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#### Analytic sheaf cohomology with compact supports

by

Yum-Tong Siu

Among many other results Andreotti and Grauert proved in [2] the following:

(1) Suppose *n* is a non-negative integer and  $\mathscr{F}$  is a coherent analytic sheaf on a Stein space X such that  $\operatorname{codh} \mathscr{F} \ge n$  (where  $\operatorname{codh} \mathscr{F} = \operatorname{homological}$  codimension of  $\mathscr{F}$ ). Then  $H^p_*(X, \mathscr{F}) = 0$  for p < n. (Cf. Prop. 25, [2]).

Reiffen proved in [6] the following:

(2) Suppose *n* is a non-negative integer and  $\mathscr{F}$  is a coherent analytic sheaf on a complex space X such that dim Supp  $\mathscr{F} \leq n$  (where Supp  $\mathscr{F} =$  support of  $\mathscr{F}$ ). Then  $H^p_*(X, \mathscr{F}) = 0$  for p > n. (Cf. Satz 3, [6]).

In this note we prove converses of these statements:

THEOREM 1. Suppose *n* is a non-negative integer. If  $\mathscr{F}$  is a coherent analytic sheaf on an open subset *G* of a Stein space *X* and  $H^p_*(G, \mathscr{F}) = 0$  for p < n, then codh  $\mathscr{F}_x \ge n$  for  $x \in G$ .

THEOREM 2. Suppose n is non-negative integer,  $\mathcal{F}$  is a coherent analytic sheaf on a Stein space X, and G is an open subset of X. If  $H^p_*(G, \mathcal{F}) = 0$  for p > n, then dim  $(G \cap \text{Supp } \mathcal{F}) \leq n$ .

For the proofs of Theorems 1 and 2 we need the following Lemmata:

LEMMA 1. Suppose G is an open subset of  $\mathbb{C}^N$ ,  $x \in G$ , and A is an at most countable subset of  $G - \{x\}$ . Then there exists a holomorphic function f on  $\mathbb{C}^N$  such that f(x) = 0 and  $f(y) \neq 0$  for  $y \in A$ .

**PROOF.** Let F be the vector space of all holomorphic functions on  $\mathbb{C}^N$  vanishing at x. F is a Fréchet space with the topology of uniform convergence on compact subsets of  $\mathbb{C}^N$ . For  $y \in A$  let  $\varphi_y: F \to \mathbb{C}$  be defined by  $\varphi_y(f) = f(y)$  for  $f \in F$ . Let  $K_y = \text{Ker } \varphi_y$ .  $K_y$  is a nowhere dense closed subspace of F. For, if we take  $g \in F$ such that  $g(y) \neq 0$ , then for any open neighborhood U in F of  $h \in K_y$  we have  $\lambda g + h \in U - K_y$  for  $\lambda \in \mathbb{C} - \{0\}$  with  $|\lambda|$  sufficiently small. By Baire category theorem  $\bigcup_{y \in A} K_y \neq F$ .  $f \in F - \bigcup_{y \in A} K_y$ satisfies the requirement. q.e.d.

**LEMMA 2.** Suppose  $\mathscr{G}$  is a coherent analytic sheaf on an open subset G of  $\mathbb{C}^N$ . There exist subvarieties  $X_p$  in G, either empty or of pure dim p,  $0 \leq p \leq N-1$ , such that, for every  $x \in G$ , if a nonidentically-zero holomorphic function-germ f at x does not vanish identically on any non-empty branch-germ of  $X_p$  at x for any p, then f is not a zero-divisor for the stalk  $\mathscr{G}_x$  of  $\mathscr{G}$  at x.

