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Z. ALTSHULER

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**A BANACH SPACE WITH A SYMMETRIC
 BASIS WHICH CONTAINS NO ℓ_p OR c_0 , AND ALL
 ITS SYMMETRIC BASIC SEQUENCES ARE EQUIVALENT**

Z. Altshuler*

Abstract

A Banach space having the properties described in the title of this paper is constructed.

In this paper we investigate the symmetric basic sequences in a Banach space with a symmetric basis. It is well known that the unit vector basis in the spaces c_0 and ℓ_p ($1 \leq p < \infty$), is a symmetric basis, and every symmetric basic sequence in each of these spaces is equivalent to it. A natural question is whether there exists any other Banach space X , with a symmetric basis $\{e_n\}_{n=1}^\infty$, which has the same property. Let us recall that by [1], it turns out that if, in addition to the assumption that every symmetric basic sequence in X is equivalent to $\{e_n\}_{n=1}^\infty$, we know that the same holds in X^* , the dual of X , with respect to $\{f_n\}_{n=1}^\infty$, the sequence of the biorthogonal functionals associated to $\{e_n\}_{n=1}^\infty$, then $\{e_n\}_{n=1}^\infty$ is equivalent to the unit vector basis of c_0 or ℓ_p , for some $1 \leq p < \infty$.

We answer the question raised above affirmatively by proving the following

THEOREM: *There exists a Banach space X , with a symmetric basis $\{e_n\}_{n=1}^\infty$, such that all symmetric basic sequences in X are equivalent to each other, and X is not isomorphic to c_0 or ℓ_p , for any $1 \leq p < \infty$.*

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Clearly a Banach space having the properties described in the theorem above contains no subspace isomorphic to c_0 or ℓ_p ($1 \leq p < \infty$). A natural candidate for such an example is the space constructed by Figiel and Johnson [2], which has a symmetric basis and no subspace of which is isomorphic to c_0 or ℓ_p . Our example is obtained by a modification of their construction. Before passing to the proof of the theorem we need some definitions and notations.

DEFINITION: Let X be a Banach space with a symmetric basis $\{e_n\}_{n=1}^\infty$. Let N_i $i = 1, 2, \dots$ be subsets of the set of natural numbers N , so that $\bar{N}_i = \bar{N}_j$ for every i , $N = \bigcup_{i=1}^\infty N_i$ and $N_i \cap N_j = \emptyset$ for all $i \neq j$. For any $0 \neq \alpha = \sum_i \alpha_i e_i \in X$ put $u_i^{(\alpha)} = \sum_{j=1}^\infty \alpha_j e_{i,j}$ where $N_i = \{i, j\}_{j=1}^\infty$ for $i = 1, 2, \dots$. The sequence $\{u_i^{(\alpha)}\}_{i=1}^\infty$ is called a basic sequence generated by α .

Clearly for any $\alpha \in X$ $\{u_i^{(\alpha)}\}_{i=1}^\infty$ is a symmetric basic sequence in X .

If $\{u_n\}_{n=1}^\infty$ and $\{v_n\}_{n=1}^\infty$ are basic sequences in Banach spaces X , respectively Y , we say that $\{u_n\}_{n=1}^\infty$ and $\{v_n\}_{n=1}^\infty$ are equivalent when a series $\sum_i \alpha_i u_i$ converges if and only if $\sum_i \alpha_i v_i$ converges. We write in this case $\{u_n\} \sim \{v_n\}$. We say that a basic sequence $\{u_n\}_{n=1}^\infty$ is bounded if there exists an $M > 0$ such that $M^{-1} < \inf_n \|u_n\| \leq \sup_n \|u_n\| < M$.

The first example of a Banach space which contains no ℓ_p or c_0 is due to Tsirelson [4]. Figiel and Johnson [2], described the dual of this space, which will be denoted by T , and showed that T contains no subsymmetric basic sequence. We also recall that the unit vector basis $\{x_n\}_{n=1}^\infty$ of T is an unconditional basis.

