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SOME REMARKS ON SYMMETRIC BASIC SEQUENCES IN L_1

B. Maurey and G. Schechtman*

Summary

Every subspace of L_1 with an unconditional basis is isomorphic to a complemented subspace of a subspace of L_1 with a symmetric basis.

1. Introduction

The purpose of this paper is to show that, in spite of the quite simple representation of subspaces of L_1 with symmetric bases, given in [2], such spaces can be very complicated from the point of view of the structure of their complemented subspaces.

THE MAIN THEOREM: *Let $(x_i)_{i=1}^{\infty}$ be an unconditional basic sequence in L_p, $1 \leq p \leq 2$. There exists a symmetric basic sequence $(s_i)_{i=1}^{\infty}$ in L_p such that $(x_i)_{i=1}^{\infty}$ is equivalent to a block basis with constant coefficients of $(s_i)_{i=1}^{\infty}$. In particular $[x_i]_{i=1}^{\infty}$ is isomorphic to a complemented subspace of $[s_i]_{i=1}^{\infty}$.*

Theorems of such a nature were previously discovered by Lindenstrauss [8], Szankowski [15] and Davis [4]. The proof of the main theorem, given in §2, uses the technique of Davis' proof as well as some results concerning the space X_p of Rosenthal [12].

Section 3 is devoted mainly to an alternative proof of the following reduction (cf. Dacunha-Castelle and Krivine [3]): every subspace of L_1 contains some l_p if and only if every subspace of L_1 with a symmetric basis contains one.

Notions which are not explained here can be found in [9] or [10].

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2. The main result

We begin with two known lemmas:

Lemma 1 (Rosenthal [12]): *Let $1<p<2$. There exists a constant K_p such that if $m\geq 1$ is a real number and $(y_{i,m})_{i=1}^{\infty}$ is a sequence of independent, identically distributed, symmetric random variables* (*on* $[0,1]$) *each taking three values in such a manner that $\|y_{i,m}\|_{L_p}=m^{-1}$ and $\|y_{i,m}\|_{L_2}=m$, then*:

$$K_p^{-1}\Big\|\sum_{i=1}^{\infty} c_i y_{i,m}\Big\|_{L_p}\leq \inf\Big\{\Big(\sum_{i=1}^{\infty}|a_i|^2+\Big(\sum_{i=1}^{\infty}|b_i|^p\Big)^{2/p}\Big)^{1/2};\ \forall i\geq 1,\ m^{-1}a_i+mb_i=c_i\Big\}\leq K_p\Big\|\sum_{i=1}^{\infty} c_i y_{i,m}\Big\|_{L_p}$$

Proof: Let $q=p/(p-1)$ and let $z_{i,m}=m^{-2}y_{i,m}$, $i=1,2,\ldots$. It is easily checked that $\|z_{i,m}\|_{L_2}=m^{-1}$ and $\|z_{i,m}\|_{L_q}=m$, $i=1,2,\ldots$. Thus, by the main result of [12]:

$$A_p^{-1}\Big\|\sum_{i=1}^{\infty} c_i z_{i,m}\Big\|_{L_q}\leq \max\Big\{m^{-1}\Big(\sum_{i=1}^{\infty}|c_i|^2\Big)^{1/2},\ m\Big(\sum_{i=1}^{\infty}|c_i|^q\Big)^{1/q}\Big\}\leq A_p\Big\|\sum_{i=1}^{\infty} c_i z_{i,m}\Big\|_{L_q}$$

and $P\colon L_q\to L_q$ defined by:

$$Pf=\sum_{i=1}^{\infty}\Big(\int_0^1 f\frac{z_{i,m}}{\|z_{i,m}\|_{L_2}^2}\Big)z_{i,m}=\sum_{i=1}^{\infty}\Big(\int_0^1 f y_{i,m}\Big)z_{i,m}$$

is a projection of norm $\|P\|\leq A_p$, where A_p depends only on p. It follows that $(y_{i,m})_{i=1}^{\infty}$ is equivalent, with constant depending on p only, to sequence of the functionals biorthogonal to the unit vector basis in the space

$$X_{q,m}=\Big\{(c_i)_{i=1}^{\infty};\ \|(c_i)_{i=1}^{\infty}\|=\max\Big\{m^{-1}\Big(\sum_{i=1}^{\infty}|c_i|^2\Big)^{1/2},\ m\Big(\sum_{i=1}^{\infty}|c_i|^q\Big)^{1/q}\Big\}<\infty\Big\}$$

and the result follows. ■

Let $1\leq p<2$, $m\geq 1$ and define a norm on ℓ_2 by

$$|x|_m=\inf\{(\|y\|_{\ell_2}^2+\|z\|_{\ell_p}^2)^{1/2};\ m^{-1}y+mz=x\}.$$

The unit vectors $(e_i)_{i=1}^{\infty}$ in ℓ_2 clearly form a 1-symmetric basis for