PROOF. For  $0 \leq p \leq N-1$ , define a subsheaf  $\mathscr{G}_p$  of  $\mathscr{G}$  on Gas follows: for  $x \in G$ ,  $(\mathscr{G}_p)_x = \{s \in \mathscr{G}_x | \text{ for some subvariety } A_s \text{ of}$ dimension  $\leq p$  in some open neighborhood  $U_s$  of x in G there exists  $t \in \Gamma(U_s, \mathscr{G})$  such that  $t_x = s$  and  $t_y = 0$  for  $y \notin A_s\}$ .  $\mathscr{G}_p$  is a coherent analytic subsheaf of  $\mathscr{G}$  and dim  $\text{Supp } \mathscr{G}_p \leq p$ . For, if  $\varphi : {}_N \mathscr{O}^q \to \mathscr{G}$  is a sheaf-epimorphism on an open subset D of G (where  ${}_N \mathscr{O} = \text{structure-sheaf}$  of  $\mathbb{C}^N$ ) and  $(\text{Ker } \varphi)_p$  is the  $p^{\text{th}}$  step gap-sheaf of Ker  $\varphi$  in the sense of Thimm (Def. 9, [9]), then  $\mathscr{G}_p = \varphi((\text{Ker } \varphi)_p)$  on D and by Satz 3, [9] (Ker  $\varphi)_p$  is coherent and dim  $\{x \in D | ((\text{Ker } \varphi)_p)_x \neq (\text{Ker } \varphi)_x\} \leq p$ . Let  $X_p$  be the union of p-dimensional branches of Supp  $\mathscr{G}_p$ . We claim that these satisfy the requirement.

Suppose f is a non-identically-zero holomorphic function-germ at a point x of G not vanishing identically on any non-empty branch-germ of  $X_p$  at x for any p. We have to prove that f is not a zero-divisor for  $\mathscr{G}_x$ . Suppose the contrary. Then there exist  $g \in \Gamma(U, {}_N \mathscr{O})$  and  $h \in \Gamma(U, \mathscr{G})$  for some connected open neighborhood U of x in G such that  $g_x = f$ ,  $h_x \neq 0$ , and gh = 0. Let Z = Supp h and let p be the dimension of the germ of Z at x.  $0 \leq p \leq N-1$ . By shrinking U we can assume that dim Z = p.  $h \in \Gamma(U, \mathscr{G}_p)$  and  $Z \subset \text{Supp } \mathscr{G}_p$ . Since dim Supp  $\mathscr{G}_p \leq p$  and at x Z has dimension p, Z and  $X_p$  have a branch-germ A in common at x. gh = 0 implies that f vanishes identically on A. Contradiction. q.e.d.

**LEMMA 3.** Suppose  $\mathscr{S}$  is a torsion-free coherent analytic sheaf on a normal reduced irreducible complex space  $Z_0$ . Then the set Eof points in  $Z_0$  where  $\mathscr{S}$  is not locally free is a subvariety of codimension  $\geq 2$ .

**PROOF.** Let  $m = \dim Z_0$ . *D* is a subvariety in  $Z_0$  (Prop. 8, [1]). Suppose the Lemma is false. Then *D* contains an (m-1)-dimensional branch *A*. Let *M* be the set of all regular points of  $Z_0$ . Since dim  $(Z_0 - M) \leq m-2$ , there exists  $x \in M \cap A$ . There is a non-identically-zero holomorphic function f on some connected open neighborhood U of x in M such that f vanishes identically on  $A \cap U$ . Since  $\mathscr{S}$  is torsion-free, for  $y \in U$   $f_y$  is not a zerodivisor for  $\mathscr{S}_y$ . Let  $\mathscr{I} = \mathscr{S}/\mathfrak{f} \mathcal{S}$  on U.  $F = \{y \in U | \operatorname{codh} \mathscr{I}_y \leq m-2\}$ is of dimension  $\leq m-2$  (Satz 5, [7]). There exists  $z \in U \cap A - F$ . codh  $\mathscr{S}_z = m$ .  $\mathscr{S}$  is locally free at z, contradicting that  $z \in D$ . q.e.d.

LEMMA 4. Suppose P is an m-dimensional complex manifold. Suppose O is the structure-sheaf of P, S is a locally free sheaf on P, and  $\mathcal{L}$  is the sheaf of germs of holomorphic (m, 0)-forms on P. If  $H^{*}_{*}(P, S) = 0$ , then  $\Gamma(P, \operatorname{Hom}_{\mathcal{O}}(S, \mathcal{L})) = 0$ .