We are ready now to construct our example. First we define a sequence of norms, $|\cdot|_n$, on c_0 , by

$$(1) \quad |\alpha|_n = \sup_j \left[\sum_{i=1}^j \hat{\alpha}_i \omega_i / (2^n + 2^{-n} s_j) \right] \quad \text{where } \alpha = \{\alpha_i\}_{i=1}^\infty \in c_0$$

$\{\hat{\alpha}_i\}_{i=1}^\infty$ is the rearrangement in non-increasing order of $\{|\alpha_i|\}_{i=1}^\infty$, $\omega_i = i^{-1}$, and $s_j = \sum_{i=1}^j \omega_i$. Notice that since

$$(2) \quad 2^{-n-1} \sup_j |\alpha_j| \leq |\alpha|_n \leq \left(\sup_j |\alpha_j| \right) \cdot s_j / (2^n + 2^{-n} s_j) \leq 2^n \sup_j |\alpha_j|$$

we have that for all $n = 1, 2, \dots$, $|\cdot|_n$ is equivalent to the sup norm on c_0 . We put now $Y = \{y \in c_0; \|\sum_n |y|_n x_n\|_T < \infty\}$ where $\{x_n\}_{n=1}^\infty$ is the unit vector basis of T . The space Y is a subspace (called the diagonal) of the direct sum $Z = (\sum_{n=1}^\infty \oplus (c_0, |\cdot|_n))_T$. Since for any unit vector e_j , $j = 1, 2, \dots$ we get $|e_j|_n = (2^n + 2^{-n})^{-1}$, we deduce that the sequence of

unit vectors $\{e_j\}_{j=1}^\infty$ belong to Y , and they clearly form a symmetric basis, with symmetric constant 1. We also remark that we may assume, without loss of generality, that every symmetric basic sequence in Y is equivalent to a symmetric block basic sequence of $\{e_n\}_{n=1}^\infty$. So in order to prove the theorem it suffices to check the block bases of $\{e_n\}_{n=1}^\infty$.

LEMMA 1: *Let $y_m = \sum_{i=p_m+1}^{p_{m+1}} \alpha_i e_i$ be a normalized block basis in Y . If $\lim_{i \rightarrow \infty} \alpha_i = 0$ then there exists a subsequence $\{y_{m_j}\}_{j=1}^\infty$ of $\{y_m\}_{m=1}^\infty$ which is equivalent to a block basis of $\{x_i\}_{i=1}^\infty$, the unit vector basis of T .*

PROOF: For fixed m and N we have by (2) that

$$\sum_{n=1}^{N-1} |y_m|_n \leq \sum_{n=1}^{N-1} 2^n \cdot \max \{|\alpha_i|; p_m < i \leq p_{m+1}\}.$$

Therefore we can construct inductively two increasing sequences of integers $\{m_j\}_{j=1}^\infty$, and $\{N_j\}_{j=1}^\infty$ such that $\|\sum_{n=N_j}^\infty |y_{m_j}|_n x_n\|_T < 2^{-j-1}$ for all $j \geq 1$ and $\|\sum_{n=1}^{N_{j-1}-1} |y_{m_j}|_n x_n\|_T < 2^{-j-1}$ for all $j > 1$. The block basis $\{y_{m_j}\}_{j=1}^\infty$ can be identified with the basic sequence $\{\hat{y}_{m_j}\}_{j=1}^\infty$ in Z where $\hat{y}_{m_j} = (y_{m_j}, y_{m_j}, \dots, y_{m_j}, \dots) \in Z$ $j = 1, 2, \dots$. Put $v_j = (0, 0, \dots, 0, \overset{N_{j-1}}{y_{m_j}}, \overset{N_{j-1}+1}{y_{m_j}}, \dots, y_{m_j}^{N_{j-1}-1}, 0, 0, \dots) \in Z$ $j = 1, 2, \dots$ and notice that for each j ,

$$\|\hat{y}_{m_j} - v_j\|_Z = \left\| \sum_{n=1}^{N_{j-1}-1} |y_{m_j}|_n x_n + \sum_{n=N_j}^\infty |y_{m_j}|_n x_n \right\|_T < 2^{-j}.$$

Hence the basic sequence $\{y_{m_j}\}_{j=1}^\infty$ in Y is equivalent to $\{\hat{v}_j\}_{j=1}^\infty$ which, in turn, is equivalent to the block basis $z_j = \sum_{n=N_{j-1}}^{N_j-1} |y_{m_j}|_n x_n$ $j = 1, 2, \dots$ of $\{x_n\}_{n=1}^\infty$.

