J.-L. LIEUTENANT

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APPLICATIONS OF DECOMPOSITIONS OF HOLOMORPHIC FUNCTIONS TO PARTIAL DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

by J.-L. LIEUTENANT (University of Liège)

(Research Assistant F.N.R.S.)

NOTATIONS.

We consider $\mathbb{R}^n$ endowed with the usual scalar product defined by $\langle y,\xi\rangle = \Sigma y_j \xi_j$ and the euclidean norm $|y| = \sqrt{\langle y,y\rangle}$ as a closed submanifold of $\mathbb{C}^n$. We shall denote by S^{n-1} the unit sphere of $\mathbb{R}^n$ and for any cone $\Gamma \subset \mathbb{R}^n$ we define the polar of Γ by $\Gamma^{\perp} = \{\xi \in \mathbb{R}^n \setminus \{0\} : \langle y,\xi\rangle \geq 0, \forall y \in \Gamma\}$.
By a salient cone, we mean a cone that does not contain any straight line. Given an open subset Ω of $\mathbb{R}^n$ and an open convex cone $\Gamma \subset \mathbb{R}^n$, a subset A of $\mathbb{C}^n$ will be called of profile $\Omega + i\Gamma$ if for every compact sets $K \subset \Omega$ and $\mathcal{K} \subset \Gamma \cap S^{n-1}$, there exists $\rho_o > 0$ such that the wedge

$$\{x + i\rho y : x \in K,\ y \in \mathcal{K},\ \rho \in]0,\rho_o]\}$$

is contained in A. We are going to represent by $\mathcal{D}$ the ring of linear partial differential operators with constant complex coefficients. It is well known that $\mathcal{D}$ is unitary and noetherian. If $P = \sum_{|\alpha|\leq m} a_\alpha D^\alpha$ belongs to $\mathcal{D}$, we shall write $\overset{\circ}{P}$ the principal symbol of P and $\mathrm{car}(P)$ the characteristic variety of P, i.e. the set $\{\xi + i\eta \in \mathbb{C}^n : |\xi|^2 + |\eta|^2 = 1,\ \overset{\circ}{P}(\xi + i\eta) = 0\}$. Finally, let us denote by $\mathcal{O}$ the sheaf of holomorphic functions on $\mathbb{C}^n$ and by $\mathcal{A}$ the linear space of $\mathbb{C}$-valued analytic functions on $\mathbb{R}^n$.

Let us first recall two decomposition theorems proved in [5] and [6].

Theorem 1: For any $F \in \mathcal{A}$ and any finite family of open convex salient cones Γ_j of $\mathbb{R}^n$ whose polars cover S^{n-1}, there exist domains of holomorphy V_j containing $\mathbb{R}^n$ and an open convex tube $\mathbb{R}^n + i\Omega_j$ of profile $\mathbb{R}^n + i\Gamma_j$ and $F_j \in \mathcal{O}(V_j)$ such that $F = \Sigma F_j$ holds on a neighborhood of $\mathbb{R}^n$. Moreover, if the interiors of the polars of the Γ_j's cover S^{n-1}, given $r \in]0,+\infty[$, one can assume that the V_j's are open pseudoconvex neighborhoods of the closed tubes

$$\mathbb{R}^n + i\{y \in \overline{\Gamma}_j : |y| \leq r\} .$$

Theorem 2: Let Γ be an open convex cone of $\mathbb{R}^n$, Ω an open subset of $\mathbb{R}^n$ and V an open subset of $\mathbb{C}^n$ of profile $\Omega + i\Gamma$. For any $r \in]0,+\infty[$, any $F \in \mathcal{O}(V)$ and any open subcone Γ' of Γ whose intersection with S^{n-1} is relatively compact in Γ, there exist an open convex neighborhood Ω' of $\{y \in \overline{\Gamma}' : 0 < |y| \leq r\}$ in $\mathbb{R}^n$, an open pseudoconvex neighborhood W of Ω contained in $\Omega + i\,\mathbb{R}^n$, $A \in \mathcal{O}(W)$ and $G \in \mathcal{O}(\mathbb{R}^n + i\Omega')$ such that $W \cap (\mathbb{R}^n + i\Omega') \subset V$ and $F = G + A$ on $W \cap (\mathbb{R}^n + i\Omega')$.

