COMPOSITIO MATHEMATICA

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Compositio Mathematica, tome 23, nº 3 (1971), p. 347-356

<http://www.numdam.org/item?id=CM_1971__23_3_347_0>

COMPOSITIO MATHEMATICA, Vol. 23, Fasc. 3, 1971, pag. 347–356
Wolters-Noordhoff Publishing
Printed in the Netherlands

FORMATIONS AND π-CLOSURE IN FINITE GROUPS

by

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1. Introduction

The theory of formations of finite solvable groups, developed by Gaschütz [6], Carter and Hawkes [5], and Schunck [12], provides some general methods for investigating conjugate classes of subgroups in solvable groups. In some situations their methods depend on the Theorem of Galois on primitive solvable groups, hence their methods can not be applied to the class of all finite groups. However, using formations of π-closed groups, Lausch [11] has extended some of the results of Gaschütz [6] and Carter and Hawkes [5] to π'-solvable groups.

Let $\mathscr{F}$ be a homomorph and let π denote a set of prime numbers. Denote by $\mathscr{F}(\pi)$ the class of all finite groups G which contain a normal Hall π'-subgroup and a normal Hall π-subgroup belonging to $\mathscr{F}$. Lausch [10] investigated $\mathscr{F}(\pi)$ in the case when $\mathscr{F}$ is a formation. It is our purpose in the present paper to consider $\mathscr{F}(\pi)$ in some detail. Among other things, we extend several results of Lausch [10] and Gaschütz [6].

The main purpose of this paper is to prove the following theorem: *Let $\mathscr{F}$ be a homomorph such that $\mathscr{F}(\pi)$ is a saturated formation. If G is a π-solvable group, then G possesses an $\mathscr{F}(\pi)$-covering subgroup and any two such subgroups are conjugate.*

Let π denote the set of all primes. Then $\mathscr{F}(\pi) = \mathscr{F}$ and our theorem is a generalization of the following theorem of Gaschütz (see Satz 7.10 of [9, p. 700]) which states: *Let $\mathscr{F}$ be a saturated formation. If G is a solvable group, then G possesses an $\mathscr{F}$-covering subgroup and all such subgroups are conjugate.*

In the final section we consider the formation $\mathscr{N}(\pi)$, where $\mathscr{N}$ is the saturated formation of nilpotent groups. A subgroup H of a π-solvable group G is termed a K_π-subgroup of G provided 1) $H \in \mathscr{N}(\pi)$, 2) $N_G(H) = H$, and 3) H contains a Hall π'-subgroup of G. We prove: *If L is a K_π-subgroup of the π-solvable group G, then L is $\mathscr{N}(\pi)$-covering subgroup of G.* This result generalizes Proposition 1 of [10].

[1] The author was supported by National Science Foundation Grant G.P. 8627.

2. Preliminaries

The only groups considered here are finite. If H is a subset of the group G, then

$\{H\}$ is the subgroup generated by H,
$N_G(H)$ is the normalizer of H in G,
$C_G(H)$ is the centralizer of H in G,
$H^x = xHx^{-1}$ for each x belonging to G,
$H < G$ means H is a proper subgroup of G,
$H \leqq G$ means H is a subgroup of G,
$|H|$ is the order of H,
H_π is the set of π-elements of H, π is a set of prime numbers.

If $H \leqq G$, then $[G : H]$ will denote the index of H in G, core (H) will denote the core of H in G, and $\phi(G)$ will denote the Frattini subgroup of G.

Throughout the present paper π will denote a set of prime numbers and π' will denote the complement of π in the set of all prime numbers. A positive integer n is called a *π-number* if the only prime divisors of n belong to π. A subgroup H of the group G is termed a *Hall π-subgroup* if $|H|$ is a π-number and $[G : H]$ is a π'-number. Throughout this paper $\text{Hall}_\pi(G)$ will denote the set of all Hall π-subgroups of G. The group G is termed π-closed if it posesses a normal Hall π-subgroup. The group G is π-closed if and only if G_π is a normal subgroup of G. The group G is termed *π-separated* if all the chief factors of G are either π-groups or π'-groups. In [4] and [8] π-separated groups are called π-serial. The group G is termed *π-solvable* if G is π-separated and the π-chief factors are solvable. We mention that π-separated groups are discussed in [2], [3] and [9, p. 659–661].