**PROOF.** Let *B* and *B*<sup>\*</sup> be respectively the holomorphic vectorbundles canonically associated with the locally free sheaves  $\mathscr{S}$ and  $\operatorname{Hom}_{\mathscr{O}}(\mathscr{S}, \mathscr{L})$ . For  $0 \leq p \leq m$  let  $\lambda(0, p)$  denote the vectorbundle of (0, p)-forms on *P*. Let  $\mathscr{A}^{(0, p)}(B)$  denote the sheaf of germs of infinitely differentiable sections in  $B \otimes \lambda(0, p)$  and let  $\mathscr{D}^{(0, p)}(B^*)$  denote the sheaf of germs of distribution-sections in  $B^* \otimes \lambda(0, p)$ . Let  $\Gamma_*(P, \mathscr{A}^{(0, p)}(B))$  denote the set of all global sections in  $\mathscr{A}^{(0, p)}(B)$  with compact supports.

$$0 \to \mathscr{S} \to \mathscr{A}^{(0,0)}(B) \xrightarrow{\overline{\mathfrak{d}}} \cdots \xrightarrow{\overline{\mathfrak{d}}} \mathscr{A}^{(0,m-1)}(B) \xrightarrow{\overline{\mathfrak{d}}} \mathscr{A}^{(0,m)}(B) \to 0$$

and

$$0 \to \operatorname{Hom}_{\mathscr{O}}(\mathscr{S}, \, \mathscr{L}) \to \mathscr{D}^{(0,0)}(B^*) \xrightarrow{\overline{\mathfrak{d}}} \mathscr{D}^{(0,m)}(B^*) \xrightarrow{\overline{\mathfrak{d}}} \mathscr{D}^{(0,m)}(B^*) \to 0$$

are fine-sheaf-resolutions for  $\mathscr{S}$  and  $\operatorname{Hom}_{\mathscr{O}}(\mathscr{S}, \mathscr{L})$  respectively.  $H^m_*(P, \mathscr{S}) = 0$  means that

$$\alpha: \Gamma_*(P, \mathscr{A}^{(0,m-1)}(B)) \to \Gamma_*(P, \mathscr{A}^{(0,m)}(B))$$

induced by

$$\overline{\partial}: \mathscr{A}^{(0,m-1)}(B) \to \mathscr{A}^{(0,m)}(B)$$

is surjective.  $\Gamma(P, \mathscr{D}^{(0,0)}(B^*))$  and  $\Gamma(P, \mathscr{D}^{(0,1)}(B^*))$  are respectively the duals of  $\Gamma_*(P, \mathscr{A}^{(0,m)}(B))$  and  $\Gamma_*(P, \mathscr{A}^{(0,m-1)}(B))$ .

$$\beta: \Gamma(P, \mathscr{D}^{(0,0)}(B^*)) \to \Gamma(P, \mathscr{D}^{(0,1)}(B^*))$$

induced by  $\overline{\partial}: \mathscr{D}^{(0,0)}(B^*) \to \mathscr{D}^{(0,1)}(B^*)$  is the transpose of  $\alpha$  (Cf. [8]).  $\beta$  is therefore injective.  $\Gamma(P, \operatorname{Hom}_{\mathscr{O}}(\mathscr{S}, \mathscr{L})) = 0$ . q.e.d.

**PROOF OF THEOREM 1:** Since X is Stein, by imbedding X and extending  $\mathscr{F}$  trivially we can assume w.l.o.g. that  $X = \mathbb{C}^N$  and

n > 0. Fix  $x \in G$ . For  $0 \le m \le n$  we are going to construct by induction on m holomorphic functions  $f_0 \equiv 0, f_1, \dots, f_m$  on G such that  $f_1(x) = \dots = f_m(x) = 0, (f_1)_x \ne 0, \dots, (f_m)_x \ne 0$ , and for  $1 \le k \le m$ 

$$(3) \qquad 0 \to \mathscr{F}/\sum_{i=0}^{k-1} f_{i}\mathscr{F} \xrightarrow{\varphi_{k}} \mathscr{F}/\sum_{i=0}^{k-1} f_{i}\mathscr{F} \to \mathscr{F}/\sum_{i=0}^{k} f_{i}\mathscr{F} \to 0$$

is an exact sequence on G, where  $\varphi_k$  is defined by multiplication by  $f_k$ .

