homeomorphisms of a space X, and let M be the monoid of immersions of X. Suppose that the open sets $R\,m(X)$, $m \in M$, form a basis for the topology of X. Then :*

i) for any action $\Gamma\backslash Y$ of Γ on a space Y, the functor $M\backslash Y \to \Gamma\backslash Y$ induces a weak equivalence $|M\backslash Y| \to |\Gamma\backslash Y|$;

ii) in particular, there is a weak equivalence $|M\backslash X| \to |\Gamma|$.

Our applications use Corollary 2.8 below.

2.6. Definition. Let Γ be a groupoid of homeomorphisms of X and let $U \subset X$ be open. The *stabilizer* Γ^U of U is the groupoid of homeomorphisms of U whose space of morphisms is

$$\Gamma^U_1 = \{g \in \Gamma_1 \mid Dg\,,\ Rg \in U\}\ .$$

The *monoid of immersions of* U is

$$M^U = \{m : U \to \Gamma_1 \mid Dm = \mathrm{id} \text{ and } RmU \subset U\}.$$

2.7. Proposition [5]. *Suppose the open sets* RsU, $s : U \to \Gamma_1$ *a section of the domain map, form a basis for the topology of* X. *Then the inclusion functor* $\Gamma^U \to \Gamma$ *induces a weak equivalence* $|\Gamma^U| \to |\Gamma|$.

2.8. Corollary. *Let* Γ *be a groupoid of homeomorphisms of* X, *let* $U \subset X$, *and suppose the open sets* RsU, $s : U \to \Gamma_1$ *a section of the domain map, form a basis for the topology of* X. *Then there is a weak equivalence* $|M^U\backslash U| \to |\Gamma^U|$.

Proof. By 2.7 there is a weak equivalence $|\Gamma^U| \to |\Gamma|$ and by 2.5 there is a weak equivalence $|M^U\backslash U| \to |\Gamma^U|$.

2.9. Examples. i) Let G be a discrete group acting on a space X. Define a groupoid of homeomorphisms $G\backslash X$ of X whose space of morphisms is GxX, with D, R : GxX $\to$ X defined by

$$D(g, x) = x, \qquad R(g, x) = gx\ .$$

There is a weak equivalence $|G\backslash X| \to EG\times_G X$.

Let $U \subset X$ be an open set, and suppose that the images $\{gU\}$, $g \in G$ form a basis for the topology of X. Let M^U be the monoid of elements of g taking U into U. Then there is a weak equivalence $|M\backslash U| \to |G\backslash X|$. If U is contractible there is a weak equivalence $BM \to |G\backslash X|$.

ii) Let N^k be the "final k-dimensional submanifold of R^n" defined as $N^k = \coprod U_f/\sim$ where we take one copy U_f for every open subset U of R^k and every C^∞ immersion $f : U \to R^n$, where if $u \in U_f$, $v \in V_g$ we set $u \sim v$ if there is a neighborhood W of u in U_f and a C^∞ embedding $h : W \to V_g$ such that $g \circ h = f$ on W. N^k is a non-Hausdorff k-dimensional manifold with an immersion $N^k \to R^n$.

Let Γ^n be the groupoid of diffeomorphisms of R^n; Γ^n_1 is the space of germs of diffeomorphisms of R^n with the sheaf topology. There is an obvious action $\Gamma\backslash N^k$. Picking a standard embedding $R^k \to R^n$, we can regard R^k as a submanifold of N^k. The stabilizer $(\Gamma\backslash N^k)^{R^k}$ is the groupoid of diffeomorphisms of R^k with germs of extension to R^n; by (2.7) there is a weak equivalence

$$|(\Gamma\backslash N^k)^{R^k}| \longrightarrow |\Gamma\backslash N^k|$$

The monoid $M^{\mathbf{R}^k}$ is the monoid of immersions of $\mathbf{R}^k$ with germs of extensions to $\mathbf{R}^n$. By (2.9), there is a weak equivalence $BM^{\mathbf{R}^k} \to |\Gamma\backslash N^k|$.

This example is exploited in [4].

