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## ON AN ADDITIVE PROBLEM OF ERDŐS AND STRAUS, 2

by

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**Abstract.** — We denote by  $s^{\wedge}A$  the set of integers which can be written as a sum of  $s$  pairwise distinct elements from  $A$ . The set  $A$  is called **admissible** if and only if  $s \neq t$  implies that  $s^{\wedge}A$  and  $t^{\wedge}A$  have no element in common.

P. Erdős conjectured that an admissible set included in  $[1, N]$  has a maximal cardinality when  $A$  consists of consecutive integers located at the upper end of the interval  $[1, N]$ . The object of this paper is to give a proof of Erdős' conjecture, for sufficiently large  $N$ .

Let  $\mathcal{A}$  be a set of positive integers having the property that each time an integer  $n$  can be written as a sum of distinct elements of  $\mathcal{A}$ , the number of summands is well defined, in that the integer  $n$  cannot be written as a sum of distinct elements of  $\mathcal{A}$  with a different number of summands. This notion has been introduced by P. Erdős in 1962 (cf. [2]) and called **admissibility** by E.G. Straus in 1966 (cf. [5]). In other words, if we denote by  $s^{\wedge}\mathcal{A}$  the set of integers which can be written as a sum of  $s$  pairwise distinct elements from  $\mathcal{A}$  then  $\mathcal{A}$  is **admissible** if and only if  $s \neq t$  implies that  $s^{\wedge}\mathcal{A}$  and  $t^{\wedge}\mathcal{A}$  have no element in common.

Erdős conjectured that an admissible subset  $\mathcal{A}$  included in  $[1, N]$  has a cardinality which is maximal when  $\mathcal{A}$  consists of consecutive integers located at the upper end of the interval  $[1, N]$ . As it was computed by E.G. Straus, the set

$$\{N - k + 1, N - k + 2, \dots, N\}$$

is admissible if and only if  $k \leq 2\sqrt{N + 1/4} - 1$ .

Straus himself proved that  $\sqrt{N}$  is the right order of magnitude for the cardinality of a maximal admissible subset from  $[1, N]$ . More precisely, he proved the inequality  $|\mathcal{A}| \leq (4/\sqrt{3} + o(1))\sqrt{N}$ . The constant involved has been slightly reduced by P. Erdős, J.-L. Nicolas and A. Sárközy (cf. [3]) and we proved (cf. [1]) the inequality

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$|A| \leq (2 + o(1))\sqrt{N}$ . The object of this paper is to give a proof of Erdős conjecture, at least when  $N$  is sufficiently large.

**Theorem 1.** — *There exists an integer  $N_0$ , effectively computable, such that for any integer  $N \geq N_0$  and any admissible subset  $A \subset [1, N]$  we have*

$$\text{Card } A \leq 2\sqrt{N + 1/4} - 1.$$

The proof is based on the description of the structure of large admissible sets we obtained previously, namely :

**Theorem 2 (J.-M. Deshouillers, G.A. Freiman [1]).** — *Let  $A$  be an admissible set included in  $[1, N]$ , such that  $\text{Card } A > 1.96\sqrt{N}$ . If  $N$  is large enough, there exist  $C \subset A$  and an integer  $q$  having the following properties :*

- (i)  $\text{Card } C \leq 10^5 N^{5/12}$ ,
- (ii) for some  $t$  the set  $t^\wedge C$  contains at least  $3N^{5/6}$  terms in an arithmetic progression modulo  $q$ ,
- (iii)  $A \setminus C$  is included in an arithmetic progression modulo  $q$  containing at most  $N^{7/12}$  terms.

Although we do not develop this point, it will be clear from the proof that our arguments may be used to describe the structure of maximal admissible subsets of  $[1, N]$ , leading for example to the fact that when  $N$  has the shape  $n^2$  or  $n^2 + n$  (and  $n$  sufficiently large), the Erdős - Straus example is the only maximal subset of  $[1, N]$ .

1. We first establish a lemma expressing the fact that if a set of integers  $\mathcal{D}$  is part of a finite arithmetic progression with few missing elements, then the same is locally true for  $s^\wedge \mathcal{D}$ .