We can already state some consequences of Lemma 1.

PROPOSITION 1: *Let Y and $\{e_i\}_{i=1}^\infty$ be as above. Then the following assertions are true:*

- (i) *There is no symmetric block basis $y_m = \sum_{i=p_m+1}^{p_{m+1}} \alpha_i e_i$ $m = 1, 2, \dots$ of $\{e_i\}_{i=1}^\infty$ such that the coefficients $\{\alpha_i\}_{i=1}^\infty$ tend to zero.*
- (ii) *Y contains no subspace isomorphic to c_0 or ℓ_p for any $1 \leq p < \infty$.*

PROOF: The first assertion follows from Lemma 1 and the fact that T contains no subsymmetric basic sequence. To prove the second assertion we assume first that there is a block basis $\{u_j\}_{j=1}^\infty$ of $\{e_i\}_{i=1}^\infty$ which is equivalent to the unit vector basis of ℓ_p , for some $p \geq 1$. Since $\|\sum_{j=1}^n u_j\|_Y \rightarrow \infty$ as $n \rightarrow \infty$ it is easy to construct a block basis $\{v_m\}_{m=1}^\infty$ of $\{u_j\}_{j=1}^\infty$ with coefficients, in the expansion with respect to

$\{e_i\}_{i=1}^\infty$, tending to zero. The proof of this case can be then completed by using (i).

Suppose now that there is a block basis $\{u_j\}_{j=1}^\infty$ of $\{e_i\}_{i=1}^\infty$ which is equivalent to the unit vector basis of c_0 . If the coefficients of the y_j 's form a sequence tending to zero, then we complete the proof of (ii) by (i). Otherwise, it follows easily that $\{e_i\}_{i=1}^\infty$ itself is equivalent to the unit vector basis of c_0 , hence for all $k = 1, 2, \dots$ $\|\sum_{i=1}^k e_i\|_Y \leq M$, for some $M > 0$. On the other hand for any $k \geq 4$ we pick an integer $n = n(k)$ such that $s_k/2 < 2^{2n} \leq 2s_k$. For these values of k and $n = n(k)$ we have

$$\left\| \sum_{i=1}^k e_i \right\|_Y \geq \left| \sum_{i=1}^k e_i \right|_n = s_k/(2^n + 2^{-n}s_k) \geq \sqrt{s_k}/6 = \left(\sum_{i=1}^k i^{-1} \right)^{1/2} / 6 \rightarrow \infty \quad \text{as } k \rightarrow \infty.$$

We consider now block bases generated by one vector in Y .

LEMMA 2: *Every block basis $\{u_i^{(\alpha)}\}_{i=1}^\infty$ of $\{e_j\}_{j=1}^\infty$ generated by a vector $\alpha \in Y$, is equivalent to $\{e_i\}_{i=1}^\infty$.*

