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of riemannian metrics. II”**

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Correction to: ON ISOSPECTRAL DEFORMATIONS OF RIEMANNIAN METRICS. II

Ruishi Kuwabara

The proof of Lemma 3.3: (1) given in the paper in Vol. 47 [p. 201] is incorrect. We here give a complete proof of the lemma.

Define a differential operator $\delta_g^k: S_{k+1} \rightarrow S_k$ by

$$(\delta_g^k a)^{i_1 \dots i_k} = -(k+1) \nabla_{\rho} a^{\rho i_1 \dots i_k},$$

∇ being the covariant differentiation with respect to g . Then, δ_g^k is the formally adjoint operator of $\hat{\nabla}_g^k$ with respect to the inner products in S_k 's naturally defined by g . Set

$$D_g^k = \frac{1}{k+1} \delta_g^k \hat{\nabla}_g^k.$$

Then D_g^k is a non-negative, self-adjoint, elliptic differential operator of order 2, and the equation $D_g^k a = 0$ is equivalent to $\hat{\nabla}_g^k a = 0$ (see [2]).

Next, let us introduce various norms on the space of tensor fields on M . A (fixed) C^∞ Riemannian metric g_0 naturally defines a norm, $|\cdot|$, on each fibre of the tensor bundle over M . Various global norms for a tensor field T are defined by

$$|T|_k = \max_{0 \leq r \leq k} \sup_{x \in M} \left\{ \underbrace{|\nabla \dots \nabla T(x)|}_r \right\},$$

$$\|T\|_k^2 = \sum_{r=0}^k \left(\int_M \underbrace{|\nabla \dots \nabla T|^2}_{r} dV_{g_0} \right),$$

for $k = 0, 1, 2, \dots$, where ∇ is the covariant differentiation with respect to g_0 .

Using these notations, we have for every $a \in S_k$,

$$\|D_g^k a - D_{g_0}^k a\|_0 \leq C_1 |g - g_0|_1 \|a\|_2 \quad (\text{when } |g - g_0|_1 < 1), \quad (1)$$

C_1 being a constant, because D_g^k is a second order differential operator

whose coefficients consist of g and its first derivatives. On the other hand, since $D_{g_0}^k$ is an elliptic operator of order 2, there is a constant C_2 such that

$$\|a\|_2 \leq C_2 (\|a\|_0 + \|D_{g_0}^k a\|_0), \quad (2)$$

for every $a \in S_k$.

Now we prove that $\mathcal{U}_k = \{g \in \mathcal{R}; (D_g^k)^{-1}(0) = \{0\}\}$ is an open subset of \mathcal{R} . Suppose g_0 belongs to \mathcal{U}_k . Noting that $D_{g_0}^k$ has a discrete spectrum consisting of non-negative real eigenvalues, we have

$$\|D_{g_0}^k a\|_0 \geq \lambda \|a\|_0 \quad (\lambda > 0), \quad (3)$$

for every $a (\neq 0) \in S_k$, where λ is the least eigenvalue. We show g_0 is an interior point of \mathcal{U}_k . If the contrary holds, there are sequences $\{g_n\}_{n=1}^\infty$ in \mathcal{R} and $\{a_n\}_{n=1}^\infty$ in S_k such that $D_{g_n}^k a_n = 0$, $\|a_n\|_0 = 1$, and $g_n \rightarrow g_0$ with respect to the C^∞ topology (i.e. $|g_n - g_0|_k \rightarrow 0$ for every $k \geq 0$) as $n \rightarrow \infty$. Using (1) and (2), we have

$$\begin{aligned} \|D_{g_0}^k a_n\|_0 &= \|D_{g_0}^k a_n - D_{g_n}^k a_n\|_0 \leq C_1 |g_0 - g_n|_1 \|a_n\|_2 \\ &\leq C_1 C_2 |g_0 - g_n|_1 (\|a_n\|_0 + \|D_{g_0}^k a_n\|_0) \\ &= C_1 C_2 |g_0 - g_n|_1 (1 + \|D_{g_0}^k a_n\|_0). \end{aligned}$$

Hence, for sufficiently large n ,

$$\|D_{g_0}^k a_n\|_0 \leq \frac{C_1 C_2 |g_0 - g_n|_1}{1 - C_1 C_2 |g_0 - g_n|_1}.$$

Therefore, we get $\|D_{g_0}^k a_n\|_0 \rightarrow 0$ as $n \rightarrow \infty$. This contradicts (3).

References

- [1] R. KUWABARA: On isospectral deformations of Riemannian metrics. II. *Comp. Math.* 47 (1982) 195–205.
 [2] C. BARBANCE: Sur les tenseurs symétriques. *C.R. Acad. Sc. Paris* 276 (1973) 387–389.

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