The group G is said to satisfy *condition D_π* (see [4] and [8]) if 1) $\text{Hall}_\pi(G)$ is nonempty, 2) $\text{Hall}_\pi(G)$ is a class of conjugate subgroups of G, and 3) if M is a π-sugbroup of G, then $M \leqq H$ for some $H \in \text{Hall}_\pi(G)$. Because of Satz 5 of [4] a π-separated group satisfies D_π.

Let $\mathscr{F}$ denote a nonempty class of finite groups. Then $\mathscr{F}$ is called a *homomorph* if $\mathscr{F}$ is closed under epimorphic images. Schunck [12] has investigated homomorphs of solvable groups. A homomorph $\mathscr{F}$ is called a *saturated homomorph* if $G/\phi(G)$ belongs to $\mathscr{F}$ implies G belongs to $\mathscr{F}$. A subgroup H of the group G is termed a *$\mathscr{F}$-covering subgroup* of G if and only if $H \in \mathscr{F}$ and for $H \leqq L \leqq G$ and K a normal subgroup of L such that $L/K \in \mathscr{F}$ implies $L = KH$.

Let $\mathscr{F}$ be a homomorph. The *$\mathscr{F}$-commutator*, denoted $[G, \mathscr{F}]$, of G is the intersection of all normal subgroups K of G such that $G/K \in \mathscr{F}$ (see [1, p. 126]). $\mathscr{F}$ is called a *formation* if $G/[G, \mathscr{F}] \in \mathscr{F}$ for each group

G. The reader is referred to [5], [6], [7] and [9, p. 696–711] for a study of the basic properties of formations of finite solvable groups.

A formation is called a *saturated formation* if it is a saturated homomorph [9, p. 696]. We note that the concept of saturated formation used is slightly weaker than that used by Lausch [10]. Lausch [10] called the formation $\mathscr{F}$ saturated if $G \notin \mathscr{F}$ but $G/N \in \mathscr{F}$, N a minimal normal subgroup of G, implies N is complemented in G. However, in the case of formations of solvable groups the above definitions of saturated formation are equivalent. This fact is included in Satz 4.2 of [14].

Let $\mathscr{F}$ be a homomorph and let π be a set of prime numbers. Throughout this paper $\mathscr{F}(\pi)$ will denote the class of finite groups which are π-closed and π'-closed and G_π belongs to $\mathscr{F}$. The class $\mathscr{F}(\pi)$ has recently been studied by Lausch [10] in the case $\mathscr{F}$ is a formation of finite groups. In the present paper we provide several more general theorems than given by Lausch [10] and also extend Satz 2.1 of [6].

LEMMA 2.1. *If $\mathscr{F}$ is a homomorph, then $\mathscr{F}(\pi)$ is also a homomorph.*

PROOF. Let G belong to $\mathscr{F}(\pi)$ and let H be a normal subgroup of G. Since $(G/H)_\pi = G_\pi H/H$ and $(G/H)_{\pi'} = G_{\pi'} H/H$, it follows that G/H is both π-closed and π'-closed. We also note that $G_\pi H/H \in \mathscr{F}$ since $\mathscr{F}$ is a homomorph. Thus $G/H \in \mathscr{F}(\pi)$ and the proof is complete.

The next result follows from the proof of Theorem 1 of [10].

LEMMA 2.2. *If $\mathscr{F}$ is a formation, then $\mathscr{F}(\pi)$ is also a formation.*

A homomorph $\mathscr{F}$ is said to be *subgroup-inherited* if $G \in \mathscr{F}$ and $H < G$ implies $H \in \mathscr{F}$.

LEMMA 2.3. *If $\mathscr{F}$ is a homomorph which is subgroup-inherited, then $\mathscr{F}(\pi)$ is also a homomorph which is subgroup-inherited.*

PROOF. Because of Lemma 2.1 $\mathscr{F}(\pi)$ is a homomorph. Let $G \in \mathscr{F}(\pi)$ and let H be a subgroup of G. Then $H_\pi = H \cap G_\pi$ and so H is π-closed. Similarly, H is π'-closed. Since $G_\pi \in \mathscr{F}$ it follows that H_π also belongs to $\mathscr{F}$. This completes the proof.

A homomorph $\mathscr{F}$ is said to be *closed under normal products* if M and N are normal subgroups of G both of which belong to $\mathscr{F}$, then MN also belongs to $\mathscr{F}$. Let θ denote the formation of all π-closed groups. Then θ is closed under normal products (see [1, p. 129]). We also note that θ is closed under direct products. Hence, we have the following lemma.