Remark 3: This last statement constitutes in fact a slight improvement of the result obtained in [6]. To establish it, one only needs (besides evident modifications) to remark that lemma 6 of [5] can be precised as follows: if the U_j's are stricly pseudoconvex tubes with C_2-boundaries $\partial U_j \supset \mathbb{R}^n$, for any complex neighborhood W of an open set Ω of $\mathbb{R}^n$, there exists an open pseudoconvex neighborhood V_W of $\Omega \cup (\cap U_j)$ such that $V_W \setminus (\cap U_j) \subset W$.

Lemma 4: Let Γ be an open convex non void cone of $\mathbb{R}^{n'}$. The polar of $\mathbb{R}^n \times \Gamma$ coincides with $0 \times \Gamma^{\perp}$ and is a convex salient cone closed in $\mathbb{R}^{n+n'} \setminus \{0\}$. Moreover, for any open cone $\gamma \supset 0 \times \Gamma^{\perp}$, there exists $\rho > 0$ such that $\gamma^{\perp}$ is contained in

$$\tilde{\Gamma}_\rho = \{(x,y) : |x| \leq \frac{|y|}{\rho},\ y \neq 0,\ d(\frac{y}{|y|}, S^{n'-1} \setminus \Gamma) \geq \rho\} ,$$

where d denotes the euclidean distance.

Proof: One obtains immediately the equality $(\mathbb{R}^n \times \Gamma)^{\perp} = 0 \times \Gamma^{\perp}$ and as Γ is open and non void, this is a salient cone closed in $\mathbb{R}^{n+n'} \setminus \{0\}$.

For the second part of the statement, let us first choose $\varepsilon > 0$ such that

$$\omega_\varepsilon = \{(\xi,\eta) \in S^{n+n'-1} : |\xi| < \varepsilon,\ d(\frac{\eta}{|\eta|}, \Gamma^{\perp} \cap S^{n'-1}) < \varepsilon\} \subset \gamma ,$$

hence such that $\gamma^{\perp}$ is contained in

$$\Gamma' = \bigcap_{(\xi,\eta)\in\omega_\varepsilon} \{(x,y) : \langle x,\xi\rangle + \langle y,\eta\rangle \geq 0\} .$$

As it is clear by definition of ω_ε that $\Gamma' \setminus \{0\}$ is disjoint from $\{(x,o) : x \in \mathbb{R}^n\}$, we are going to show that the existence of sequences (x_m,y_m) or $(x'_m,y'_m) \in \Gamma' \setminus \{0\}$ such that $|x_m| > m|y_m|$ or such that $|y'_m| = 1$ and $d(y'_m, S^{n'-1} \setminus \Gamma) < \frac{1}{m}$ leads to a contradiction in each case. As a matter of fact, for any $\eta_o \in \Gamma^{\perp} \cap S^{n'-1}$, the points $(\xi_m,\eta_m) = \frac{1}{\sqrt{1+m^2}} \left(- \frac{x_m}{|x_m|}, m\eta_o\right)$ belong to ω_ε for m sufficiently large and one obtains

$$0 \leq \langle x_m,\xi_m\rangle + \langle y_m,\eta_m\rangle < - \frac{m|y_m|}{\sqrt{1+m^2}} + \frac{m|y_m|}{\sqrt{1+m^2}} = 0 ,$$

hence a first contradiction. In the second case, we can find $y''_m \in S^{n'-1} \setminus \overline{\Gamma}$ such that $|y'_m - y''_m| < \frac{1}{m}$ and by convexity of $\overline{\Gamma}$, there are points $\eta'_m \in \Gamma^{\perp} \cap S^{n'-1}$ verifying $\langle y''_m,\eta'_m\rangle < 0$.