**Proposition 1.** — *Let us consider integers  $r, s, t$  and  $a, q$  such that  $t \geq 2s - q$ ,  $s \geq 4r + 3 + q$  and  $0 \leq a < q$ .*

*Let further  $\mathcal{D} = \{d_1 < d_2 < \dots < d_t\}$  be a set of  $t$  distinct integers congruent to  $a$  modulo  $q$  such that  $d_t - d_1 = (t - 1 + r)q$ , and denote by  $m$  (resp.  $M$ ) the smallest (resp. largest) element in  $s^\wedge \mathcal{D}$ . Then, among  $2r + 1$  consecutive integers congruent to  $a$  modulo  $q$  and laying in the interval  $[m, M]$ , at least  $r + 1$  belong to  $s^\wedge \mathcal{D}$ .*

*Proof.* — We treat the special case when  $a = 0, q = 1$  and  $\mathcal{D}$  is included in  $[1, t]$ . We notice that the general case reduces to this one by writing

$d_i = d_1 + q(\delta_i - 1)$  and considering the set  $\{\delta_1, \dots, \delta_t\}$ .

Let  $x$  be an integer in  $s^\wedge \mathcal{D} \cap [m, (m + M)/2]$ . We first show that the interval  $[x, x + 3r]$  contains at least  $2r + 1$  elements from  $s^\wedge \mathcal{D}$ . Since  $x$  is in  $s^\wedge \mathcal{D}$ , we can find  $d(1) < \dots < d(s)$ , elements in  $\mathcal{D}$ , the sum of which is  $x$ .

Let us show that  $d(1)$  is less than  $t - s - 3r$ . On the one hand we have

$$m + M \leq (r + 1) + \dots + (r + s) + (t + r - s + 1) + \dots + (t + r) = \frac{s}{2}(2t + 4r + 2),$$

and on the other hand we have

$$x \geq d(1) + (d(1) + 1) + \dots + (d(1) + s - 1) = \frac{s}{2}(2d(1) + s - 1).$$

The inequality  $x \leq (m + M)/2$  implies that we have

$$2d(1) + s - 1 \leq t + 2r + 1,$$

whence

$$2d(1) \leq 2(t - s - 3r) - (t - s - 4r - 2),$$

and we notice that  $t - s - 4r - 2$  is positive, by the assumptions of Proposition 1.

Since  $d(1)$  is less than  $t - s - 3r$ , the interval  $[d(1), t + r]$  contains at least  $s + 4r + 1$  integers. We denote by  $i_1 < \dots < i_l$  the indexes of those  $d$ 's such that  $d(i_k + 1) - d(i_k) \geq 2$ , with the convention that  $d(i_l + 1) = 3Dt + r + 1$  in the case when  $d(s) < t + r$ . The set

$$\bigcup_{k=1}^l ]d(i_k) + 1, d(i_k + 1) - 1[$$

contains at least  $4r + 1$  integers. We now suppress from those intervals those which contain no element from  $\mathcal{D}$ , and we rewrite the remaining ones as

$$]d(j_1) + 1, d(j_1 + 1) - 1[, \dots, ]d(j_h) + 1, d(j_h + 1) - 1[.$$

They contain at least  $3r + 1$  integers, among which at most  $r$  are not in  $\mathcal{D}$ .

Let us define  $u_1$  to be the largest integer such that  $d(j_1) + u_1$  is in  $\mathcal{D}$  and is less than  $d(j_1 + 1)$ , and let us define  $u_2, \dots, u_h$  in a similar way. We consider the integers

$$x = y + d(j_1) + \dots + d(j_h) \quad (\text{which defines } y),$$

$$x + 1 = y + d(j_1) + 1 + d(j_2) + \dots + d(j_h),$$

...

$$x + u_1 = y + d(j_1) + u_1 + d(j_2) + \dots + d(j_h),$$

...

$$x + u_1 + \dots + u_h = y + d(j_1) + u_1 + d(j_2) + u_2 + \dots + d(j_h) + u_h.$$

One readily deduces from this construction that the interval

$$[x, x + \min(3r, u_1 + \dots + u_h)]$$

contains at most  $r$  elements which are not in  $s^\wedge \mathcal{D}$ .

