Astérisque

AST

Pages préliminaires

Astérisque, tome 113-114 (1984), p. 1-14

INTRODUCTION

Ce volume rassemble les contributions écrites des participants aux Journées qui se sont tenues au C.I.R.M. de Luminy du 1er au 5 juin 1982.

L'organisation de ces Journées est le fruit d'une amitié scientifique, née à Louvain en 1980... avec la synthèse des points de vue covariant et contravariant en homotopie rationnelle ! L'année suivante à Bonn, les "Stockholmiens" sont venus apporter leurs préoccupations et convaincre les topologues qu'elles ressemblaient étonnement aux leurs.

A Luminy, les quelque soixante topologues et algébristes présents ont parlé le même langage : d'ailleurs, deux de leurs maîtres communs étaient là pour les y encourager, Henri CARTAN et Jean-Louis KOSZUL.

Les séances du matin étaient réservées à des conférences de synthèse, "commandées" à leurs auteurs par les organisateurs. Les séances de l'après-midi ont été consacrées à des séries d'exposés des participants et ont été animées par Jean-Louis KOSZUL, Willy MEIER, Jan-Erik ROOS et Michel ZISMAN.

Sur le plan matériel, l'organisation de ces Journées dans le cadre exceptionnel du C.I.R.M. a été possible grâce à un soutien financier important de l'Etablissement Public Régional Provence-Alpes-Côte d'Azur, ainsi que de la Société Mathématique de France (R.C.P. 365), de l'U.E.R. de Mathématiques de Lille et du Département de Mathématiques de Nice, auquel il convient d'ajouter le Conseil Scientifique du C.I.R.M. qui nous a accordé le tarif de pension préférentiel.

Les organisateurs tiennent à exprimer leur gratitude à tous ceux qui ont contribué au succès de ces Journées, en particulier, au Secrétariat Scientifique de l'U.E.R. de Mathématiques de Lille I qui a assuré une grande partie de la dactylographie de ce volume.

TABLE DES MATIERES

RESUMES DES CONFERENCES DE SYNTHESES

AVRAMOV, Luchezar - *Local Algebra and Rational Homotopy.*

Outline of the construction of a functor, from commutative DG rings with divided powers to graded Lie algebras, whose properties are similar to the rational homotopy of finite CW complexes. Applications to the determination of the rate of growth of Betti numbers of local rings, and to the determination of the rational homotopy Lie algebra of a formal manifold in terms of the one of the punctured manifold.

BAUES, Hans Joachim - *The chains on the loops and 4-dimensional homotopy types.*

The connection of the chain complex of the universal covering with the chain algebra of the loop space is studied. This chain algebra is used as a classifying invariant for the 4-dimensional homotopy types.

BURGHELEA, Dan - *Rational homotopy theory, group actions and algebraic K-theory of topological spaces.*

One reports on some applications of the algebraic methods of rational homotopy theory and minimal model theory to the rational homotopy theory of G-spaces (G-discrete, mostly infinite) and to the computation of the rational algebraic K-theory of 1-connected topological spaces.

CHEN, Kuo-Tsai - *Loop Spaces and Differential Forms.*

This is a brief account of path space differential forms, called iterated path integrals, and their topological and geometrical significance.

FELIX, Yves - *Formal and π-formal spaces.*

Summary and synthesis of the theory of formal and π-formal spaces : Définitionand properties, obstructions to formality and study of spaces with Poincaré duality are the three main aspects of this lecture.

HALPERIN, Stephen - *The structure of $\pi_*(\Omega S)$.*

For a simply connected space, S, the Samelson product makes $\pi_*(\Omega S)\otimes\mathbb{Q}$ into a graded connected Lie algebra, and a theorem of Quillen shows that all graded connected Lie algebras/$\mathbb{Q}$ arise this way. If S has finite category then serious restrictions are imposed on this Lie algebra - in particular if it is infinite dimensional then it cannot be solvable and it grows exponentially.