$(\ell_2, |\cdot|_m)$. We define

$$\lambda_m(n) = \left|\sum_{i=1}^{n} e_i\right|_m \qquad n = 1, 2, \ldots$$

In the following lemma we collect some properties of the $\lambda_m(n)$'s. For a proof see [4] or [10, lemma 3.b.3 and the proof of proposition 3.b.4].

LEMMA 2: (1) $\lambda_m(n) = (m^{-2}n^{-1} + m^2 n^{-2/p})^{-1/2}$ *for every integer n and every* $m \geq 1$.

(2) *For a fixed n*:

$$\max_{m\geq 1} \lambda_m(n) = 2^{-1/2} n^{(1/2+1/p)(1/2)}$$

and the maximum is attained at $m = m(n) = n^{(1/p-1/2)(1/2)}$

(3) *For any sequence of positive numbers* $(\epsilon_i)_{i=1}^{\infty}$ *there exists a subsequence* $(n_i)_{i=1}^{\infty}$ *of the integers such that, putting* $m_i = m(n_i)$, $i = 1, 2, \ldots$, *we get*:

$$\sum_{j=1, j\neq i}^{\infty} \lambda_{m_i}(n_j)/\lambda_{m_j}(n_j) < \epsilon_k$$

and

$$\sum_{i=1}^{\infty} \frac{1}{m_i} \leq 1$$

We are ready now for the proof of the main theorem. The proof is written for the case $p = 1$ only, the case $1 < p < 2$ requires only minor changes (and the case $p = 2$ is trivial).

PROOF OF THE MAIN THEOREM: Let $(x_k)_{k=1}^{\infty}$ be a normalized unconditional basic sequence in L_1 and let $1 < p < 2$. $(\epsilon_i)_{i=1}^{\infty}$ will denote a sequence of positive numbers to be specified later and $(n_i)_{i=1}^{\infty}$ will denote subsequence of the integers with the properties stated in Lemma 2.3.

Let $(y_{i,k})_{i=1,k=1}^{\infty}$ be a double sequence of independent, symmetric, three-valued random variables on [0, 1] such that

$$\|y_{i,k}\|_{L_p} = m_k^{-1} \text{ and } \|y_{i,k}\|_{L_2} = m_k \qquad i, k = 1, 2, \ldots.$$

By Lemma 1:

$$\text{(1)} \qquad K_p^{-1}|(c_i)_{i=1}^{\infty}|_{m_k} \leq \left\|\sum_{i=1}^{\infty} c_i y_{i,k}\right\| \leq K_p|(c_i)_{i=1}^{\infty}|_{m_k}$$

for all scalars $(c_i)_{i=1}^{\infty}$, where K_p depends only on p.

Define $s_i \in L_p([0,1]^2)$, $i = 1, 2, \dots$ by:

$$s_i(r,t) = \sum_{k=1}^{\infty} y_{i,k}(r) x_k(t)$$

Then:

$$\|s_i\|_{L_1(L_p)} = \int \left(\int |s_i(r,t)|^p \, \mathrm{d}r \right)^{1/p} \mathrm{d}t \le \sum_{k=1}^{\infty} \|y_{i,k}\|_{L_p} \le 1$$

and $s_i(r,t)$, $i = 1, 2, \dots$ clearly constitutes a 1-symmetric basic sequence in $L_1(L_p)$.

Choose a sequence $(\Gamma_\ell)_{\ell=1}^{\infty}$ of disjoint subsets of the integers with $\bar{\bar{\Gamma}}_\ell = n_\ell$ ($\bar{\bar{A}}$ denotes the cardinality of the set A), and put

$$w_\ell = \sum_{i \in \Gamma_\ell} s_i / \lambda_{m_\ell}(n_\ell) \qquad \ell = 1, 2, \dots$$

$(w_\ell)_{\ell=1}^{\infty}$ is a block basis with constant coefficients of $(s_i)_{i=1}^{\infty}$; we are going to show that, if the ϵ_i's are chosen properly, $(w_\ell)_{\ell=1}^{\infty}$ is equivalent to $(x_\ell)_{\ell=1}^{\infty}$.