3. PROOF OF 2.5.

From now on, we assume the conditions of 2.5 : Γ is a groupoid of homeomorphisms of a space X, such that the images RmX, $m \in M$, form a basis for the topology of X, $p : Y \to X$ is a map, and $\Gamma\backslash Y$ is an action of Γ on Y over X.

3.1. Definition [8]. Let M be a monoid acting on a set S on the right, and on the space W on the left. Define a topological category $S/M\backslash W$ with objects $S\times W$, topologized as a disjoint union of copies $s\times W$ of W, and morphisms $S\times N\times W$, topologized as a disjoint union of copies $s\times n\times W$ of W. The structure maps are given by

$$D(s, n, w) = (sn, w) \qquad \text{and} \qquad R(s, n, w) = (s, nw)\ .$$

If S is a set, let Δ_S denote the "simplex on S ", the simplicial set whose set of n-simplices is S^{n+1}, with

$$d_i(x_n, \dots, x_o) = (x_n, \dots, \hat{x}_i, \dots, x_o) \quad \text{and} \quad s_i(x_n, \dots, x_o) = (x_n, \dots x_i, x_i, \dots x_o).$$

If a monoid N acts on S, then N acts on S^{n+1} by the diagonal action, and in fact N acts on Δ_S by simplicial maps.

3.2. Definition. Let N be a monoid acting on a set S on the right and on a space W on the left. We define a simplicial topological category $\Delta_S/M\backslash W$ by

$$(\Delta_S/M\backslash W)_n = S^{n+1}/M\backslash W,$$

with functors

$$d_i : S^{n+1}/M\backslash W \to S^n/M\backslash W \quad \text{and} \quad s_i : S^n/M\backslash W \to S^{n+1}/M\backslash W$$

induced by the simplicial structure of Δ_S.

Since Δ_S is contractible, there is a homotopy equivalence

$$|\Delta_S/M\backslash W| \to |M\backslash W|\ .$$

3.3. Definition. We define a simplicial topological category $\underline{\Gamma\backslash Y}$ with a homotopy equivalence $|\underline{\Gamma\backslash Y}| \to \Gamma\backslash Y$. Let $(\underline{\Gamma\backslash Y})_n$ be the topological category with space of objects $(\Gamma\backslash Y)_n$, space of morphisms $(\Gamma\backslash Y)_n$ and all structure maps the identity. The simplicial maps between the $(\Gamma\backslash Y)_n$ define the functors between the $(\underline{\Gamma\backslash Y})_n$.

Since $(\underline{\Gamma\backslash Y})_n$ is just $(\Gamma\backslash Y)_n$ crossed with a simplex $|\Delta_S|$

on $N = \{0, 1, 2, \ldots\}$, there is an equivalence $|\underline{\Gamma\backslash Y}| \to |\Gamma\backslash Y|$.

3.4. Lemma. Let M (the monoid of immersions of Γ) act on itself on the right, by composition. There is a simplicial functor $F : \Delta_M/M\backslash Y \to \underline{\Gamma\backslash Y}$ such that each $F_n : M^{n+1}/M\backslash Y \to (\underline{\Gamma\backslash Y})_n$ induces a weak equivalence.

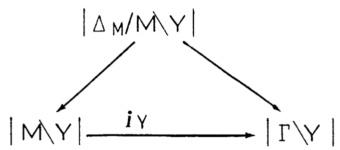
Lemma 3.4 is proved in Sections 4 and 5.

3.5. Proof of 2.5. By the remark after 3.2 there is a homotopy equivalence $|\Delta_M/M\backslash Y| \to |M\backslash Y|$. By 3.3 there is a weak equivalence

$$|\Delta_M/M\backslash Y| \to |\Gamma\backslash Y|.$$

Therefore, there is a weak equivalence $|M\backslash Y| \to |\Gamma\backslash Y|$.

With more work one can show that



commutes up to weak homotopy.