LEMMA 2.4. *Let $\mathscr{F}$ be a homomorph.*

(a) *If $\mathscr{F}$ is closed under normal products, then $\mathscr{F}(\pi)$ is also closed under normal products.*

(b) *If $\mathscr{F}$ is closed under direct products, then $\mathscr{F}(\pi)$ is also closed under direct products.*

THEOREM 2.1. *If $\mathscr{F}$ is a saturated homomorph, then $\mathscr{F}(\pi)$ is also a saturated homomorph.*

PROOF. Because of Lemma 2.1 $\mathscr{F}(\pi)$ is a homomorph. Let $G/\phi(G)$ belong to $\mathscr{F}(\pi)$. Because of Proposition 3 of [1, p. 132] G is π-closed and π'-closed. Hence, $G = G_\pi \times G_{\pi'}$ and because of Theorem 7.3.23 of [13] it follows that $\phi(G) = \phi(G_\pi) \times \phi(G_{\pi'})$. From this we conclude that $(\phi(G))_\pi = \phi(G_\pi)$ and since $G_\pi \phi(G)/\phi(G)$ belongs to $\mathscr{F}$, it follows that $G_\pi/\phi(G_\pi)$ also belongs to $\mathscr{F}$. This shows that $G \in \mathscr{F}(\pi)$ and the proof is complete.

Because of Lemma 2.2 and Theorem 2.1 we obtain the theorem which follows.

THEOREM 2.2. *If $\mathscr{F}$ is a saturated formation, then $\mathscr{F}(\pi)$ is also a saturated formation.*

We conclude this section with a lemma which is useful in the following sections.

LEMMA 2.5. *Let M be a normal subgroup of G and let Ω denote a class of conjugate subgroups of G. Let $S \in \Omega$ and assume that the members of Ω which are contained in SM are conjugate in SM. Then $N_G(SM) = N_G(S)M$.*

PROOF. We note that $N_G(S)M \leqq N_G(SM)$. Let $y \in N_G(SM)$. Then $S^y M = SM$, hence $S^{ym} = S$ for some $m \in M$. From this it follows that $ym \in N_G(S)$ and so $y \in N_G(S)M$. This completes the proof.

3. The main theorem

In this section we establish the main theorem which was mentioned in the introduction. We begin with the following lemma.

LEMMA 3.1. *Let $\mathscr{F}$ be a homomorph and let G be a π-separated group. If G is π'-closed and contains a Hall π-subgroup H from $\mathscr{F}$, then $N_G(H)$ is an $\mathscr{F}(\pi)$-covering subgroup of G.*

PROOF. We use induction on $|G|$. Let $N = N_G(H)$ and note that since G is π'-closed it follows that N is also π'-closed. Hence, N belongs to $\mathscr{F}(\pi)$ since $H \in \mathscr{F}$.

Let $N \leqq B \leqq G$ and let M be a normal subgroup of B such that $B/M \in \mathscr{F}(\pi)$. Assume that $B < G$. Then $N_B(H) = N_G(H)$, $H \in \mathrm{Hall}_\pi(B)$ and B is π'-closed. By induction N is an $\mathscr{F}(\pi)$-covering subgroup of the π-separated group B. Hence, we can assume that $B = G$. We can also assume that $M \neq 1$. For if $M = 1$, then $G \in \mathscr{F}(\pi)$ and it follows that $N_G(H) = G$.

Next we note that G/M is π'-closed and HM/M is a Hall π-subgroup of the π-separated group G/M. Since $\mathscr{F}$ is a homomorph it follows that $HM/M \in \mathscr{F}$. By induction it follows that $N_{G/M}(HM/M)$ is an $\mathscr{F}(\pi)$-covering subgroup of G/M. Since $G/M \in \mathscr{F}(\pi)$, $G/M = N_{G/M}(HM/M)$. Because of Satz 5 of [4] and Lemma 2.5, it follows that $N_{G/M}(HM/M) = N_G(HM)/M = N_G(H)M/M$, hence $G = NM$. This shows that $N = N_G(H)$ is an $\mathscr{F}(\pi)$-covering subgroup of G and the proof is complete.