Let

$$v_\ell(n,t) = \sum_{i \in \Gamma_\ell} y_{i,\ell}(r) \cdot x_\ell(t) / \lambda_{m_\ell}(n_\ell) \qquad \ell = 1, 2, \dots$$

and note that, by the triangle inequality, (1) and Lemma 2:

$$\tag{2} \begin{aligned} \left\| \sum_{\ell=1}^{\infty} a_\ell w_\ell \right\| &= \left\| \sum_{\ell=1}^{\infty} a_\ell \sum_{k=1}^{\infty} \sum_{i \in \Gamma_\ell} y_{i,k}(r) x_k(t) / \lambda_{m_\ell}(n_\ell) \right\|_{L_1(L_p)} \\ &\le \left\| \sum_{\ell=1}^{\infty} a_\ell v_\ell(n,t) \right\|_{L_1(L_p)} \\ &\qquad + \sum_{\ell=1}^{\infty} |a_\ell| \sum_{k \ne \ell} \left\| \sum_{i \in \Gamma_\ell} y_{i,k} \right\|_{L_p} \Big/ \lambda_{m_\ell}(n_\ell) \\ &\le \left\| \sum_{\ell=1}^{\infty} a_\ell v_\ell(r,t) \right\|_{L_1(L_p)} \\ &\qquad + K_p \sum_{\ell=1}^{\infty} |a_\ell| \sum_{k \ne \ell} \lambda_{m_k}(n_\ell) \Big/ \lambda_{m_\ell}(n_\ell) \\ &\le \left\| \sum_{\ell=1}^{\infty} a_\ell v_\ell(r,t) \right\|_{L_1(L_p)} + K_p \left(\sum_{\ell=1}^{\infty} \epsilon_\ell \right) \max_{i \le \ell < \infty} |a_\ell| \end{aligned}$$

for all scalars $(a_\ell)_{\ell=1}^\infty$. And similarly:

$$\left\|\sum_{\ell=1}^\infty a_\ell w_\ell\right\| \geq \left\|\sum_{\ell=1}^\infty a_\ell v_\ell(r,t)\right\|_{L_1(L_p)} - K_p\Big(\sum_{i=1}^\infty \epsilon_i\Big)\max_{1\leq\ell<\infty}|a_\ell| \tag{3}$$

Now

$$\Big(\sum_{i\in\Gamma_\ell} y_{i,\ell}\Big)_{\ell=1}^\infty$$

is a sequence of symmetric independent random variables and thus forms a 1-unconditional basic sequence in L_p [cf. 12]; it follows that $(v_\ell)_{\ell=1}^\infty$ is a 1-unconditional basic sequence in $L_1(L_p)$, thus

$$\left\|\sum a_\ell v_\ell(r,t)\right\|_{L_1(L_p)} \geq \max_{1\leq\ell<\infty}|a_\ell|\cdot\min_{1\leq\ell<\infty}\|v_\ell(r,t)\|_{L_1(L_p)} \geq K_p^{-1}\max_{1\leq\ell<\infty}|a_\ell| \tag{4}$$

Hence, if the ϵ_i's satisfy

$$\sum_{i=1}^\infty \epsilon_i < \tfrac{1}{2}K_p^{-2}$$

we get from (2), (3) and (4) that

$$\tfrac{1}{2}\left\|\sum a_\ell v_\ell\right\| \leq \left\|\sum a_\ell w_\ell\right\| \leq \tfrac{3}{2}\left\|\sum a_\ell v_\ell\right\|$$

for all choices of scalars $(a_\ell)_{\ell=1}^\infty$, so it is enough to prove that $(v_\ell)_{\ell=1}^\infty$ is equivalent to $(x_\ell)_{\ell=1}^\infty$.