As the points $(\xi_m,\eta_m) = \frac{1}{\sqrt{1+m^2}} (0,m\eta'_m - y'_m)$ belong to ω_ε for m sufficiently large, we obtain another contradiction:

$$0 \leq \langle x'_m,\xi_m\rangle + \langle y'_m,\eta_m\rangle \leq \frac{1}{\sqrt{1+m^2}} [m \langle y'_m - y''_m,\eta'_m\rangle + m \langle y''_m,\eta'_m\rangle - 1]$$

$$< \frac{1}{\sqrt{1+m^2}} [m |y'_m - y''_m| - 1] \leq 0 .$$

Definition 4: Adapting a definition due to Bony and Schapira (cf. [1]) to the particular situation we have in mind, we shall say that a point z_o of the boundary of an open subset V of $\mathbb{C}^n$ verifies the condition $c(z_o,\gamma,V)$ with respect to a convex salient cone γ closed in $\mathbb{R}^n\setminus\{0\}$ if for any $\varepsilon > 0$, there exists $\eta > 0$ such that

$$\{z \in V : |z-z_o| < \eta\} - i\{y \in (\gamma_\varepsilon)^\perp : |y| < \eta\}$$

is contained in V, where γ_ε denotes the conic hull of the set of points of S^{n-1} whose distance to $\gamma \cap S^{n-1}$ is less than ε.

Lemma 5: a) If $\Gamma \subset \mathbb{R}^n$ is an open convex non void cone and $V = \mathbb{R}^n + i\Omega$ an open tube of $\mathbb{C}^n$, then for any $z_o \in \partial V$, the condition $c(z_o,-\Gamma^\perp,V)$ is equivalent to the cone condition $C(z_o,(0\times\Gamma^\perp) \cap S^{2n-1})$ stated in 4.1 of [1].

b) Let r belong to $]0,+\infty[$, Γ be an open convex non void cone of $\mathbb{R}^n$ and V denote the open tube $\{z \in \mathbb{R}^n + i\Gamma : |y| < r\}$. Every $x \in \mathbb{R}^n \subset \partial V$ verifies then the condition $c(x,-\Gamma^\perp,V)$. Moreover, if γ is an open conic neighborhood of $(-\Gamma^\perp)\setminus\{0\}$ and if $V' = \mathbb{R}^n + i\Omega$ is an open tube of profile $\mathbb{R}^n + i\Gamma$, there exists an open convex subcone Γ' of Γ such that $-\Gamma'^\perp \subset \gamma$ and any $x \in \mathbb{R}^n \subset \partial[V' \cap (\mathbb{R}^n+i\Gamma')]$ verifies the condition $c[x,-\Gamma'^\perp, V' \cap (\mathbb{R}^n+i\Gamma')]$

Proof: a) We first show that c implies C. Let I' be an open neighborhood of $(0 \times \Gamma^\perp) \cap S^{2n-1}$ in S^{2n-1}. By Lemma 4, the polar of I' in the sense of Bony and Schapira (which is the opposite of ours) is contained in $-\widetilde{\Gamma}_\rho$ for some $\rho > 0$. There exists $\varepsilon > 0$ such that $(\Gamma^\perp{}_\varepsilon)^\perp$ contains $\{y \in S^{n-1} : d(y,S^{n-1}\setminus\Gamma) \geq \rho\}$. Now let η be the number which corresponds to ε by application of $c(z_o,-\Gamma^\perp,V)$. For $V' = \{z : |z-z_o| < \frac{\eta}{2}\}$ and the set A of the points of the polar of I' (in the sense of [1]) whose module is less than $\frac{\eta}{2}$, the inclusion $(V' \cap V) + A \subset V$ is easy to obtain because V is a tube.

The proof of $C \rightarrow c$ is similar and as we shall not use this implication in what follows, we do not give further details.

b) The first assertion follows immediately from the inclusion $[(\Gamma^{\perp})_{\varepsilon}]^{\perp} \subset \Gamma$, which is easy to obtain. For the second one, let us denote by γ' an open convex cone of $\mathbb{R}^n$ verifying

$$-\Gamma^{\perp} \subset \gamma' \subset \overline{\gamma'} \setminus \{0\} \subset \gamma$$

and set $\Gamma' = -\gamma'^{\perp o}$. One has evidently $\overline{\Gamma'} = -\gamma'^{\perp} = -\overline{\gamma'}^{\perp} \subset \Gamma^{\perp\perp o} = \Gamma$ and $-\Gamma'^{\perp} = \overline{\gamma'} \setminus \{0\} \subset \gamma$ and as $\mathbb{R}^n + i\Omega$ is of profile $\mathbb{R}^n + i\Gamma$, there exists $\rho_o > 0$ such that $\{y \in \Gamma' : |y| < \rho_o\} \subset \Omega$. Hence the conclusion by application of the first part of this result.