What we have proven so far easily implies that any interval  $[z - r, z]$  with  $m \leq z \leq (M + m)/2$  contains at least one element in  $s^\wedge \mathcal{D}$ . Let us consider an interval  $[y, y + 2r]$  with  $m \leq y \leq (M + m)/2$ . By what we have just said, the interval  $[y - r, y]$  contains an element in  $s^\wedge \mathcal{D}$ , let us call it  $x$ . As we have shown the interval  $[x, x + 3r]$  contains at most  $r$  integers not in  $s^\wedge \mathcal{D}$ , so that  $[y, y + 2r]$  contains at most  $r$  integers not in  $s^\wedge \mathcal{D}$ , which is equivalent to say that it contains at least  $r + 1$  elements from  $s^\wedge \mathcal{D}$ .

A similar argument taking into account decreasing sequences and starting with  $M$  shows that any interval  $[y - 2r, y]$  with  $(m + M)/2 \leq y \leq M$  contains at least  $r + 1$  elements from  $s^\wedge \mathcal{D}$ .

2. We now prove the following result concerning the structure of a large admissible finite set.

**Theorem 3.** — *Let  $\mathcal{A} = \{a_1 < \dots < a_A\}$  be an admissible subset of  $[1, N]$  with cardinality  $A = 2N^{1/2} + O(N^{5/12})$ , and let us define  $q$  to be the largest integer such that  $\mathcal{A}$  is contained in an arithmetic progression modulo  $q$ . We have  $q = O(N^{5/12})$  and there exists an integer  $u$  in  $[N^{11/24}, 2N^{11/24}]$  such that*

$$a_{A-u} - a_{u+1} = q(2N^{1/2} + O(N^{11/24})).$$

*Proof.* — The proof is based on the structure result we quoted in the introduction as Theorem 2. We keep its notation and first show that an integer  $q$  satisfying (ii) and (iii) is indeed the largest integer such that  $\mathcal{A}$  is contained in an arithmetic progression modulo  $q$ . We let  $\mathcal{B}$  denote  $\mathcal{A} \setminus \mathcal{C}$ .

A simple counting argument will show that  $\mathcal{A}$  is included in the same arithmetic progression as  $\mathcal{B}$ . Otherwise, let us consider an element  $a \in \mathcal{A}$  which is not in the same arithmetic progression as  $\mathcal{B}$  modulo  $q$ . The set  $s^\wedge \mathcal{A}$  contains the disjoint sets  $s^\wedge \mathcal{B}$  and  $a + (s-1)^\wedge \mathcal{B}$ . We thus have  $|s^\wedge \mathcal{A}| \geq |s^\wedge \mathcal{B}| + |(s-1)^\wedge \mathcal{B}|$ . It is well-known (cf. [4] for example) that  $|s^\wedge \mathcal{B}| \geq s(|\mathcal{B}| - s)$  for  $s \leq |\mathcal{B}|$ , and since  $\mathcal{A} \subset [1, N]$  is admissible we have

$$\begin{aligned} N(|\mathcal{B}| + 1) &\geq \text{Card} \left( \bigcup_s (s^\wedge \mathcal{B} \cup (a + (s-1)^\wedge \mathcal{B})) \right) \\ &\geq 2 \sum_s |s^\wedge \mathcal{B}| \geq 2 \sum_s s = 20(|\mathcal{B}| - s) = \frac{1}{3}|\mathcal{B}|^3 + O(N), \end{aligned}$$

which implies  $|\mathcal{B}| \leq (\sqrt{3} + o(1))\sqrt{N}$ , so that we have  $|\mathcal{A}| = |\mathcal{B}| + |\mathcal{C}| \leq (\sqrt{3} + o(1))\sqrt{N}$ , a contradiction.

We have so far proven that  $q$  divides  $g := \gcd(a_2 - a_1, \dots, a_A - a_1)$ . Property (ii) implies that  $q$  is a multiple of  $g$ , so that we have  $q = g$ , as we wished to show.