PROOF: Let $\{u_i^{(\alpha)}\}_{i=1}^\infty$ be a block basis generated by a vector $0 \neq \alpha = \sum_i \alpha_i e_i \in Y$. Then, for any $\beta = \sum_i \beta_i e_i \in Y$, we have $\|\sum_i \beta_i u_i^{(\alpha)}\| \geq (\sup_j |\alpha_j|) \|\sum_i \beta_i e_i\|$, so in order to prove that $\{u_i^{(\alpha)}\} \sim \{e_i\}$ we have to show that $\sum_i \beta_i u_i^{(\alpha)}$ converges, for any $\beta \in Y$. We first observe that it is enough to prove this for $\beta = \alpha$ i.e. to show that $\sum_i \alpha_i u_i^{(\alpha)}$ is a convergent series for any $0 \neq \alpha = \sum_i \alpha_i e_i \in Y$. Indeed, if this is true for any $\alpha = \sum_i \alpha_i e_i \in Y$ then for any $\beta = \sum_i \beta_i e_i \in Y$ we would get that $\sum_i (\alpha_i + \beta_i) u_i^{(\alpha+\beta)}$, and therefore also that $\sum_i \beta_i u_i^{(\alpha)}$, is a convergent series. Fix $\alpha = \{\alpha_i\}_{i=1}^\infty \in c_0$ with $1 \geq \alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_i \geq \dots \geq 0$, and notice that in order to check whether $\sum_i \alpha_i u_i^{(\alpha)}$ converges in Y we have to compute the $|\cdot|_n$ -norms of the double sequence $\{\alpha_i \alpha_j\}_{i,j=1}^\infty$, which is the expansion of $\sum_i \alpha_i u_i^{(\alpha)}$ with respect to $\{e_i\}_{i=1}^\infty$. Let $\alpha(t)$ be a non-increasing function on $[1, \infty)$ such that $\alpha(i) = \alpha_i$ for all i . If, for some integer m , $i \cdot j = m$ then at least one of the integers i or j is greater than or equal to $m^{1/2}$, and therefore $\alpha_i \alpha_j \leq \alpha(m^{1/2})$. It follows that the non-increasing rearrangement of $\{\alpha_i \alpha_j\}_{i,j=1}^\infty$ (as a one indexed sequence) is majorated by the sequence $\beta = \{\beta_i\}_{i=1}^\infty$ whose explicit form is

$$\beta = (\overbrace{\alpha(1^{1/2})}^{\tau(1) \text{ times}}, \overbrace{\alpha(2^{1/2}), \alpha(2^{1/2})}^{\tau(2) \text{ times}}, \dots, \overbrace{\alpha(m^{1/2}), \dots, \alpha(m^{1/2})}^{\tau(m) \text{ times}}, \dots),$$

where $\tau(m)$ is the number of distinct divisors of m . Thus, for every n , we have $|\sum_i \alpha_i u_i^{(\alpha)}|_n \leq |\beta|_n$.

For each integer m , let $\varphi(m)$ be the first place where $\alpha(m)$ appears in the sequence β . Then, for $\varphi(m) \leq k < \varphi(m+1)$ we have

$$\begin{aligned} \left(\sum_{i=1}^k \beta_i i^{-1} \right) / (2^n + 2^{-n} s_k) &\leq \left(\sum_{j=1}^m \sum_{i=\varphi(j)}^{\varphi(j+1)-1} \beta_i i^{-1} \right) / (2^n + 2^{-n} s_k) \\ &\leq \left(\sum_{j=1}^m \beta_{\varphi(j)} \varphi(j)^{-1} (\varphi(j+1) - \varphi(j)) \right) / (2^n + 2^{-n} s_{\varphi(m)}) \\ &\leq \left(\sum_{j=1}^m \alpha_j \varphi(j)^{-1} (\varphi(j+1) - \varphi(j)) \right) / (2^n + 2^{-n} s_{\varphi(m)}) \end{aligned}$$

Since $s_{\varphi(m)} \geq \log \varphi(m)$ we get that

$$(3) \quad |\beta|_n \leq \sup_m \left[\sum_{j=1}^m \alpha_j \varphi(j)^{-1} (\varphi(j+1) - \varphi(j)) \right] / (2^n + 2^{-n} \log \varphi(m))$$

for all n .

