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COCYCLES OVER PARTIALLY HYPERBOLIC MAPS

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SOCIÉTÉ MATHÉMATIQUE DE FRANCE

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COCYCLES OVER PARTIALLY HYPERBOLIC MAPS

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Abstract. — The works collected in this volume, while addressing quite different goals, are focused on the same type of mathematical object: cocycles over partially hyperbolic diffeomorphisms. We begin with a preliminary overview giving background on the history and applications of the study of dynamical cocycles and partially hyperbolic theory and elucidating the connections between the two main articles. The first one investigates effective conditions which ensure that the Lyapunov spectrum of a (possibly non-linear) cocycle over a partially hyperbolic dynamical system is non-trivial. In the second one, the classical Livšic theory of the cohomological equation for Anosov diffeomorphisms is extended to accessible partially hyperbolic diffeomorphisms.

Résumé (Des cocycles au-dessus d'applications partiellement hyperboliques.) — Les travaux réunis dans ce volume, ayant pourtant des objectifs très différents, sont axés sur le même genre d'objet mathématique : les cocycles au dessus d'un difféomorphisme partiellement hyperbolique. On commence par des rappels sur l'histoire et les applications de l'étude des cocycles dynamiques et la théorie partiellement hyperbolique; ils mettent en évidence des liens entre les deux articles principaux. Dans le premier, on étudie des conditions efficaces qui garantissent la non-trivialité du spectre de Lyapunov d'un cocycle (éventuellement non-linéaire) au dessus d'un difféomorphisme partiellement hyperbolique. Dans le second, la théorie classique de Livšic sur l'équation cohomologique pour les difféomorphismes d'Anosov est étendue au cas d'un difféomorphisme partiellement hyperbolique et accessible.

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