4. DEFINITION OF F_n.

We define the functors $F_n : M^{n+1}/M\backslash Y \to (\underline{\Gamma\backslash Y})_n$ and prove (4.2) that F_n is "onto" in a certain sense. We write elements of $(\Gamma\backslash Y)_n$ as $(f_n, \ldots, f_1, y)$ where

$$f_i \in \Gamma_1 \quad \text{and} \qquad D f_i = R f_{i-1}, \quad i > 1, \quad \text{and} \quad D f_1 = py.$$

Recall that if $m \in M$, $m(x)$ is an element of Γ_1 for $x \in X$.

4.1. Definition. On objects, F_n is the map $F_{n,0} : M^{n+1} \times Y \to (\Gamma\backslash Y)_n$ given by

$$F_{n,0}\,(m_n, \ldots, m_0, y) = (m_n(py) \circ m_{n-1}(py)^{-1}, \ldots, m_1(py) \circ m_0(py)^{-1}, R\, m_0(y)).$$

On morphisms, $F_{n,1} : M^{n+1} \times M \times Y \to (\Gamma\backslash Y)_n$ is defined by

$$F_{n,1}\,(m_n, \ldots, m_0, k, y) = F_{n,0}\,(m_n k, \ldots, m_0 k, y).$$

It's not hard to verify that the F_n define a simplicial functor

$$F_* : \Delta_M/M\backslash Y \to \underline{\Gamma\backslash Y}.$$

4.2. Lemma. Let $(f_n, \ldots, f_1, y) \in (\Gamma\backslash Y)_n$. Then there exists

$(m_n, \dots, m_o, z) \in M^{n+1} \times Y$ such that $F_n(m_n, \dots, m_o, z) = (f_n, \dots, f_1, y)$.

Proof. We prove the result for $n = 1$. For $n > 1$ the result follows similarly, using induction. Let $(f_1, y) \in (\Gamma \backslash Y)_1$. Let $x = Df_1 = py$. There is a section $f_1 : U \to \Gamma_1$ of D on a neighborhood U of x, such that $f_1(x) = f_1$ and Rf_1 is one-to-one on U. Let

$$m_o \in M \quad \text{such that} \quad x \in Rm_oX \subset U ;$$

such a m_o exists because the RmX, $m \in M$, form a basis for the topology of X. Let $x' \in X$ such that $Rm_o(x') = x$. Define $m_1 \in M$ by

$$m_1(x) = f_1(Rm_ox) \circ m_o(x).$$

Since $x = py$, $y \in Rm_oY$. Pick $z \in Y$ so that $Rm_o(z) = y$. Then

$$F_{1,0}(m_1, m_o, z) = (f_1, y).$$

5. PROOF OF LEMMA 3.4.

We have defined functors $F_n : M^{n+1}/M \backslash Y \to (\underline{\Gamma \backslash Y})_n$. There is a projection map $|(\underline{\Gamma \backslash Y})_n| \to (\Gamma \backslash Y)_n$. Let T be the composition

$$T_n : |M^{n+1}/M \backslash Y| \to (\Gamma \backslash Y)_n .$$

To prove 3.4, it is enough that each T_n be a weak equivalence by (1.8).

First we show that the maps T_n are almost locally trivial ([10], Appendix). By ([13], A.1) it then suffices to prove that $T_n^{-1}(x)$ is contractible for every $x \in (\Gamma \backslash Y)_n$.

5.1. Definition [13]. A map $f : B \to A$ of spaces is *almost locally trivial* if for every $a \in A$ there is a neighborhood of $f^{-1}(a)$ in B which is homeomorphic as a space to a neighborhood of $f^{-1}(a) \times a$ in $f^{-1}(a) \times A$.