The case m = 0 is trivial. Suppose we have constructed  $f_0 \equiv 0$ ,  $f_1, \dots, f_m$  for some  $0 \leq m < n$ . (3) implies that

(4) 
$$H^p_*(G, \mathscr{F}/\sum_{i=0}^{k-1} f_i \mathscr{F}) \to H^p_*(G, \mathscr{F}/\sum_{i=0}^k f_i \mathscr{F})$$
  
 $\to H^{p+1}_*(G, \mathscr{F}/\sum_{i=0}^k f_i \mathscr{F})$  is exact for  $p \ge 0$ .

Since  $H^p_*(G, \mathscr{F}) = 0$  for p < n, by induction on k we obtain from (4) that, for  $0 \leq k \leq m$ 

$$(5)_k \qquad \qquad H^p_*(G, \mathscr{F}/\sum_{i=0}^k f_i \mathscr{F}) = 0 \quad \text{for} \quad p < n-k.$$

Let  $\mathscr{G} = \mathscr{F} / \sum_{i=0}^{m} f_i \mathscr{F}$ . For the coherent analytic sheaf  $\mathscr{G}$  on Gwe have in G subvarieties  $X_p$ , of pure dim p or empty,  $0 \leq p \leq N-1$ , satisfying the requirement of Lemma 2. Since  $H^0_*(G, \mathscr{G}) = 0$  by  $(5)_m$ , from the construction in the proof of Lemma 2 we can choose  $X_0 = \emptyset$ . Let  $X_p = \bigcup_{i \in I_p} X_p^i$  be the decomposition into irreducible branches,  $1 \leq p \leq N-1$ . For  $X_p \neq \emptyset$  take  $x_p^i \in X_p^i - \{x\}$ . Let  $G - \{x\} = \bigcup_{i \in J} G_i$  be the decomposition into topological components. Take  $x_j \in G_j$ . Let

$$A = \{x_p^i | i \in I_p, 1 \leq p \leq N-1, X_p \neq \emptyset\} \cup \{x_j | j \in J\}.$$

A is at most countable. There exists by Lemma 1 a holomorphic function f on G such that f(x) = 0 and  $f(y) \neq 0$  for  $y \in A$ . For  $z \in G$   $f_z$  cannot vanish identically in any non-empty branch-germ of  $X_p$  at z for any p. Therefore for  $z \in G$   $f_z$  is not a zero-divisor for  $\mathscr{G}_z$ . Set  $f_{m+1} = f$ . The sequence  $f_0 \equiv 0, f_1, \dots, f_m, f_{m+1}$  satisfies the construction requirement. The construction is complete.  $(f_1)_x, \dots, (f_n)_x$  is an  $\mathscr{F}_x$ -sequence in the sense of (27.1), [5]. codh  $\mathscr{F}_x \geq n$ .

**PROOF OF THEOREM 2.** Again w.l.o.g. we can assume that  $X = \mathbb{C}^N$ . Let  $Y = \text{Supp } \mathscr{F}$ ,  $D = G \cap Y$ , and dim D = m. We have to prove that  $m \leq n$ . Suppose the contrary. Then n < m and  $H^p_*(G, \mathscr{F}) = 0$  for  $p \geq m$ .