The second step in the proof consists in showing that for  $0 < k \leq |\mathcal{B}| - q$ , any element in  $k^\wedge \mathcal{B}$  is less than any element in  $(k+q)^\wedge \mathcal{B}$ . Let us call  $J$  the  $3N^{5/6}$  consecutive terms of the arithmetic progression modulo  $q$ , the existence of which is asserted in (ii). Since  $\mathcal{B}$  is included in an arithmetic progression modulo  $q$  with less than  $3N^{5/6}$  terms, the sets  $k^\wedge \mathcal{B} + J$  and  $(k+q)^\wedge \mathcal{B} + J$  consists of consecutive terms of arithmetic progressions modulo  $q$ , and moreover, they are in the *same* class modulo  $q$ . Since  $\mathcal{A}$  is admissible, the sets  $k^\wedge \mathcal{B} + J$  (included in  $(k+t)^\wedge \mathcal{A}$ ) and  $(k+q)^\wedge \mathcal{B} + J$  (included in  $(k+q+t)^\wedge \mathcal{A}$ ) do not intersect. To prove that any element of  $k^\wedge \mathcal{B}$  is less than any element of  $(k+q)^\wedge \mathcal{B}$ , it is now sufficient to notice that  $k^\wedge \mathcal{B}$  contains an element (we can consider the smallest element of  $k^\wedge \mathcal{B}$ ), which is smaller than some element of  $(k+q)^\wedge \mathcal{B}$ .

We now prove that  $q = O(N^{5/12})$ . The cardinality of  $\mathcal{A}$  and Theorem 2 imply that  $|\mathcal{B}| = 2N^{1/2} + O(N^{5/12})$ . We choose  $k$  so that  $2k+q$  is  $|\mathcal{B}|$  or  $|\mathcal{B}| - 1$ . (We notice that this is always possible since  $\mathcal{A}$  contains at least  $N^{1/2}$  integers from  $[1, N]$  in an arithmetic progression modulo  $q$ , so that  $q \leq N^{1/2}$ ). By the second step, the largest element in  $k^\wedge \mathcal{B}$  is smaller than the largest element in  $(k+q)^\wedge \mathcal{B}$ . Let  $z$  be  $(k+q)$ -th element from  $\mathcal{B}$ , in the increasing order. We have

$$z \leq N - (k-1)q$$

and

$$(z + q) + \cdots + (z + qk) \leq z + (z - q) + \cdots + (z - (k + q - 1)q) \quad ;$$

by an easy computation, we get

$$(q + 2k)^2 \leq 2N + 2k^2 + 3q,$$

but  $2k + q = |\mathcal{B}| + O(1) = |\mathcal{A}| + O(N^{5/12})$ , which implies

$$2k^2 \geq 2N(1 + O(N^{-1/12})),$$

so that we have

$$k = N^{1/2} + O(N^{5/12}).$$

We now use again the same argument, being more precise. Let us write  $\mathcal{B} = \{b_1 < \cdots < b_{k+q} < b_{k+q+1} < \cdots < b_{2k+q} \leq b_B\}$ . We have

$$b_{k+q+1} + \cdots + b_{2k+q} < b_1 \cdots + b_k + b_{k+1} + b_{k+q}.$$

Let  $t$  be any integer in  $[1, k]$ . We have

$$b_{k+1} + \cdots + b_{k+q} > (b_{2k+q} - b_1) + \cdots + (b_{2k+q-t+1} - b_t) + \cdots + (b_{k+q+1} - b_k).$$

We clearly have the inequalities

$$\begin{aligned} b_{k+q+1} - b_k &\geq (q + 1)q, \\ b_{k+q+2} - b_{k-1} &\geq (q + 3)q, \\ &\dots \\ b_{2k+q-t-1} - b_{t+2} &\geq (q + 1 + 2(k - t - 2))q, \\ b_{2k+q-t} - b_{t+1} &\geq b_{2k+q-t} - b_{t+1}, \\ b_{2k+q-t+1} - b_t &\geq (b_{2k+q-t} - b_{t+1}) + 2q, \\ &\dots \\ (b_{2k+q} - b_1) &\geq (b_{2k+q-t} - b_{t+1}) + 2tq. \end{aligned}$$

