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INTRODUCTION TO STOCHASTIC FIELD THEORY *

(Abstract)

Francesco Guerra

We consider classical dynamical systems with a finite or infinite number of degrees of freedom and with Lagrangian of canonical type which arise in many areas of physics. Typical examples are the harmonic and anharmonic oscillator, the free Klein-Gordon scalar field, the vector meson Proca field, the electromagnetic Maxwell field, the Klein-Gordon field with polynomial self-interaction, etc. To each of these classical dynamical systems we associate some stochastic processes following the prescriptions of Nelson stochastic mechanics [9,10] extended to general Lagrangian systems [7,4]. In particular we focus our attention on the ground state stochastic process of lowest energy and study its properties.

As a rather unexpected and surprising result [7] we find that the ground state stochastic process so constructed coincides with the stochastic process introduced in the so-called Euclidean formulation of quantum field theory. We recall here that the Euclidean theory, initiated by Schwinger [14] and Nakano [8], after the work of Symanzik [17] and Nelson [11,12] has become an essential tool [18,15] for the constructive quantum field theory program of Glimm and Jaffe and

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their followers [3, 18]. In the usual interpretation the Euclidean theory is introduced starting from quantum field theory in the Wightman formulation[16] and then performing an analytic continuation in the time variables from real Minkowski time to the imaginary "Euclidean" time [14, 17]. The inverse analytic continuation is necessary in order to recover Wightman field theory from the Euclidean theory [11,13,15,1]. According to the proposed new interpretation [7] we can introduce "Euclidean"-Markov fields directly inside the framework of physical space-time and no analytic continuation is necessary both for the mathematical construction and the physical interpretation.

An attractive feature of our way of introducing Euclidean fields via stochastic mechanics is a simple derivation [7] of the Feynman-Kac-Nelson formula (see [6]) from the theory of stochastic differential equations [2].

The role of Euclidean field theory and stochastic mechanics as possible tools for the quantization of general relativity is stressed in [5].

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