To estimate further the norm of β we use the quite known fact (see e.g. [3, p. 118]) that $\sum_{i=1}^k \tau(i) = k \log k + (2\gamma - 1)k + O(k^{1/2})$ where $\gamma = 0.57721 \dots$ is the Euler constant. Notice that $\varphi(1) = 1$, and for $j > 1$ we have $\varphi(j) = \sum_{i=1}^{j^2-1} \tau(i) = (j^2 - 1) \log(j^2 - 1) + (2\gamma - 1)(j^2 - 1) + O(j)$, consequently

$$(4) \quad \varphi(j) \geq 1 + c_1 j^2 \cdot \log j \quad \text{for all } j \geq 1, \text{ and some constant } c_1 > 0.$$

We also have

$$\begin{aligned} (5) \quad \varphi(j+1) - \varphi(j) &= \sum_{i=1}^{(j+1)^2-1} \tau(i) - \sum_{i=1}^{j^2-1} \tau(i) \\ &\leq (j^2 + 2j) \log(j^2 + 2j) - (j^2 - 1) \log(j^2 - 1) + (2\gamma - 1)(2j + 1) + O(j) \\ &\leq c_2(1 + j \log j) \quad \text{for some constant } c_2 > 0. \end{aligned}$$

Using the fact that s_m behaves asymptotically as $\log m$ and substituting (4) and (5) in (3) we deduce that

$$|\beta|_n < c_3 \sup_m \left(\left(\sum_{j=1}^m \alpha_j j^{-1} \right) / (2^n + 2^{-n} s_m) \right) = c_3 |\alpha|_n,$$

for all n and some $c_3 > 0$.

Since $\alpha \in Y$ implies that $\alpha \in c_0$ we get that $\|\beta\|_Y \leq c_3 \|\alpha\|_Y$, i.e. $\|\sum_i \alpha_i u_i^{(\alpha)}\|_Y \leq c_3 \|\alpha\|_Y$ for all $\alpha \in Y$.

We are ready to give the proof of the theorem. Let $y_m = \sum_{i=p_m+1}^{p_{m+1}} \alpha_i e_i$ be a symmetric normalized block basic sequence in Y . We may assume without loss of generality that $\alpha_{p_m+1} \geq \alpha_{p_m+2} \geq \dots \geq \alpha_{p_{m+1}} \geq 0$, for all $m = 1, 2, \dots$. If $\sup_m (p_{m+1} - p_m) < +\infty$ then clearly $\{y_m\} \sim \{e_m\}$,

hence we may assume also that $\sup_m (p_{m+1} - p_m) = +\infty$.

Suppose now that for any $\epsilon > 0$ there exists an integer $N = N(\epsilon)$ such that $\|\sum_{i=p_m+1}^{p_{m+1}} \alpha_i e_i\| < \epsilon$ for all m with $p_{m+1} - p_m \geq N$. In this case $\{y_m\}_{m=1}^\infty$ is equivalent to a block basis generated by one vector and thus, by Lemma 2, $\{y_m\} \sim \{e_m\}$. Indeed, for any $\epsilon > 0$, let $u_m = \sum_{i=1}^{p_{m+1}-p_m} \alpha_{i+p_m} e_i$ and $u'_m = \sum_{i=1}^{N(\epsilon)} \alpha_{i+p_m} e_i$ $m = 1, 2, \dots$. Using the fact that for all $m = 1, 2, \dots$ u'_m have at most the first $N(\epsilon)$ coordinates distinct from zero, and $\|u_m - u'_m\| < \epsilon$, we deduce that $\{u_m\}_{m=1}^\infty$ is a relatively compact set in Y . Hence there exists a subsequence $\{u''_m\}_{m=1}^\infty$ of $\{u_m\}_{m=1}^\infty$ such that $\lim_{m \rightarrow \infty} u''_m = \beta = \sum_i \beta_i e_i \in Y$. Clearly, a subsequence $\{u_{m_j}\}_{j=1}^\infty$ of $\{u''_m\}$ can be chosen such that $\|u_{m_j} - \beta\| < 2^{-j}$, and since $\|y_m\|_Y = \|u_m\|_Y = 1$, for all m , we have $\|\beta\|_Y = 1$. Notice that $\{u_{m_j}\}_{j=1}^\infty$ is a "translation" of $\{y_{m_j}\}_{j=1}^\infty$. Hence $\{y_{m_j}\}_{j=1}^\infty$ is equivalent to a block basis generated by β .