The sequence

$$\Big(\sum_{i\in\Gamma_\ell} y_{i,\ell}/\lambda_{m_\ell}(n_\ell)\Big)_{\ell=1}^\infty$$

is, as was already mentioned, a bounded away from zero 1-unconditional orthogonal sequence and, for $\ell = 1, 2, \ldots$

$$\left\|\sum_{i\in\Gamma_\ell} y_{i,\ell}/\lambda_{m_\ell}(n_\ell)\right\|_{L_2} = \|y_{1,\ell}\|_{L_2}\cdot n_\ell^{1/2}/\lambda_{m_\ell}(n_\ell) = m_\ell\cdot n_\ell^{1/2}/\lambda_{m_\ell}(n_\ell) = 2^{1/2}$$

by Lemma 2.

It follows easily that

$$\left(\sum_{i\in\Gamma_\ell} y_{i,\ell}/\lambda_{m_\ell}(n_\ell)\right)_{\ell=1}^{\infty}$$

is equivalent, in L_p, to the unit vector basis of ℓ_2 and thus there exists a constant K_p^1 such that

$$(K_p^1)^{-1}\left\|\left(\sum_{\ell=1}^{\infty} a_\ell^2 x_\ell^2\right)^{1/2}\right\|_{L_1} \le \left\|\sum_{\ell=1}^{\infty} a_\ell v_\ell(r,t)\right\|_{L_1(L_p)} \le K_p^1\left\|\left(\sum_{\ell=1}^{\infty} a_\ell^2 x_\ell^2\right)^{1/2}\right\|_{L_1}$$

Finally, by the unconditionality of $(x_\ell)_{\ell=1}^{\infty}$ and by Khinchine's inequality, the expression

$$\left\|\left(\sum_{\ell=1}^{\infty} a_\ell^2 x_\ell^2\right)^{1/2}\right\|_{L_1} \text{ is equivalent to } \left\|\sum_{\ell=1}^{\infty} a_\ell x_\ell\right\|_{L_1}. \quad \blacksquare$$

CONCLUSION: *Let $1\le p\le 2$ there exists a 1-symmetric sequence $(s_i)_{i=1}^{\infty}$ in L_p such that every unconditional basic sequence $(x_i)_{i=1}^{\infty}$ in L_p is equivalent, with constant depending only on the unconditionality constant of $(x_i)_{i=1}^{\infty}$, to a block basis with constant coefficients of $(s_i)_{i=1}^{\infty}$.*

PROOF: By [14] there exists, in L_p, a 1-unconditional basic sequence such that any other unconditional basic sequence in L_p is equivalent, with constant depending on the unconditionality constant only, to a subsequence of it. Apply the main theorem to this universal basic sequence. ■

3. Dacunha-Castelle and Krivine's reduction

This section is devoted to the proof of the following proposition.

PROPOSITION 3: *If every subspace of L_1 with a symmetric basis contains an isomorphic copy of ℓ_p for some $1\le p\le 2$ then every subspace of L_1 contains such a subspace.*

We begin by recalling two definitions.

DEFINITION 1 ([6], [7]): An unconditional basis $(x_i)_{i=1}^{\infty}$ is said to be q-concave $(1\le q<\infty)$ if there exists a constant $c>0$ such that for every N and every N elements $y_n = \Sigma_{i=1}^{\infty} a_{i,n}x_i$, $n = 1, \ldots, N$ in $[x_i]_{i=1}^{\infty}$ we have:

$$\left\| \sum_{i=1}^{\infty} \left(\sum_{n=1}^{N} |a_{i,n}|^q \right)^{1/q} x_i \right\| \geq c \left(\sum_{n=1}^{N} \|y_n\|^q \right)^{1/q}$$

DEFINITION 2 ([5]): Given $1 \leq p \leq 2$, an 1-unconditional normalized basis $(x_i)_{i=1}^{n}$ and a sequence $m_n \nearrow \infty$ with $\Sigma_{n=1}^{\infty} m_n^{-1} < \infty$, let $Y = Y(\ell_2, \ell_p, (x_i)_{i=1}^{\infty}, (m_n)_{n=1}^{\infty})$ be the space of all sequences of scalars $\alpha \in \ell_2$ such that

$$\|\alpha\|_Y = \left\| \sum_{n=1}^{\infty} |\alpha|_{m_n} x_n \right\|_X < \infty$$

($|\cdot|_{m_n}$ is defined in the preceding section). $(e_i)_{i=1}^{\infty}$ will denote the unit vectors in this space: note that they form a 1-symmetric basis for Y.