THEOREM 3.1. *Let $\mathscr{F}$ be a homomorph such that $\mathscr{F}(\pi)$ is a saturated formation. If G is a π-solvable group, then G possesses an $\mathscr{F}(\pi)$-covering subgroup.*

PROOF. Let M be a minimal normal subgroup of G. By induction on $|G|$, it follows that the π-solvable group G/M possesses an $\mathscr{F}(\pi)$-covering subgroup L/M. Assume that $L < G$. Then by induction L possesses an $\mathscr{F}(\pi)$-covering subgroup H. Because of Hilfssatz 1.8 of [14] H is an $\mathscr{F}(\pi)$-covering subgroup of G. Hence, we can assume that $G/M \in \mathscr{F}(\pi)$ for each minimal normal subgroup M of G. Since $\mathscr{F}(\pi)$ is a formation we can also assume that M is the unique minimal normal subgroup of G. Since G is π-solvable, M is either a π-group or a π'-group. We distinguish two cases.

CASE 1. M is a π-group. Then M is an elementary abelian p-group, p a prime number from π. Since $\mathscr{F}(\pi)$ is a saturated formation we can assume $\phi(G) = 1$, hence there is a maximal subgroup B of G such that $M \not\leq B$. Hence, $G = MB$ and since M is abelian it follows that $M \cap B = 1$. Thus $B \in \mathscr{F}(\pi)$ and since B is a maximal subgroup of G and M is the unique minimal normal subgroup of G, it follows that B is an $\mathscr{F}(\pi)$-covering subgroup of G.

CASE 2. M is a π'-group. Since $G/M \in \mathscr{F}(\pi)$, there exists subgroups K and W of G such that $K/M = (G/M)_\pi$ and $W/M = (G/M)_{\pi'}$. Since M is a π'-group, it follows that $W = G_{\pi'}$, hence G is π'-closed. Further, by the Schur-Zassenhaus theorem ([13, p. 224]) it follows that $K = MH$ with $M \cap H = 1$ and $H \in \mathrm{Hall}_\pi(K)$. We note that $H \in \mathrm{Hall}_\pi(G)$ and also $H \cong K/M \in \mathscr{F}$.

Hence, G is π'-closed and contains a Hall π-subgroup H which belongs to $\mathscr{F}$. Because of Lemma 3.1 $N_G(H)$ is an $\mathscr{F}(\pi)$-covering subgroup of G.

This completes the proof of the theorem.

Because of Theorems 2.2 and 3.1 we obtain the following result.

THEOREM 3.2. *Let $\mathscr{F}$ be a saturated formation and let G be a π-solvable group. Then G possesses an $\mathscr{F}(\pi)$-covering subgroup.*

We now proceed to show that $\mathscr{F}(\pi)$-covering subgroups of π-solvable groups are conjugate. We begin with the following lemma.

LEMMA 3.2. *Let $\mathscr{F}$ be a homomorph and let G be a π-separated group which is π'-closed. Let G contain a Hall π-subgroup $H \in \mathscr{F}$. If L is an $\mathscr{F}(\pi)$-covering subgroup of G, then L is conjugate to $N_G(H)$.*

PROOF. Let L be an $\mathscr{F}(\pi)$-covering subgroup of G and let M be a minimal normal subgroup of G. Because of Hilfssatz 1.7 of [14] LM/M is an $\mathscr{F}(\pi)$-covering subgroup of G/M. We also note that $N_{G/M}(HM/M)$ is an $\mathscr{F}(\pi)$-covering subgroup of G/M by Lemma 3.1. Because of Satz 5 of [4] and Lemma 2.5 it follows that $N_{G/M}(HM/M) = N_G(H)M/M$. By induction on $|G|$, there is an element $x \in G$ such that $N_G(H^x) \leqq LM$. Hence, we can assume that $N_G(H) \leqq LM$. Assume that $LM < G$. By Hilfssatz 1.7 of [14] L and $N_G(H) = N_{LM}(H)$ are $\mathscr{F}(\pi)$-covering subgroups of LM. Hence, by induction it follows that L and $N_G(H)$ are conjugate subgroups of LM.

Hence, we can assume that $LM = G$. We note that $G/M \in \mathscr{F}(\pi)$. Since G is a π-separated group, M is either a π-group or a π'-group. Hence, we distinguish two cases.

CASE 1. M is a π-group. Because of Satz 5 of [4], $M \leqq H$ and it follows that $H = G_\pi$. From this we conclude that $G = L = N_G(H)$.

