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Correction

On a class of starlike functions

by

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Compositio Mathematica 19 (1968), 78-82

Introduction

In a recent paper (Comp. Math., 19 (1968), 78-82) with the above title we investigated the class \bar{S} of functions

$$f(z) = z + a_2 z^2 + \dots$$

that are analytic and starlike in $|z| < 1$ and satisfy the condition $|\{z f'(z)/f(z)\} - 1| \leq 1$ in $|z| < 1$. Theorem 4 of that paper, giving the radius of convexity of the class \bar{S} , contains an error and as such the result obtained is not sharp. Below we give the correct proof of that theorem.

THEOREM. *Each function $f(z) \in \bar{S}$ maps*

$$|z| < \frac{(3 - \sqrt{5})}{2}$$

onto a convex domain.

PROOF. If $f(z) \in \bar{S}$, we can write

$$(1) \quad z \frac{f'(z)}{f(z)} = 1 + \psi(z),$$

when $\psi(z)$ is analytic in $|z| < 1$ and satisfies $|\psi(z)| \leq 1$ and $\psi(0) = 0$. Logarithmic differentiation of (1) yields:

$$z \frac{f''(z)}{f'(z)} = \frac{z\psi'(z)}{1 + \psi(z)} + \psi(z).$$

Making use of the fact that $|\psi(z)| \leq |z|$ and

$$|\psi'(z)| \leq \frac{1 - |\psi(z)|^2}{(1 - |z|^2)},$$

we find that

$$\begin{aligned} \left| z \frac{f''(z)}{f'(z)} \right| &\leq \frac{|z|(1-|\psi(z)|^2)}{(1-|\psi(z)|)(1-|z|^2)} + |\psi(z)| \\ &= \frac{|z|(1+|\psi(z)|)}{(1-|z|^2)} + |\psi(z)| \\ &\leq \frac{2|z|-|z|^2}{(1-|z|)}. \end{aligned}$$

Therefore, $f(z)$ is convex if

$$\frac{2|z|-|z|^2}{1-|z|} < 1$$

or

$$|z| < \frac{(3-\sqrt{5})}{2}.$$

To show that $(3-\sqrt{5})/2$ is the exact radius of convexity, we consider the function

$$f_0(z) = z \exp(z).$$

A little calculation shows that for this function

$$1 + \frac{zf_0''(z)}{f_0'(z)} = \frac{z^2+3z+1}{1+z}$$

vanishes when $z = (\sqrt{5}-3)/2$.