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**Cats' tales**

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CATS' TALES

Omer Adelman

Consider the following: there are cats in some of the places of a two-sided infinite sequence. Then start, step by step, this process: at each step each cat jumps, independently of the others, with probability  $\frac{1}{2}$  to each of the two neighbouring places. If two cats land on the same place, they disappear (imagine each second cat to be an anti-cat).

Define  $A \equiv \{ 0 \text{ is visited an infinite number of times} \}$ .

Question:  $p(A) = ?$

(there are some versions of this problem, that can be handled in a very similar way).

The answer depends, of course, on the initial distribution of the cats.

We can immediately get  $p(A) = 1$  and  $p(A) = 0$  in the cases of odd and even number of cats, respectively.

Denote by  $i(n)$  the initial number of cats in the block  $1, \dots, n$ , and suppose the negatives are initially empty. Then in the infinite cats case in which  $\frac{i(n)}{n} \rightarrow 0$  simple examples can be found for which  $p(A) = 1$ , as well as others for which  $p(A) = 0$ .

The general case  $\overline{\lim} \frac{i(n)}{n} > 0$  is unsolved yet, but there is a large class for which the answer can be proved to be  $p(A) = 1$ . This class contains, as a typical sub-class, those sequences in which there is some  $n$  such that there are infinitely many  $n_k$ 's such that

the block  $n, \dots, n+2n_k$  is, in the beginning, symmetric with respect to reflection about  $n+n_k$ , and contains an odd number of cats (the sequence in which all the naturals are initially occupied  $(\frac{i(n)}{n} \equiv 1)$  is, of course, contained in this subclass).

The proof to the last claim is rather long, but its basic idea is the same as that in the following proof of  $p(A)$  being 1 when there is one cat only.

Suppose the cat is in the  $n$ 'th place. By symmetry, there is probability  $\frac{1}{2}$  that  $2n$  is visited before 0. If that happens, then there is probability  $\frac{1}{2}$  that  $4n$  is visited before 0, and so on. But  $(\frac{1}{2})^\infty = 0$ , so 0 will a.s. be visited, so it will a.s. be visited an infinite number of times.

In the case of finite number of cats, a similar method can be applied to the  $n$ -dimensional analogous problem ( $p(A)$  found, as is known, to vanish for  $n > 2$ ), but I don't know how to treat the general  $n$ -dimensional infinite cats problem (excluding some special cases).

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