Let  $\mathscr{I}$  be the annihilating ideal-sheaf for  $\mathscr{F}$ , i.e. for  $x \in \mathbb{C}^N$ ,  $\mathscr{I}_x = \{s \in {}_N \mathscr{O}_x | s \mathscr{F}_x = 0\}$ . Let  $\mathscr{H} = {}_N \mathscr{O} / \mathscr{I}$ . The sheaf of modules  $\mathscr{F}$  can be regarded as over the sheaf of rings  $\mathscr{H}$ . Let  $\mathscr{K}$  be the subsheaf of all nilpotent elements of  $\mathscr{H}$ . The exactness of

$$0 o \mathscr{KF} o \mathscr{F} o \mathscr{F} | \mathscr{KF} o 0$$

implies the exactness of

$$H^p_*(G,\mathscr{F}) o H^p_*(G,\mathscr{F}/\mathscr{KF}) o H^{p+1}_*(G,\mathscr{KF}) \quad ext{for} \quad p \geqq 0.$$

Since

 $\dim G \cap (\operatorname{Supp} \mathscr{KF}) \leq m, H^{p+1}_{*}(G, \mathscr{KF}) = 0 \quad \text{for} \quad p \geq m$ 

by Satz 3, [6]. Hence

$$H^p_{oldsymbol{*}}(G, \mathscr{F}/\mathscr{KF}) = 0 \quad ext{for} \quad p \geqq m.$$

Supp  $(\mathcal{F}/\mathcal{KF}) =$  Supp  $\mathcal{F}$ . For, if for some  $x \in \mathbb{C}^N$   $\mathcal{F}_x = \mathcal{K}_x \mathcal{F}_x$ , then, since  $\mathcal{K}_x$  is contained in the maximal-ideal of the local ring  $\mathcal{H}_x$ , we have  $\mathcal{F}_x = 0$  by Krull-Azumaya Lemma ((4.1), [5]).

Let  $\mathscr{G} = (\mathscr{F}/\mathscr{KF})|Y$  and  $\tilde{\mathscr{O}} = (\mathscr{H}/\mathscr{K})|Y$ .  $\mathscr{G}$  is a coherent analytic sheaf on the *reduced* Stein space  $(Y, \tilde{\mathscr{O}})$ . Supp  $\mathscr{G} = Y$ and  $H^p_*(D, \mathscr{G}) = 0$  for  $p \geq m$ .

Let  $\pi: \mathbb{Z} \to Y$  be the normalization of  $(Y, \tilde{\mathcal{O}})$ . Let  $\mathscr{G}'$  be the inverse image of  $\mathscr{G}$  under  $\pi$  (Def. 8, [3]) and let  $\mathscr{G}''$  be the zero<sup>th</sup> direct image of  $\mathscr{G}'$  under  $\pi$ . There exists a natural sheaf-homomorphism  $\lambda: \mathscr{G} \to \mathscr{G}''$  (Satz 7 (b), [3]).  $\lambda$  is bijective at regular points of Y. Let  $\mathscr{R} = \text{Ker } \lambda$  and  $\mathscr{Z} = \lambda(\mathscr{G})$ . The exactness of  $\mathbf{0} \to \mathscr{R} \to \mathscr{G} \to \mathscr{L} \to \mathbf{0}$  implies the exactness of

$$H^p_{m{\ast}}(D,\,\mathscr{G}) o H^p_{m{\ast}}(D,\,\mathscr{Z}) o H^{p+1}_{m{\ast}}(D,\,\mathscr{R}) \quad ext{for} \quad p\geqq 0.$$

Since dim  $D \cap \text{Supp } \mathscr{R} < m$ ,  $H^{p+1}_*(D, \mathscr{R}) = 0$  for  $p \ge m-1$ .  $H^p_*(D, \mathscr{Z}) = 0$  for  $p \ge m$ . The exactness of

$$0 \to \mathscr{Z} \to \mathscr{G}^{\prime\prime} \to \mathscr{G}^{\prime\prime}/\mathscr{Z} \to 0$$

implies the exactness of

$$H^p_{\boldsymbol{\ast}}(D,\, \mathscr{Z}) \to H^p_{\boldsymbol{\ast}}(D,\, \mathscr{G}^{\prime\prime}) \to H^p_{\boldsymbol{\ast}}(D,\, \mathscr{G}^{\prime\prime}/\mathscr{Z}) \quad \text{for} \quad p \ge 0.$$

Since dim  $D \cap \text{Supp } \mathscr{G}''/\mathscr{Z} < m$ ,  $H^p_*(D, \mathscr{G}''/\mathscr{Z}) = 0$  for  $p \ge m$ .  $H^p_*(D, \mathscr{G}'') = 0$  for  $p \ge m$ . Let  $L = \pi^{-1}(D)$ . Since

$$H^p_*(L, \mathscr{G}') \approx H^p_*(D, \mathscr{G}'') \text{ for } p \ge 0,$$

 $H^p_*(L, \mathscr{G}') = 0$  for  $p \ge m$ .