HUSEMÖLLER, Dale - *Loops spaces decompositions in the theory of exponents.*

A review of the recent work of Cohen, Moore and Neisendorfer on exponents of homotopy groups, and more generally on the order of the identity map in the group of homotopy classes of self-maps of some loop spaces and suspensions : the determination of this order follows from loop spaces decompositions ; emphasis is laid on the two basic techniques used to achieve such decompositions : graded Lie algebras in characteristic p and mod p-Hurewicz homomorphisms.

.../...

TANRÉ Daniel - *Fibrations and classifying spaces.*

First, we place briefly the classifying spaces in their original topological and algebraic frame. Then, we construct two algebraic models of a Serre fibration in the category of differential graded Lie algebras. The first allow the computation of the Eilenberg Moore spectral sequence from the descending central series. The second provides an algebraic model of the universal fibration for fibrations with fixed fiber, rediscovering Schlessinger Stasheff's classifying space.

RESUMES DES COMMUNICATIONS

ALLDAY, Christopher and HALPERIN, Steve - *Sullivan-de Rham theory for rational Alexander-Spanier cohomology.*

The Sullivan-de Rham-Alexander-Spanier algebra and its homotopy are defined. Two theorems are stated including a version of the Grivel-Halperin-Thomas theorem concerning minimal models in a Hurewicz fibration.

AVRAMOV, Luchezar, HALPERIN, Stephen - *On the Structure of the Homotopy Lie Algebra of a Local Ring.*

The homotopy Lie algebra of a local ring R is not nilpotent and vanishes (at most) in finitely many positive degrees, unless R is a complete intersection.

BØGVARD Rikard - *Some elementary results on the cohomology of graded Lie algebras.*

Here, I characterize solvable graded Lie algebras of finite global dimension, and prove an analogue of Serre's theorem on virtual and ordinary global dimension of torsion free groups.

BROWN, Ronald - *Non-abelian cohomology and the homotopy classification of maps.*

Let C be a crossed complex (a generalisation of chain complex and of crossed module). Let X be a CW-complex. We define cohomology $H^0(X;C)$ and a classifying space BC and prove $[X,BC] \cong H^0(X;C)$. This generalises a number of classical homotopy classification theorems to the non-simply connected case.

CENKL, Bohumil, PORTER, Richard - *Algebraic Categories and the Homotopy Theory of some C.W. Complexes.*

The category of s.c. spaces whose cohomology satisfies the c-r condition is equivalent to a homotopy theory of commutative algebras over the integers. For H^* finite dimensional, an example illustrates how to compute the set of homotopy types with cohomology H^* as the orbits of a matrix group acting on a lattice in a finite dimensional variety.

FELIX, Yves - *L.S. Category and invariant* e.

The purpose of this paper is to exhibit explicitely an example of a space whose rational L.S. category is infinite and whose invariant e is two. e is the length of E_∞ in the Milnor-Moore spectral sequence

$$\operatorname{Tor}^{H_*(\Omega X;\mathbb{Q})}(\mathbb{Q};\mathbb{Q}) \Longrightarrow H^*(X;\mathbb{Q}).$$

FELIX, Yves and LÖFWALL, Clas - *Sur le rayon de convergence de la série de Poincaré des anneaux locaux gradués.*

For a graded local ring of embedding dimension at least $p+1$ it is proved that the radius of convergence of the Poincaré series of the ring is at most $2^{-1/p}$.

GOLASINSKI, Marek - *Some remarks on the rational homotopy type of diagrams and reduced* K_o.

For a discrete group G by I is denoted the full subcategory of G-Set determined by G/H as H varies over all subgroups of G. A generalization of Sullivan's theorem on the rational homotopy equivalence of the functors category I SS on the one hand and I DGA on the other, where SS denotes the category of simplicial sets and DGA - the category of differential graded Q-algebras is considered. Moreover, it is proved that for a finite group G and a finite G-simplicial set X the functor $K_O^{\sim G} \Omega_k^O(X)$ is representable if k is a field such that $\chi(k) \nmid |G|$.