CASE 2. M is a π'-group. Then $L \cap M$ is a π'-subgroup of L and $G/M = LM/M \cong L/M \cap L$. Hence, L contains a Hall π-subgroup W of G. Because of Satz 5 of [4] there is an element x of G such that $H^x = W$, and so we can assume that $H \leqq L$. Since L is an $\mathscr{F}(\pi)$-covering subgroup of $G, L \in \mathscr{F}(\pi)$ and so H is a normal subgroup of L. Therefore, $L \leqq N_G(H) \in \mathscr{F}(\pi)$, and it now follows that $L = N_G(H)$. This completes the proof.

THEOREM 3.3. *Let $\mathscr{F}$ be a homomorph and let G be a π-solvable group. Then any two $\mathscr{F}(\pi)$-covering subgroups of G are conjugate.*

PROOF. Let K and L be $\mathscr{F}(\pi)$-covering subgroups of G and let M be a minimal normal subgroup of G. Because of Hilfssatz 1.7 of [14] KM/M and LM/M are $\mathscr{F}(\pi)$-covering subgroups of G/M. By induction on $|G|$, there is an element $x \in G$ such that $L^x \leqq KM$. Assume that $KM < G$. Because of Hilfssatz 1.7 of [14] L^x and K are $\mathscr{F}(\pi)$-covering subgroups of KM, hence by induction L^x and K are conjugate subgroups of KM. Hence, we can assume that $KM = LM = G$ for each minimal normal subgroup M of G. We note that $G/M \in \mathscr{F}(\pi)$ for each minimal subgroup M of G. Further, we can assume $\text{core}(L) = 1$. For if $\text{core}(L) \neq 1$, then $L = L\,\text{core}(L) = G$, hence $G \in \mathscr{F}(\pi)$ and so $G = K = L$.

Let M be a minimal normal subgroup of G. Then M is either a π-group or a π'-group. We distinguish two cases.

CASE 1. M is a π'-group. Let W be the normal subgroup of G such that $W/M = (G/M)_{\pi'}$. Then $W = G_{\pi'}$. Let X be a subgroup of G such that $X/M = (G/M)_{\pi}$. Then $X/M \in \mathscr{F}$ and by the Schur-Zassenhaus theorem ([13, p. 224]) it follows that there is a subgroup H of X such that $X = MH$ and $M \cap H = 1$. We note that $H \in \mathrm{Hall}_{\pi}(G)$ and $H \cong X/M \in \mathscr{F}$, hence because of Lemma 3.2 K and L are both conjugate to $N_G(H)$. Therefore, K and L are conjugate subgroups of G.

CASE 2. M is a π-group. Then M is an elementary abelian p-group with $p \in \pi$. We note that $\mathrm{core}(L) = \mathrm{core}(K) = 1$ and $G = KM = LM$. Since M is abelian, it follows that K and L are maximal subgroups of G and $K \cap M = L \cap M = 1$.

Assume that $K_{\pi} = 1$. Since $K \simeq G/M \simeq L$, K and L are Hall π'-subgroups of G. Because of Satz 5 of [4], K and L are conjugate in G. Hence, we can now assume that $K_{\pi} \neq 1$, and it follows that K is a maximal subgroup of G of core 1 which contains a solvable normal subgroup $K_{\pi} \neq 1$. Because of Lemma 4, part (a), of [1, p. 123] K contains a normal subgroup, not 1, whose order is relatively prime to $[G : K]$. Since L is a maximal subgroup of G of core 1, it follows by Lemma 4, part (b), of [1, p. 123] that L and K are conjugate subgroups of G. This completes the proof.

REMARK 3.1. Let $\mathscr{F}$ be a homomorph and let π denote the set of all prime numbers. Then $\mathscr{F}(\pi) = \mathscr{F}$ and G is π-solvable if and only if G is solvable. Therefore, Theorem 3.3 is a generalization of Satz 3.5 of [12].

Because of Theorems 3.1 and 3.3 we obtain the main theorem of the present paper.

THEOREM 3.4. *Let $\mathscr{F}$ be a homomorph such that $\mathscr{F}(\pi)$ is a saturated formation. If G is π-solvable, then G possesses an $\mathscr{F}(\pi)$-covering subgroup and all such subgroups are conjugate.*

REMARK 3.2. Let $\mathscr{F}$ be a saturated formation and let π denote the set of all prime numbers. Then $\mathscr{F}(\pi) = \mathscr{F}$ and we note that Theorem 3.4 is a generalization of part (b) of Satz 7.10 of [9, p. 700].