PROPOSITION 4: *Let $q < 2$ and let $(x_i)_{i=1}^{\infty}$ be a q-concave 1-unconditional basic sequence in L_1, let $q < p < 2$, and let $(m_n)_{n=1}^{\infty}$ and $(s_i)_{i=1}^{\infty}$ be as in the proof of the main theorem. Then $(s_i)_{i=1}^{\infty}$ is equivalent to $(e_i)_{i=1}^{\infty}$.*

PROOF: We use the same notation as in the proof of the main theorem. By Khinchine's inequality and the triangle inequality in $\ell_{2/p}$ we get that:

$$\begin{aligned}
\left\| \sum_{i=1}^{\infty} a_i s_i \right\|_{L_1(L_p)} &= \int \left(\int \left| \sum_{k=1}^{\infty} \sum_{i=1}^{\infty} a_i y_{i,k}(r) x_k(t) \right|^p \mathrm{d}r \right)^{1/p} \mathrm{d}t \\
&\geq A_p \int \left(\int \left(\sum_{k=1}^{\infty} \left| \sum_{i=1}^{\infty} a_i y_{i,k}(r) x_k(t) \right|^2 \mathrm{d}r \right)^{p/2} \mathrm{d}t \\
&\geq A_p \int \left(\sum_{k=1}^{\infty} \left(\int \left| \sum_{i=1}^{\infty} a_i y_{i,k}(r) \right|^p \mathrm{d}r \right)^{2/p} |x_k(t)|^2 \right)^{1/2} \mathrm{d}t \\
&\geq A_p K_p^{-1} \|(a_i)_{i=1}^{\infty}\|_Y
\end{aligned}$$

A_p denotes here the Khinchine's constant of L_p: note that we have used the fact that for a fixed t, $(\Sigma_{i=1}^{\infty} a_i y_{i,k}(r) x_k(t))_{k=1}^{\infty}$ constitutes a 1-unconditional basic sequence in L_p.

For the other side inequality we need the fact [cf. 1] that there exists an isometry (into) $T : L_p \to L_1$ and a constant C such that

$$C\|f\|_{L_p} = C\|Tf\|_{L_1} \geq \|Tf\|_{L_q} \geq \|Tf\|_{L_1} = \|f\|_{L_p}$$

for all $f \in L_p$. Using this fact and the q-concavity of $(x_i)_{i=1}^{\infty}$ (with constant c, say) we get that,

$$\begin{aligned}\Big\|\sum_{i=1}^{\infty} a_i s_i\Big\|_{L_1(L_p)} &= \iint\Big|\sum_{k=1}^{\infty}\sum_{i=1}^{\infty} a_i(Ty_{i,k})(r)x_k(t)\Big|\,dt\,dr \\ &\le \Big(\int\Big\|\sum_{k=1}^{\infty}\sum_{i=1}^{\infty} a_i(Ty_{i,k})(r)x_k\Big\|_{L_1}^{q} dr\Big)^{1/q} \\ &\le c^{-1}\Big\|\sum_{k=1}^{\infty}\Big(\int\Big|\sum_{i=1}^{\infty} a_i Ty_{i,k}(r)\Big|^{q} dr\Big)^{1/q} x_k\Big\|_{L_1} \\ &\le c^{-1}C\Big\|\sum_{k=1}^{\infty}\Big\|\sum_{i=1}^{\infty} a_i y_{i,k}\Big\|_{L_p} x_k\Big\|_{L_1} \\ &\le c^{-1}CK_p\|(a_i)_{i=1}^{\infty}\|_Y. \quad \blacksquare\end{aligned}$$

We shall need also the following standard.

LEMMA 5: *Let U be an infinite dimensional subspace of $Y = Y(\ell_2, \ell_p, (x_i)_{i=1}^{\infty}, (m_n)_{n=1}^{\infty})$. Then U contains a subspace which embeds isomorphically in $[x_i]_{i=1}^{\infty}$.*

PROOF: Using standard arguments (compare for example the proof of proposition 3.b.4 in [10]) the proof reduces to showing that the identity map from Y to ℓ_2 is strictly singular.