GRIVEL Pierre-Paul - *Algebraic fibrations with a given fiber.*

We study the algebraic account of the Dold-Lashof'theorem on the homotopy classification of fibrations.

HALPERIN, Stephen - *Spaces whose rational homology and De Rham cohomology are both finite dimensional.*

If a simply connected space S satisfies the hypotheses of the title let n be the top integer for which $H^n(S;\mathbb{Q}) \neq 0$. It was known that $\sum_{i=n+1}^{2n-1} \dim \pi_i(S) \otimes \mathbb{Q} \leq 1$; the case when equality holds is analyzed. We also show that $\dim H^*(S;Q) \leq 2^n$, and derive a formula for the Lefschetz number of a map.

HAEFLIGER, André - *The homology of nilpotent Lie groups made discrete.*

As a consequence of Malcev theory, we note that the reduced integral homology of a simply connected nilpotent Lie group (considered as a discrete group) is isomorphic to the homology of its Lie algebra considered as an algebra over the rationnals.

HOPKINS Michael J. - *Formulations of cocategory and the iterated suspension.*

In this note, I will present new formulations of category and cocategory closer in spirit to the origininal definition of category.

JACOBSON, Calle - *On local flat homomorphisms and the Yoneda* Ext-*algebra of the fibre.*

Let $A \to B$ be a flat homomorphism of local rings with fibre $\bar{B}$. M André has shown $P_A(z).P_B(z) = P_B(z).(1-z)^{-\delta_2}$ with $\delta_2 = \text{edim } A + \text{edim } \bar{B} - \text{edim } B$, if there are no "special variables" of $\bar{B}$ of degree > 2. We show that a "special variable" of $\bar{B}$ corresponds to a central element of the graded Lie algebra underlying the Yoneda Ext-algebra of $\bar{B}$. We also show that the centre of this Lie algebra is concentrated in degrees 1 and 2 if $\bar{B}$ is "Golod-attached", which proves the formula in this case.

LEGRAND André - π_1 *and* d_2.

Let a bundle $X \to E \to V$ with $\pi_1(V)$ not zero. Beside the monodromy of local systems $H_*(X)$ or $\pi_*(X)$ associated to E and which appear in E^2-terms of Serre spectral sequence (or generalized Shih spectral sequence, [L]) the differential d_2 of these spectral sequences bring us $\pi_1(V)$ actions on $H_*(X)$ or $\pi_*(X)$. We explain here these actions.

LEMAIRE, J.M. - *Sur le type d'homotopie rationnelle des espaces de Ganêa.*

Ganêa has defined a sequence of fibrations $p_n : X(n) \to X$ such that the L.S. category of the space X is $\leq n$ iff p_n has a homotopy section. Halperin and Félix showed that $X(n)$ has the rational homotopy type of the wedge of $X\langle n\rangle$ and a wedge of spheres, where a DGA model of $X\langle n\rangle$ is the quotient of the DGA model of X by the $n+1$-st power of the augmentation ideal. We show that $X\langle n\rangle$ rationally is again a wedge of a space $X<n>$ and spheres, a DGL model of $X<n>$ being the n-th term in the filtered DGL model of X. This yields a characterization of rational L.S. category in terms of DGL models.

LÖFWALL, Clas - *A change of rings theorem for local rings.*

The notion of the (homotopy) Lie algebra of a local ring is extended to differential graded algebras by means of derivations of the acyclic closure. In an appendix it is proved that the definition coïncide with one made by Avramov. The notion is used to prove the existence of an exact sequence of Lie algebras connected to an arbitrary homomorphism of local rings. The characteristic of the residue field is supposed to be zero. The situation in positive characteristic has later been fully explored by Avramov.

LÖFWALL, Clas - *On the centre of graded Lie algebras.*

The (homotopy) Lie algebra of a class of local rings is proved to have trivial centre. As an application it is proved that the deviations for local rings with the cube of the maximal ideal equal to zero are strictly positive unless the ring is a complete intersection.