Theorem 6: Let $P \in \mathscr{D}$, $r \in]0,+\infty[$ and Γ_j be a finite family of open convex non void cones of $\mathbb{R}^n$. If $\mathrm{car}(P) \cap [-i \cup \Gamma_j^{\perp}]$ is empty, the equation $Pu = f$ is solvable in the subspace of $\mathscr{A}$, whose elements can be written $\Sigma F_j|_{\mathbb{R}^n}$ with $F_j \in \mathscr{A} \cap \mathscr{O}[\mathbb{R}^n + i\{y \in \Gamma_j : |y| < r\}]$.

If there moreover exists an open convex conic neighborhood γ of $-\cup\Gamma_j^{\perp}$ such that $\mathrm{car}(P) \cap i\gamma$ is empty, the same equation is solvable in the subspace of $\mathscr{A}$, whose elements can be written $\Sigma F_j|_{\mathbb{R}^n}$ with $F_j \in \mathscr{A} \cap \mathscr{O}(V_j)$, where V_j denotes an open convex tube of profile $\mathbb{R}^n + i\Gamma_j$.

In particular, when P is ξ_o-hyperbolic in the sense that it verifies the following two conditions

a) $\overset{\circ}{P}(\xi_o) \neq 0$

b) P does not vanish on $\mathbb{R}^n + i\{\lambda\xi_o : \lambda > c\}$ for some $c \geq 0$,

the second situation occurs if $-\cup\Gamma_j^{\perp}$ is contained in the open convex cone

$$\gamma_P = \bigcap_{\lambda \geq 0} \{\xi \in \mathbb{R}^n : P(\xi + \lambda\xi_o) \neq 0\}$$

Proof: According to the Malgrange-Ehrenpreis theorem (cf. [4] or [7]), the equations $PU_j = F_j$ are solvable in both cases in a convex tube of profile

$\mathbb{R}^n + i\Gamma_j$. By the precedent lemma, we can apply theorem 4.1 of [1] and therefore suppose that the U_j's are also holomorphic on a neighborhood of $\mathbb{R}^n$. Hence the conclusion by linearity of P .

The third assertion is a direct consequence of a slight modification of Gårding's well known result on hyperbolic polynomials (cf. [2]) which asserts under our hypothesises that $\mathring{P}$ does not vanish on $\mathbb{R}^n + i\gamma_P$.

Remark 7: a) Our definition of ξ_o-hyperbolicity differs from Gårding's one because we interchange the roles of the real and imaginary parts of the complex directions.

b) When P is ξ_o-hyperbolic, it is easy to prove by Hurwitz's theorem that γ_P coincides with the connected component of

$$\{\xi \in \mathbb{R}^n : \mathring{P}(\xi) = 0\}$$

that contains ξ_o .

c) Combining theorems 1 and 6, one obtains immediately the following well known result:

Corollary 8: If $P \in \mathscr{D}$ is elliptic, one has $P(D)\mathscr{A} = \mathscr{A}$.

Proof: One only needs to point out that car(P) does not meet $i\mathbb{R}^n$.