We treat now the case when such an $N(\epsilon)$ does not exist for all $\epsilon > 0$. In this case there exists an $\epsilon > 0$ and an increasing sequence of integers $\{p_{m_j}\}_{j=1}^\infty$ such that $p_{m_j+1} - p_{m_j} > j$ and $\|\sum_{i=p_{m_j}+1}^{p_{m_j+1}} \alpha_i e_i\|_Y \geq \epsilon$ for all j . Put $v_j = \sum_{i=p_{m_j}+1}^{p_{m_j+1}} \alpha_i e_i$ and $u_j = \sum_{i=p_{m_j}+1}^{p_{m_j+1}} \alpha_i e_i$. Notice that $\alpha_{p_{m_j}+j} > c$ for some constant c and every j imply $1 \geq \|u_j\|_Y \geq c \|\sum_{i=1}^j e_i\|$ $j = 1, 2, \dots$ i.e. $c = 0$. Thus, $\lim_{j \rightarrow \infty} \alpha_{p_{m_j}+j} = 0$ which means that $\{v_j\}_{j=1}^\infty$ is a bounded block basis of $\{e_i\}_{i=1}^\infty$ with coefficients tending to zero. By Lemma 1 (and passing to a subsequence if necessary) we can assume that $\{v_j\}_{j=1}^\infty$ is equivalent to a block basis $\{z_j\}_{j=1}^\infty$ of $\{x_n\}_{n=1}^\infty$, the unit vector basis of T . The definition of the norm in T implies the existence of a constant $A_1 > 0$ such that, for every k , we have $\|\sum_{i=k+1}^{2k} z_i\|_T \geq A_1 k$. It follows that for every integer k and some constant $A_2 > 0$ $\|\sum_{j=1}^{2k} y_{m_j}\|_Y \geq \|\sum_{j=1}^{2k} v_j\|_Y \geq A_2 \|\sum_{j=1}^{2k} z_j\|_T \geq A_1 A_2 k$. Since $\{y_{m_j}\}_{j=1}^\infty$ is a symmetric basic sequence we get that $\{y_{m_j}\}_{j=1}^\infty$, and therefore $\{y_m\}_{m=1}^\infty$, is equivalent to the unit vector basis of ℓ_1 , contrary to Proposition 1. This completes the proof of the theorem.

REMARK: One can check that the unit balls determined by the norms $|\cdot|_n$ $n = 1, 2, \dots$, are the sets $2^{-n}B_0 + 2^n B_d$, where B_0 and B_d are the unit balls of c_0 , respectively of the Lorentz space $d(i^{-1}, 1)$. (Recall that $d(i^{-1}, 1)$ is the space of all sequences $\{\alpha_i\}_{i=1}^\infty \in c_0$ such that $\|\alpha\|_d = \sum_{i=1}^\infty \hat{\alpha}_i i^{-1} < \infty$, where $\{\hat{\alpha}_i\}_{i=1}^\infty$ is the non-increasing rearrangement of $\{\alpha_i\}_{i=1}^\infty$). Similarly, it can be shown that the sequence of norms $\|\cdot\|_n$ $n = 1, 2, \dots$ defined by Figiel and Johnson in [2] can be given explicitly by the formulas

$$\|\alpha\|_n = \sup_j \left[\left(\sum_{i=1}^j \hat{\alpha}_i \right) / (2^n + 2^{-n}j) \right] \quad n = 1, 2, \dots$$

In this case it is not true any more that for any $\alpha = \{\alpha_i\}_{i=1}^\infty \in c_0$ we have $\|\{\alpha_{ij}\}_{i,j=1}^\infty\|_n \leq c \|\{\alpha_i\}_{i=1}^\infty\|_n$ for all n and some $c > 0$. Indeed, for the sequence $\alpha_i = i^{-1/2}$ we can find constants $A, B > 0$ such that $\|\{i^{-1/2}\}_{i=1}^\infty\|_n \leq A$ but $\|\{(ij)^{-1/2}\}_{i,j=1}^\infty\|_n \geq Bn^{1/2}$ for all $n = 1, 2, \dots$

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Institute of Mathematics
Hebrew University
Jerusalem