Let  $\mathscr{I}$  be the torsion subsheaf of  $\mathscr{G}'$  and let  $\mathscr{S} = \mathscr{G}'/\mathscr{I}$ . On Z  $\mathscr{S}$  is coherent and torsion-free (Prop. 6, [1]). Since Supp  $\mathscr{G} = Y$ ,

[5]

Supp  $\mathscr{S} = Z$ . The exact sequence  $0 \to \mathscr{I} \to \mathscr{G}' \to \mathscr{S} \to 0$  gives rise to the exact sequence

$$H^p_{m{\ast}}(L,\,\mathscr{G}') o H^p_{m{\ast}}(L,\,\mathscr{S}) o H^{p+1}_{m{\ast}}(L,\,\mathscr{I}) \quad ext{for} \quad p\geqq 0.$$

Since dim  $L \cap \text{Supp } \mathscr{I} < m$ ,  $H_*^{p+1}(L, \mathscr{I}) = 0$  for  $p \ge m-1$ .  $H_*^p(L, \mathscr{S}) = 0$  for  $p \ge m$ . Let  $Z_0$  be an *m*-dimensional branch of Z intersecting L.  $H_*^p(L \cap Z_0, \mathscr{S}) = 0$  for  $p \ge m$ . Let M be the set of all regular points of  $Z_0$  and let E be the set of points in  $Z_0$  where  $\mathscr{S}$  is not locally free. By Lemma 3 dim  $E \le m-2$ . Since  $Z_0$  is normal, dim  $(Z_0 - M) \le m-2$ . By Satz 3, [6],

$$H^p_*(L \cap (M-E), \mathscr{S}) = 0 \text{ for } p \ge m.$$

Let  $\mathscr{O}$  be the structure-sheaf of  $Z_0$  and let  $\mathscr{L}$  be the sheaf of germs of holomorphic (m, 0)-forms on M. By Lemma 4  $\Gamma(L \cap (M-E),$  $\operatorname{Hom}_{\mathscr{O}}(\mathscr{S}, \mathscr{L})) = 0$ . Take  $x \in L \cap (M-E)$ . Since  $\mathscr{S}_x \neq 0$  and  $Z_0$  is Stein, there exists  $s \in \Gamma(Z_0, \operatorname{Hom}_{\mathscr{O}}(\mathscr{S}, \mathscr{O}))$  such that  $s_x \neq 0$ . Since  $Z_0$  is Stein, there exist holomorphic functions  $g_1, \dots, g_m$  on  $Z_0$  such that the map  $(g_1, \dots, g_m) : Z_0 \to \mathbb{C}^m$  has rank m at x.  $dg_1 \wedge \dots \wedge dg_m$  defines an element f of  $\Gamma(M, \mathscr{L})$ .  $f_x \neq 0$ . Since  $\operatorname{Hom}_{\mathscr{O}}(\mathscr{S}, \mathscr{L}) \approx \operatorname{Hom}_{\mathscr{O}}(\mathscr{S}, \mathscr{O}) \otimes_{\mathscr{O}} \mathscr{L}$  on M,  $s \otimes f|L \cap (M-E)$  is a nonzero element of  $\Gamma(L \cap (M-E),$  $\operatorname{Hom}_{\mathscr{O}}(\mathscr{S}, \mathscr{L}))$ . Contradiction. q.e.d.

**REMARK.** In Theorems 1 and 2 the assumption that X is Stein cannot be dropped altogether. Counter-examples can easily be constructed by letting X be a complex projective space and by using Theorem von Serre in [3]. However, easy modifications in the proof can show that Theorem 1 holds under the weaker assumption that holomorphic functions on X separate points.

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