Now any infinite dimensional subspace of ℓ_2 contains a norm one vector with arbitrarily small ℓ_∞ norm. Fix n and let x be a norm one vector in ℓ_2 such that $\|x\|_{\ell_\infty} \le m_n^{-2p/(2-p)}$. Let y and z be elements of ℓ_2 such that

$$|x|_{m_n} \ge (\|y\|_2^2 + \|z\|_p^2)^{1/2} - 1 \quad \text{and} \quad m_n^{-1}y + m_n z = x$$

We may clearly assume also that $\|m_n z\|_{\ell_\infty} \le \|x\|_{\ell_\infty}$. If $\|m_n^{-1}y\|_{\ell_2} \ge \frac{1}{2}$ then $\|x\|_Y \ge |x|_{m_n} \ge m_n/2 - 1$. If $\|m_n^{-1}y\|_{\ell_2} < \frac{1}{2}$ then $\|m_n z\|_{\ell_2} \ge \frac{1}{2}$ and we get:

$$\tfrac{1}{2} \le \|m_n z\|_{\ell_2} \le \|m_n z\|_{\ell_p}^{p/2} \|m_n z\|_{\ell_\infty}^{(2-p)/2}$$

and

$$\|m_n z\|_{\ell_p} \ge m_n^2/2^{2/p}$$

so that

$$\|x\|_y \ge |x|_{m_n} \ge m_n/2^{2/p} - 1$$

Hence every infinite dimensional subspace of Y contains vectors with norm one in ℓ_2 and with arbitrarily big norm in Y. ■

PROOF OF PROPOSITION 3: Assume that every subspace of L_1 with a symmetric basis contains some ℓ_p. Notice first that if $(x_i)_{i=1}^{\infty}$ is a 1-unconditional basic sequence in L_1 which is q-concave for some $q<2$ then, by the main theorem, Proposition 4, Lemma 5 and the fact [cf. 11] that every infinite dimensional subspace of ℓ_p contains an isomorph of ℓ_p, $[x_i]_{i=1}^{\infty}$ contains ℓ_p isomorphically for some p (necessarily $1 \le p \le q$).

Now let X be a subspace of L_1. If X does not contain ℓ_1 then by Rosenthal's theorem [13] X is isomorphic to a subspace of L_r for some $1<r\le 2$ and since L_r has an unconditional basis the image of X in L_r contains an unconditional basic sequence $(y_i)_{i=1}^{\infty}$. It is clearly enough to show that $[y_i]_{i=1}^{\infty}$ contains some ℓ_p.

Let $(g_i)_{i=1}^{\infty}$ be a sequence in L_1 isometrically equivalent to the unit vector basis of $\ell_{2/r}$ (for instance a sequence of L_1-norm one, independent $2/r$ stable random variables), and define, for $i=1,2,\dots$ and $0\le s,\ t\le 1$

$$x_i(s,t) = g_i(s)|y_i(t)|^r$$

Then $(x_i)_{i=1}^{\infty}$ is 1-unconditional in $L_1((0,1)^2)$ and by the triangle inequality in $\ell_{2/r}$,

$$\begin{aligned}\left\|\sum_{i=1}^{\infty}\left(\sum_{j=1}^{\infty}|a_{i,j}|^{2/r}\right)^{r/2} x_i\right\| &= \int_0^1\left(\sum_{j=1}^{\infty}\sum_{i=1}^{\infty}|a_{i,j}|^{2/r}|y_i(t)|^2\right)^{r/2} dt\\ &\ge \left(\sum_{j=1}^{\infty}\left(\int_0^1\left(\sum_{i=1}^{\infty}|a_{i,j}|^{2/r}|y_i(t)|^2\right)^{r/2} dt\right)^{2/r}\right)^{r/2}\\ &= \left(\sum_{j=1}^{\infty}\left\|\sum_{i=1}^{\infty}a_{i,j}x_i\right\|^{2/r}\right)^{r/2}\end{aligned}$$

i.e. $(x_i)_{i=1}^{\infty}$ is $2/r$ concave. Thus, it follows from the first part of the proof that $[x_i]_{i=1}^{\infty}$ contains a subspace isomorphic to ℓ_p for some $1\le p\le 2/r$. Passing to a smaller subspace we may assume that there exist disjoint blocks

$$b_n = \sum_{i\in E_n} a_i x_i \qquad n=1,2,\dots$$

of $(x_i)_{i=1}^{\infty}$ which are equivalent to the usual basis of ℓ_p. It is easily checked that the blocks

$$c_n = \sum_{i \in E_n} |a_i|^{1/r} y_i \qquad n = 1, 2, \ldots$$

of $(y_i)_{i=1}^{\infty}$ are equivalent to the usual basis of ℓ_{pr}. ■

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