PAPADIMA, Stephan - *Poincaré duality algebras and the rational classification of differentiable manifolds.*

We give a way of classifying Poincaré duality algebras over $\mathbb{Q}$ and then indicate how it leads to the classification of the $\mathbb{Q}$-types of closed manifolds in some intrinsic formal cases.

PROUTÉ Alain - *Vers un $\mathbb{Z}/p$-lemme de Hirsch.*

We try to construct a Hirsch lemma, as the one used by Sullivan in his rational minimal model theory, but with $\mathbb{Z}/p$ coefficients. Our starting point is Brown's twisted Eilenberg-Zilber theorem. In our tentative to reduce the twisted tensor product to a minimal form, we are led to introduce $A(\infty)$-structures on homology.

PUPPE, Volker, *P A Smith-theory via deformations.*

Methods from deformation theory of algebraic structures (cochain complexes, associative algebras, Lie algebras) are used to study the relationship between algebraic invariants attached to a topological space on which a group acts and those attached to the fixed point set of that action.

SBAÏ, Mohamed - *Rational cocategory of a topological space.*

We give a dual definition in the sense of Eckmann-Hilton of the Félix-Halperin definition for rational category.

SHIBATA, Katsuyuki - *Sullivan-Quillen mixed type model for fibrations and the Haefliger model for the Gelfand-Fuks cohomology.*

A graded Lie algebra of certain type over a differential graded algebra can be used as an algebraic model for a fibration, so that the model for the space of sections can be well-described. Application to the Gelfand-Fuks cohomology is stated.

SHIGA, Hiroo - *Rational homotopy types of cofibrations.*

We show that a cofiber of a map between formal spaces is a formal space, if the map is formal and the source of the map is a finite complex.

THOMAS, Jean-Claude - *Lie algebra of derivations and fibration.*

We show how the composition of two fibrations yields a Lie algebra of derivations, linked to the classifying algebra in the sense of Sullivan. In the particular case of the free paths fibration of a space S, this construction gives us a Quillen model of S.

TRIANTAFILLOU, Georgia - *An algebraic model for G-homotopy types.*

Let G be a finite group and let X be a G-CW-complex. In earlier work we constructed an equivariant minimal model $\mathcal{M}_X$ for X which is a generalization of Sullivan's minimal model and classifies G-rational homotopy types. In this paper we use $\mathcal{M}_X$ to classify (integral) G-homotopy types of G-CW-complexes up to finite ambiguity. For this purpose we prove that $\mathrm{aut}_G(X_0)$ (the group of G-homotopy classes of G-self homotopy equivalence of a rationalization X_0 of X) is an algebraic $\mathbb{Q}$-matrix group and that $\mathrm{aut}_G(X)$ is commensurable to an arithmetic subgroup of $\mathrm{aut}_G(X_0)$.

UNSÖLD, Hans Michael - *Topological minimal algebra and Sullivan De Rham equivalence.*

We prove the existence of functors M and G inducing an equivalence of homotopy categories

$$M : H_o(\mathbb{Q}S_1) \rightleftarrows H_o(\mathrm{Min}_1) : G$$

where $\mathbb{Q}S_1$ denote the category of 1 reduced rational simplicial sets and Min_1 denote the category of 1-connected minimal topological algebras over $\mathbb{Q}$.

VIGUÉ-POIRRIER, Micheline - *On the growth of the Betti numbers of the free loops space.*

We recall some theorems proved by the author about the growth of the Betti numbers of the free loop space, and their applications in geometry. We prove, by elementary methods, that the sequence of Betti numbers of X^{S^1} (where $X = \bigvee_{i=1}^{r} S^{m+1}$, $r \geq 2$, $m > 1$) grows exponentially.

LISTE DES PARTICIPANTS

AUBRY Marc - Université de Nice - Département de Mathématiques
Parc Valrose
06034 - NICE CEDEX (France)

AVRAMOV Luchezar - Bulgarian Acadamy of Sciences - Institute of Mathematics
1113 - SOFIA (Bulgaria)

BACKELIN Jörgen - Stockholm Universitet - Matematiska institutionen
Box 5701, S-112 60 - STOCKHOLM (Suède)

BAUES Hans Joachim - Max Plamck Inst.
5300 - BONN 3 (R.F.A.)