We thus obtain

$$\begin{aligned} b_{k+1} + \cdots + b_{k+q} &> (t + 1)(b_{2k+q-t} - b_{t+1}) \\ &\quad + q \sum_{l=0}^{k-t-2} (q + 1 + 2l) + q \sum_{h=0}^t 2h. \end{aligned}$$

Taking into account that  $b_{k+q} \leq N - kq$ , a dull computation leads to

$$(t + 1)(b_{2k+q-t} - b_{t+1}) \leq q(N - k^2 + 2kt + O(N^{11/12})),$$

when  $t = O(N^{11/24})$ . This in turn leads to

$$b_{2k+q-t} - b_{t+1} \leq q(2k + O(N^{11/24})),$$

when  $t = \frac{3}{2}N^{11/24} + O(1)$ .

Let  $\mathcal{C}$  the cardinality of  $\mathcal{C}$ . Since  $\mathcal{A} = \mathcal{B} \cup \mathcal{C}$ , we have

$$b_{t+1} \leq a_{t+\mathcal{C}} \leq a_{A+t+\mathcal{C}-2k-q+1} \leq a_{2k+q-\mathcal{C}-t} \leq b_{2k+q-t} \quad ;$$

we choose  $u = A + t + \mathcal{C} - 2k - q$  and recall that  $A - 2k - q \leq \mathcal{C} + 1 = O(N^{5/12})$ , so that Theorem 3 is proven.

**3.** We now embark on the proof of Theorem 1 which will follow from Theorem 3 and Proposition 1. Let  $\mathcal{A}$  be an admissible subset of  $[1, N]$  with maximal cardinality. By [1], we know that  $A = 2\sqrt{N} + O(N^{5/12})$ , so we can apply Theorem 3 : there exists integers  $u$  and  $r$  such that

$$a_{A-u} - a_{u+1} = q(A - 2u + r),$$

with  $u \in [N^{11/24}, 2N^{11/24}]$  and  $r = O(N^{11/24})$ .

We let

$$\mathcal{D} := \mathcal{A} \cap [a_{u+1}, a_{A-u}], \quad t := A - 2u, \quad \sigma := [(t - q)/2],$$

and we shall apply Proposition 1 with  $s = \sigma$  and  $s = \sigma + q$  (one readily checks that the conditions of application of Proposition 1 are fulfilled). Let us further denote by  $m(s)$  (resp.  $M(s)$ ) the smallest (resp. largest) element in  $s^\wedge \mathcal{D}$ .

As a first step, we show that  $a_1 + a_2 + \dots + a_q$  cannot be too small. We have

$$\begin{aligned} M(\sigma) - m(\sigma) &\geq (a_{A-u-\sigma+1} - a_{u+\sigma}) + \dots + (a_{A-u} - a_{u+1}) \\ &\geq q(2 + 4 + \dots + 2(\sigma - 1)) = q\sigma(\sigma - 1) \\ &= qN + O(qN^{23/24}). \end{aligned}$$

If  $\alpha_q := a_1 + \dots + a_q$  were less than  $M(\sigma) - m\sigma - (2r + 1)q$ , the intersection of  $[m(\sigma), M(\sigma)]$  and  $[m(\sigma) + \alpha_q, M(\sigma) + \alpha_q]$  would be an interval containing at least  $(2r + 1)$  integers in each class modulo  $q$ . By the property of  $\sigma^\wedge \mathcal{D}$  established in Proposition 1, property obviously shared by  $\alpha_q + \sigma^\wedge \mathcal{D}$ , the pigeon-hole principle would imply that  $\sigma^\wedge \mathcal{D}$  and  $\alpha_q + \sigma^\wedge \mathcal{D}$  have an element in common, and this would contradict the admissibility of  $\mathcal{A}$ . (We may notice that this implies that  $a_1$  itself is not too small, but we shall not use this fact).