Proposition 9: Let $\Gamma \subset \mathbb{R}^n$ be a closed convex cone with non void interior and $\mathfrak{M}$ be a finitely generated $\mathscr{D}$-module. For any $r \in]0,+\infty[$, one has $\mathrm{Ext}^j_{\mathscr{D}}[\mathfrak{M}, \mathscr{O}(F_{\Gamma,r})] = 0$ for all $j \geq 2$, where $F_{\Gamma,r}$ denotes

$$\{x + iy \in \mathbb{R}^n + i\Gamma : |y| \leq r\} .$$

Proof: The $K_m = \{x + iy \in \mathbb{R}^n + i\Gamma : |x| \leq m, |y| \leq r\}$ $(m \in \mathbb{N})$ form a sequence of compact convex sets which increases towards $F_{\Gamma,r}$ in such a way that one has $\mathscr{O}(F_{\Gamma,r}) = \varprojlim_m \mathscr{O}(K_m)$ and consequently $\varprojlim_m \mathrm{Hom}_{\mathscr{D}}[\mathscr{D}^N, \mathscr{O}(K_m)] = \mathrm{Hom}_{\mathscr{D}}[\mathscr{D}^N, \mathscr{O}(F_{\Gamma,r})]$ for any $N \in \mathbb{N}$. Let

$$0 \leftarrow \mathcal{M} \leftarrow \mathcal{D}^{r_o} \xleftarrow{{}^t\psi_o} \mathcal{D}^{r_1} \xleftarrow{{}^t\psi_1} \cdots \xleftarrow{{}^t\psi_{q-1}} \mathcal{D}^{r_q} \leftarrow 0 \quad (q \leq n)$$

be a free projective resolution of $\mathcal{M}$ and consider the commutative diagram

$$\begin{array}{ccccccc}
0 \to \mathrm{Hom}_{\mathcal{D}}[\mathcal{D}^{r_o}, \mathcal{O}(F_{\Gamma,r})] & \to & \cdots & \to & \mathrm{Hom}_{\mathcal{D}}[\mathcal{D}^{r_q}, \mathcal{O}(F_{\Gamma,r})] \to 0 \\
\downarrow & & & & \downarrow \\
\vdots & & & & \vdots \\
\downarrow & & & & \downarrow \\
0 \to \mathrm{Hom}_{\mathcal{D}}[\mathcal{D}^{r_o}, \mathcal{O}(K_{m+1})] & \to & \cdots & \to & \mathrm{Hom}_{\mathcal{D}}[\mathcal{D}^{r_q}, \mathcal{O}(K_{m+1})] \to 0 \\
\downarrow & & & & \downarrow \\
0 \to \mathrm{Hom}_{\mathcal{D}}[\mathcal{D}^{r_o}, \mathcal{O}(K_m)] & \to & \cdots & \to & \mathrm{Hom}_{\mathcal{D}}[\mathcal{D}^{r_q}, \mathcal{O}(K_m)] \to 0 \\
\downarrow & & & & \downarrow \\
\vdots & & & & \vdots
\end{array}$$

A well known result (cf. [3], p. 410, for example) asserts that the canonical maps

$$\Phi^j : \mathrm{Ext}^j_{\mathcal{D}}[\mathcal{M}, \mathcal{O}(F_{\Gamma,r})] \to \varprojlim_m \mathrm{Ext}^j_{\mathcal{D}}[\mathcal{M}, \mathcal{O}(K_m)] \ , \quad \forall j \geq 2$$

are isomorphisms because Mittag-Leffler's condition is satisfied since we have for every $j \geq 1$ and $m \in \mathbb{N}$

$$H^j(0 \to \mathrm{Hom}_{\mathcal{D}}[\mathcal{D}^{r_o}, \mathcal{O}(K_m)] \to \cdots \to \mathrm{Hom}_{\mathcal{D}}[\mathcal{D}^{r_q}, \mathcal{O}(K_m)] \to 0) = \mathrm{Ext}^j_{\mathcal{D}}[\mathcal{M}, \mathcal{O}(K_m)] = 0$$

by virtue of the Malgrange-Ehrenpreis theorem. Hence the conclusion.

<u>Notation 10</u>: If Γ is an open convex non void cone of $\mathbb{R}^n$, we set

$$\tilde{\mathcal{A}}_\Gamma = \varinjlim_V \mathcal{O}(V) \quad ,$$

where V runs over the open subsets of $\mathbb{C}^n$ of profile $\mathbb{R}^n + i\Gamma$. We shall also denote by $\mathcal{F}_\Gamma$ the family of open convex non void subcones Γ' of Γ such that $\Gamma' \cap S^{n-1}$ is relatively compact in Γ .