BØGVAD Jens Olof Rikard - Universitet of Stockholm - Dep. of Math.
Box 6701
S-11385 - STOCKHOLM (Suède)

BREEN Lawrence - Université de Rennes - U.E.R. de Mathématiques
et d'Informatique
Campus de Beaulieu
35042 - RENNES CEDEX (France)

BROWN Ronald - University College of North Wales -
School of Mathematics and Computer Science
BANGOR GWYNEDD 1157 2UW (UK)

BURGHELEA Dan - Department of Mathematics
231 West 18th
Avenue Colombus OHIO 43210 (U.S.A.)

CARTAN Henri - 95, Boulevard Jourdan
75014 - PARIS (France)

CATHELINEAU J.L. - Université de Nice
Parc Valrose
06034 - NICE CEDEX (France)

CHEN Kuo-Tsai - 403 Evergreen Court
URBANA Illinois 61801 (U.S.A.)

DIDIERJEAN André - Université de Strasbourg - U.E.R. de Mathématiques
7, rue René Descartes
67084 - STRASBOURG CEDEX (France)

DIDIERJEAN Genevieve - Université de Strasbourg - U.E.R. de Mathématiques
7, rue René Descartes
67084 - STRASBOURG CEDEX (France)

FELIX Yves - Université de Louvain-la-Neuve
2, Chemin du Cyclotron
1348 - LOUVAIN-LA-NEUVE (Belgique)

FLANCHEC Annick - Université de Nantes - Institut de Mathématiques
et d'Informatique
2, Chemin de la Houssinière
44072 - NANTES CEDEX (France)

FRÖBERG Ralf - Universitet de Stockholm - Inst. Math.
Box 6701 - STOCKHOLM (Suède)

GOLASINSKI Marek - Uniwersytet M. Kopernika - Instytut Matematyki
100 TORUN, Chopina 12/18 (Pologne)

GOTTLIEB Daniel - Purdue University - Department of Math.
W. Lafayette
INDIANA 47907 (U.S.A.)

LISTE DES PARTICIPANTS

GOUYON Luce - Université de Toulouse - U.E.R. de Mathématiques
118, route de Narbonne
TOULOUSE (France)

GOYO John Octavius - University of Toronto - Department of Mathematics
TORONTO, Ontario (Canada)

GRIVEL Pierre-Paul - Université de Genève - Institut de Mathématiques
2-4, rue du Lièvre
1211 - GENEVE (Suisse)

HAEFLIGER André - Université de Genève - Institut de Mathématiques
2-4, rue du Lièvre
1211 - GENEVE (Suisse)

HALPERIN Steve - University of Toronto - Department of Mathematics
TORONTO, Ontario (Canada)

HAUSMANN Jean-Claude - Université de Genève
Institut de Mathématiques
2-4, rue du Lièvre
1211 - GENEVE (Suisse)

HOPKINS Mickael - New College OXFORD OX13 BN, England (Grande-Bretagne)

HUSEMÖLLER Dale - Haverford College
HAVERFORD Pe 19041 (U.S.A.)

JACOBSON Calle - UPPSALA University
Department of Mathmatics
THUNBERGSV 3
S-75238 - UPPSALA (Suède)

JAMES Ioan - Mathematical Institute 24-29 ST Giles
OXFORD OX 15 BJ (Grande-Bretagne)

KÖHLER Wilfried - Universität Gesamthochschule siegen FB Mathematik
Postfach 210209 D 5900 SIEGEN 21 (R.F.A.)