By using the same pigeon-hole argument, we see that the admissibility of  $\mathcal{A}$  implies

$$M(\sigma) + a_{A-u+1} + \dots + a_A \leq m(\sigma + q) + a_1 + \dots + a_u + (2r - 1)q,$$

that is to say

$$a_{A-u-\sigma+1} + \dots + a_{A-u} + \dots + a_A \leq a_1 + \dots + a_u + a_{u+1} + \dots + a_{u+\sigma+q} + (2r - 1)q,$$

whence we deduce

$$(a_A - a_1) + (a_{A-1} - a_2) + \dots + (a_{A-u-\sigma+1} - a_{u+\sigma}) \leq a_{u+\sigma+1} + \dots + a_{u+\sigma+q} + (2r - 1)q.$$

We have  $a_{A-u-\sigma+1} - a_{u+\sigma} \geq q(A - u - \sigma + 1 - u - \sigma) = q(A - 2u - 2\sigma + 1)$  and, by the definition of  $\sigma$ , we can write

$$A - 2u - 2\sigma = q + \theta,$$

where  $\theta = 0$  if  $A - q$  is even and  $\theta = 1$  if  $A - q$  is odd. We thus have

$$\begin{aligned} urq + q(1 + q + \theta) + q(3 + q + \theta) + \dots + q(2(u + \sigma) - 1 + q + \theta) &\leq \\ a_{u+\sigma+1} + \dots + a_{u+\sigma+q} + (2r - 1)q. \end{aligned}$$

Since  $u \geq 2$  and  $r \geq 0$ , we have

$$\begin{aligned} q(u + \sigma)(u + \sigma + q + \theta) &\leq a_{u+\sigma+1} + \cdots + a_{u+\sigma+q} - q \\ &\leq N - (A - u - \sigma - 1)q + \cdots + \\ &\quad N - (A - u - \sigma - q)q - q \\ &\leq Nq - Aq^2 + uq^2 + \sigma q^2 + \frac{q^2(q+1)}{2} - q. \end{aligned}$$

We now replace  $u + \sigma$  by  $\frac{A-q-\theta}{2}$ , which leads to

$$q \left( \frac{A - q - \theta}{2} \right) \left( \frac{A + q + \theta}{2} \right) \leq Nq - q^2 \left( \frac{A + q + \theta}{2} \right) + \frac{q^2(q-1)}{2}.$$

If  $A - q$  is even, we get

$$A^2 - q^2 \leq 4N - 2Aq - 2q^2 + 2q^2 - 2q,$$

whence

$$A^2 + 2Aq + q^2 \leq 4N + 2q^2 - 2q,$$

or

$$(A + q)^2 \leq 4N + 2q^2 - 2q.$$

if  $q = 1$ , this is  $(A + 1)^2 \leq 4N$ ;

if  $q \geq 2$ , we have

$$\begin{aligned} (A + 1)^2 &\leq (A + q)^2 - (A + q)^2 + (A + 1)^2 \\ &\leq 4N + 2q^2 - 2q - A^2 - 2Aq - q^2 + A^2 + 2A + 1 \\ &\leq 4N + 2A(1 - q) + (q - 1)^2 \\ &\leq 4N - (q - 1)(2A - q + 1) \leq 4N. \end{aligned}$$

If  $A - q$  is odd, we get

$$A^2 - (1 + q)^2 \leq 4N - 2Aq - 2q^2 - 2q + 2q^2 - 2q.$$

if  $q = 1$ , this is  $A^2 + 2A + 1 \leq 4N + 1$ ;

if  $q \geq 2$ , we have

$$\begin{aligned} (A + 1)^2 &\leq A^2 - (1 + q)^2 + 2q + q^2 + 2 + 2A \\ &\leq 4N - 2A(q - 1) + q^2 - 2q + 2 \\ &\leq 4N - (q - 1)(2A - q + 1) + 1 \\ &\leq 4N + 1. \end{aligned}$$

In all cases, we thus have  $(A + 1)^2 \leq 4N + 1$ , which ends the proof of our main result.



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