5.2. The space $T_n^{-1}(x)$. Let $x \in (\Gamma \backslash Y)_n$. We describe $T_n^{-1}(x)$ as the geometric realization $|C_x|$ of a discrete category C_x. The objects of C_x are pairs (s, y) with

$$s \in M^{n+1}, y \in Y \quad \text{such that} \quad F_{n,0}(s, y) = x.$$

The morphisms of C_x are triples (s, m, y) with

$$s \in M^{n+1}, m \in M, y \in Y \quad \text{so that} \quad F_{n,0}(s, m, y) = F_{n,0}(s, Rm(y)) = x.$$

The structure maps D, R of C_x are defined as

$$D(s, m, y) = (sm, y) \quad \text{and} \quad R(s, m, y) = (s, Rm(y)).$$

5.3. Proposition. $T_n : |M^{n+1}/M \backslash Y| \to (\Gamma\backslash Y)_n$ *is almost locally trivial.*

Proof. If $(s, y) \in C_x$ let $V(s, y)$ be a neighborhood of (s, y) in $s \times Y$ such that $F_{n,0}$ is one-to-one restricted to $V(s, y)$. Denote by $N(x)$ the subcategory of $M^{n+1}/M\backslash Y$ generated by the points of the $V(s, y)$; $|N(x)|$ is a neighborhood of $|C_x|$ in $|M^{n+1}/M\backslash Y|$.

Let $M(x)$ be the subcategory of $C_x \times (\underline{\Gamma\backslash Y})_n$ generated by objects

$$((s, y),\ F_{n,0}(s, y')\ ) \quad \text{where} \quad y' \in V(s, y)\ .$$

Then $|M(x)|$ is a neighborhood of $|C_x| \times |\underline{x}|$ in $|C_x| \times |(\underline{\Gamma\backslash Y})_n|$.

The functor $G : N(x) \to M(x)$ given on objects by

$$G(s, y') = ((s, y)\ ,\ F_{n,0}(s, y'))$$

is an isomorphism of categories. Thus, T_n is almost locally trivial.

To complete the proof of Lemma 3.4 we show that the categories C_x have contractible realization.

5.4. Definition. A category C is *codirected* if :

i) for any objects A_1, A_2 of C there is an object B of C, and maps $f_i : B \to A_i$.

ii) If $f_i : B \to A$, $i = 1, 2$ are maps in C there is an object E in C and a map $g : E \to B$ in C such that $f_1 \circ g = f_2 \circ g$.

After Quillen [11], codirected categories have contractible realizations.

5.5. Proof of 3.4. Since the maps T_n are almost locally trivial, we need only show that the C_x have contractible realizations. We will prove that the C_x are codirected. Note that by 4.2, the C_x are nonempty.

Condition ii of 5.4 follows for C_x from the fact that there can be at most one morphism between any two objects in C_x . To verify i we need to show that for every (s_1, y_1), $(s_2, y_2) \in C_x$ there are $y \in Y$, $m_1, m_2 \in M$ such that

$$\text{(i)}\ \ s_2 m_2 = s_1 m_1 \quad \text{and} \quad \text{(ii)}\ \ Rm_1(y) = y_1\ ,\ Rm_2(y) = y_2\ .$$

Write

$$s_2 = (s_2^n, \dots, s_2^0) \quad \text{and} \quad s_1 = (s_1^n, \dots, s_1^0),$$

where each $s_i^j \in M$. Let U_1 be a neighborhood of y_1 on which each Rs_1^j is one-to-one ; define U_2 similarly. Let

$$m_1 \in M \quad \text{so that} \quad py_1 \in Rm_1 X \subset pU_1\ ,$$

and define $m_2 \in M$ by

$$m_2(x) = (s_2{}^o)^{-1}\ (Rs_1{}^o(Rm_1 x)) \circ s_1{}^o(Rm_1 x) \circ m_1(x).$$

It is not hard to verify that $s_2{}^o \circ m_2 = s_1{}^o \circ m_1$, and then, by induction, that $s_2{}^j \circ m_2 = s_1{}^j \circ m_1$. Therefore, $s_1 m_1 = s_2 m_2$.

Now $py_1 \in Rm\, X$, so there is some $y \in Y$ such that $Rm_1(y) = y_1$. Then it follows that

$$Rs_2{}^o(Rm_2 y) = Rs_1{}^o(y_1).$$

But $s_2{}^o$ is one-to-one on U_2, so $Rm_2 y = y_2$.

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Mathematical Sciences Division
North Dakota State University
300 Minard Hall
FARGO, North Dakota 58105. U.S.A.