KOSZUL Jean-Louis - Université Scientifique et Médicale de Grenoble
Laboratoire de Mathématiques
BP 116
38402 - SAINT-MARTIN-D'HERES (France)

LEGRAND André - Université de Toulouse - U.E.R. de Mathématiques
118, route de Narbonne
31077 - TOULOUSE CEDEX (France)

LEGRAND Claude - Université de Toulouse - U.E.R. de Mathématiques
118, route de Narbonne
31077 - TOULOUSE CEDEX (France)

LEMAIRE Jean-Michel - Université de Nice - Département de Mathématiques
Parc Valrose
06034 - NICE CEDEX (France)

LESCOT Jack - U.E.R. de Sciences
Département de Mathématiques et de Mécanique
14032 - CAEN CEDEX (France)

LODAY Jean-Louis - I.R.M.A.
7, rue René Descartes
67084 - STRASBOURG CEDEX (France)

LOFWALL Clas - Department of Mathematics
Box 6701
S-11385 - STOCKHOLM (Suède)

MEIER Willi - Universität Siegen
Holdelinstv 3
D. 5900 - SIEGEN (R.F.A.)

MERLE Pierre - Université de Nice - Département de Mathématiques
Parc Valrose
06034 - NICE CEDEX (France)

PAUGAM Michel - Université de Caen - U.E.R. de Sciences
Département de Mathématiques et de Mécanique
14032 - CAEN CEDEX (France)

PORTER Richard - Northeastern University
College of Arts and Sciences
Department of Mathematics
360, Huntington Avenue
BOSTON, Massachusetts 02115 (U.S.A.)

PRADINES Jean - Université de Toulouse - U.E.R. de Mathématiques
118, route de Narbonne
31077 - TOULOUSE CEDEX (France)

PROUTÉ Alain - Université de Nantes - U.E.R. de Mathématiques
2, Chemin de la Houssinière
44000 - NANTES (France)

PUPPE Volker - Universität Konstang
Fachbereich Mathematik
7750 KONSTANG - Postfach 5560 (R.F.A.)

RAHBAR-ROCHANDEL Hamid - U.E.R. de Sciences - Université de Caen
Département de Mathématiques et de Mécanique
14032 - CAEN (France)

ROOS Jean-Erik - Université de Stokholm - Department of Mathematics
Box 6701, S-11385 STOCKHOLM (Suède)

SBAÏ Mahamed - Université de Lille I - U.E.R de Mathématiques pures et appliquées
59655 - VILLENEUVE D'ASCQ CEDEX (France)

SCHWARTZ Lionel - Université de Paris XI - Bâtiment 425
U.E.R. de Mathématiques
91405 - ORSAY CEDEX (France)

SCHEERER Hans - Math. Inst. Arnimallee 2-6
D. 1000 BERLIN 33 (Allemagne)

SHIBATA Katsuyuki - Faculty of Liberal Arts - Department of Mathematics
Saitama University, URAWA, Saitama (Japan 338)

SHIGA Hiroo - Department of Mathematics College of Science
Ryukyu University Nakagusuku
OKINAWA (Japan)

SIGRIST François - Université de Neuchâtel - Institut de Mathématiques
Chantemerle 20, CH 2000 NEUCHATEL (Suisse)

TANRÉ Daniel - Université de Lille I
U.E.R. de Mathématiques Pures et Appliquées
59655 - VILLENEUVE D'ASCQ CEDEX (France)

THOMAS Jean-Claude - Université de Lille I
U.E.R. de Mathématiques Pures et Appliquées
59655 - VILLENEUVE D'ASCQ CEDEX (France)

TRIANTAFILOU Georgia - University of Minnesota - Department of Mathematics
MINNEAPOLIS MN 55455 (U.S.A.)

UNSÖLD Michaël - Weserstrasse 37, D-1000 BERLIN
West 44, (R.F.A.)

VIGUÉ-POIRRIER Micheline - Université de Lille I
U.E.R. de Mathématiques Pures et Appliquées
59655 - VILLENEUVE D'ASCQ CEDEX (France)

VIVIENTE José Luis - Ciudad Universitaria,
ZARAGOZA (Espagne)

ZISMAN Michel - Université de Paris VII - U.E.R. de Mathématiques,
2, Place Jussieu
75251 - PARIS CEDEX 05 (France)