Theorem 11: For any open convex non void cone Γ of $\mathbb{R}^n$ and any finitely generated $\mathscr{D}$-module $\mathfrak{M}$, one has

$$\varprojlim_{\Gamma' \in \mathscr{F}_\Gamma} \mathrm{Ext}^j_{\mathscr{D}}[\mathfrak{M}, \tilde{\mathscr{A}}_{\Gamma'}/\mathscr{A}] = 0 \ , \quad \forall j \geq 1 \ .$$

Proof: Given any $\Gamma' \in \mathscr{F}_\Gamma$, we are going to establish that for every $j \geq 1$ and every $\Gamma'' \in \mathscr{F}_\Gamma$ such that $\Gamma' \in \mathscr{F}_{\Gamma''}$ the canonical operator

$$\mathrm{Ext}^j_{\mathscr{D}}[\mathfrak{M}, \tilde{\mathscr{A}}_{\Gamma''}/\mathscr{A}] \longrightarrow \mathrm{Ext}^j_{\mathscr{D}}[\mathfrak{M}, \tilde{\mathscr{A}}_{\Gamma'}/\mathscr{A}]$$

vanishes. Using the same notations as in the precedent proof to denote the free projective resolution of $\mathfrak{M}$, we are lead to prove that the image of

$$(\tilde{\mathscr{A}}_{\Gamma''}/\mathscr{A})^{r_{j-1}} \xrightarrow{\psi_{j-1}} (\tilde{\mathscr{A}}_{\Gamma''}/\mathscr{A})^{r_j} \xrightarrow{\psi_j} (\tilde{\mathscr{A}}_{\Gamma''}/\mathscr{A})^{r_{j+1}}$$

in the complex

$$(\tilde{\mathscr{A}}_{\Gamma'}/\mathscr{A})^{r_{j-1}} \xrightarrow{\psi_{j-1}} (\tilde{\mathscr{A}}_{\Gamma'}/\mathscr{A})^{r_j} \xrightarrow{\psi_j} (\tilde{\mathscr{A}}_{\Gamma'}/\mathscr{A})^{r_{j+1}}$$

is exact. In other words, we have to prove that given any $F \in \tilde{\mathscr{A}}_{\Gamma''}^{\ r_j}$ verifying

$$\psi_j F = 0 \bmod \mathscr{A} \ , \qquad (1)$$

there exists $U \in \tilde{\mathscr{A}}_{\Gamma'}^{\ r_{j-1}}$ such that

$$\psi_{j-1} U = F \bmod \mathscr{A} \ . \qquad (2)$$

By theorem 2, we can decompose F in $G + A$ with $G \in \mathscr{O}[\mathbb{R}^n + i\{y \in \overline{\Gamma}' : 0 < |y| \leq r\}]$ and $A \in \mathscr{A}$. Therefore (1) and (2) become respectively

$$\psi_j G \in \mathscr{O}(F_{\overline{\Gamma}', r})^{r_{j+1}} \qquad (3)$$

$$\psi_{j-1} U = G \bmod \mathscr{A}. \qquad (4)$$

As we have trivially $\psi_{j+1}\ \psi_j G = 0$ the precedent lemma assures the existence of $H \in \mathcal{O}(F_{\overline{\Gamma'},r})^{r_j}$ such that $\psi_j H = \psi_j G$ and we can replace (3) and (4) respectively by

$$\psi_j(G - H) = 0 \text{ on } F_{\overline{\Gamma'},r}$$

$$\psi_{j-1}U = G - H \bmod \mathcal{A} .$$

Since $\mathbb{R}^n + i\{y \in \Gamma' : |y| < r\}$ is an open convex subset of $F_{\overline{\Gamma'},r}$, another application of the Malgrange-Ehrenpreis theorem allows to conclude.

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R E F E R E N C E S

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Jean-Louis LIEUTENANT,
Institute of Mathematics,
University of Liège,
Avenue des Tilleuls 15,
B-4000 Liège,
Belgium.