From Theorems 2.2 and 3.4 we obtain the following theorem.

THEOREM 3.5. *Let $\mathscr{F}$ be a saturated formation. Then every π-solvable group possesses an $\mathscr{F}(\pi)$-covering subgroup, and all such subgroups are conjugate.*

4. The formation $\mathscr{N}(\pi)$

In this section we take $\mathscr{F}$ to be the class $\mathscr{N}$ of finite nilpotent groups. Since $\mathscr{N}$ is a saturated formation it follows from Theorem 2.2 that

$\mathcal{N}(\pi)$ is also saturated formation. We now give a set of conditions under which a subgroup H of the π-solvable group G is an $\mathcal{N}(\pi)$-covering subgroup.

A subgroup H of the π-solvable group G is called a *K_π-subgroup* of G if and only if 1) $H \in \mathcal{N}(\pi)$, 2) $N_G(H) = H$, and 3) H contains a Hall π'-subgroup of G.

We now give a theorem which extends Proposition 1 of Lausch [10] to the class of π-solvable groups. We note that the method of proof is very similar to that used by Lausch.

THEOREM 4.1. *Let G be a π-solvable group. If L is a K_π-subgroup of G, then L is an $\mathcal{N}(\pi)$-covering subgroup of G.*

PROOF. We use induction on $|G|$. Let L be a K_π-subgroup of G and let M denote the $\mathcal{N}(\pi)$-commutator of G.

Let $L \leqq H < G$ and note that L is a K_π-subgroup of the π-solvable group H. By induction it follows that L is an $\mathcal{N}(\pi)$-covering subgroup of H, hence it suffices to show $G = LM$.

Assume that $M = 1$. Then $G \in \mathcal{N}(\pi)$, hence G_π is a nilpotent normal subgroup of G. Since $G_{\pi'}$ is a normal subgroup of G, it follows that $G_{\pi'} \leqq L$, hence we must show that $L_\pi = G_\pi$. Assume that $L_\pi < G_\pi$. Since G_π is nilpotent, there is a π-element $x \in G_\pi$, $x \notin L_\pi$, such that $x \in N_{G_\pi}(L_\pi)$. Hence, $x \in N_G(L) = L$ which is a contradiction. Hence, $L_\pi = G_\pi$ and it follows that $G = L$.

We can now assume that $M \neq 1$. Let W be a minimal normal subgroup of G such that $W \leqq M$. We also assume that $LW < G$. We now show LW/W is a K_π-subgroup of G/W. We note that $LW/W \in \mathcal{N}(\pi)$ and also LW/W contains a Hall π'-subgroup of G/W. Now let $x \in N_G(LW)$. Then L and L^x are K_π-subgroups of LW. Hence, by induction, L and L^x are $\mathcal{N}(\pi)$-covering subgroups of LW. Because of Theorem 3.5 there is an element $y \in W$ such that $L = L^{xy}$, hence $xy \in N_G(L)$. From this we conclude that $N_{G/W}(LW/W) = N_G(LW)/W = N_G(L)W/W$, hence $N_{G/W}(LW/W) = LW/W$. Thus LW/W is a K_π-subgroup of G/W and it follows by induction that LW/W is an $\mathcal{N}(\pi)$-covering subgroup of G/W. Because of Lemma 1 of [1, p. 128] M/W is the $\mathcal{N}(\pi)$-commutator of G/W, hence $G/W = (LW/W)M/W$ and it follows that $G = LM$. This completes the proof of the theorem.

We now list several corollaries which follow immediately from Theorem 4.1.

COROLLARY 4.1.1. *Let G be a π-solvable group. Then any two K_π-subgroups of G are conjugate.*

COROLLARY 4.1.2. *Let G be a π-solvable group which contains a self-*

normalizing Hall π'-subgroup L of G. Then L is an $\mathscr{N}(\pi)$-covering subgroup of G.

REMARK 4.1. We note that Corollary 4.1.2 extends the corollary to Proposition 1 of Lausch [10].

Let G denote the symmetric group on four symbols and let $\pi = \{2\}$. Then the 2-Sylow subgroups are the $\mathscr{N}(\pi)$-covering subgroups of G. Let L be a 2-Sylow subgroup of G. Then $L = N_G(L)$, but L does not contain a Hall π'-subgroup of G. Hence, L is not a K_π-subgroup.

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(Oblatum 19–V–1970)

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