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CLASSIFICATION DES AUTOMORPHISMES ERGODIQUES ET FINITAIRES

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Dans ce travail, nous montrons que chaque système dynamique ergodíque et finitaire d'entropie finie est isomorphe au système produit formé d'une rotation et un schéma de Bernoulli.

CLASSIFICATION OF ERGODIC FINITARY SHIFTS

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Introduction

Adler, Shields and Smorodinsky have shown in [1] that every irreducible finite state Markov shift is isomorphic to a direct product of a rotation and a Bernoulli shift.

In this paper we extend this result to a wider class of shifts, the so called finitary shifts. These shifts are induced by finitary processes defined by Heller in [4] . He has proved that these processes include Markov chains and also functionals of Markov chains. It is given in [4] an example of a process which is finitary and is not a functional of a Markov chain.

Robertson in [5] has shown that every mixing and finitary process is a K-process.

The first author of this paper has proved in [2] that in fact every weak mixing and finitary process is weak Bernoulli. Thus every weak mixing and finitary shift is isomorphic to a Bernoulli shift.

Using the Heller and Robertson representations of stochastic processes / cf. [4] and [6] / we classify the ergodic finitary shifts with the finite entropy up to isomorphism.

§1. Preliminaries

Let $(X, \mathcal{B}, \mu)$ be a Lebesgue probability space, T be an authomorphism of $(X, \mathcal{B}, \mu)$ with the finite entropy and U_T be the unitary operator defined on $L^2(X,\mu)$ by the formula

$$U_T f = f \circ T, \quad f \in L^2(X,\mu) .$$

Let $Q = \{B_i, i \in I\}$ be a finite measurable partition of X. The pair (T,Q) will be called a process.

Now we recall the notion of a finitary process given by Heller in [4] .

Let A_I be the free associative algebra, over the field R of real numbers, generated by I. we denote by P_I the linear functional on A_I defined as follows.

$$P_I(j_1 \dots j_r) = \mu\left(B_{j_1} \cap \dots \cap T^{-(r-1)} B_{j_r}\right),$$

$$P_I(\emptyset) = 1,$$

where $j_1, \dots, j_r \in I$, $r \geqslant 1$ and $\emptyset$ means the empty sequence in A_I. The pair (A_I, P_I) is said to be the Heller representation of the process (T,Q).

Let N_I be the largest left ideal contained in the kernel of P_I. It is easy to check that

$$N_I = \left\{\xi \in A_I; \ P_I\left(A_I \cdot \xi\right) = 0\right\} = \left\{\xi \in A_I; \ P_I\left(I^{\infty} \cdot \xi\right) = 0\right\}$$

where $I^{\infty} = \bigcup_{n=0}^{\infty} I^n$, $I^0 = \emptyset$.

The process (T,Q) is said to be finitary if the vector space A_I/N_I is finite-dimensional over R.

We say that T is a finitary shift if there exists a generator Q for T such that the process (T,Q) is finitary.

Now we recall the concept of a spectral representation of a process (T,Q) defined by Robertson in [6] .

A triple $(H, e, \{W_i;\ i \in I\})$ is said to be a spectral representation of (T,Q) if

(a) H is a complex Hilbert space,

(b) $e \in H$ and $\|e\| = 1$,

(c) for every $J \subset I$, $\sum_{i \in J} W_i$ is a contraction on H,

(d) $W_T e = W_T^* e = e$ where $W_T = \sum_{i \in I} W_i$,

(e) $(e, W_\alpha e) = \mu\,(B_{i_1} \cap T^{-1} B_{i_2} \cap \ldots \cap T^{-(n-1)} B_{i_n})$ where

$$W_\alpha = W_{i_1} W_{i_2} \ldots W_{i_n},\ \alpha = i_1 \ldots i_n \in I^n,\ n \geq 1,$$

A spectral representation $(H, e, \{W_i;\ i \in I\})$ is called a reduced spectral representation, or shortly RSR of (T,Q), if

(f) $H = \overline{Sp}\{W_\alpha e\ ;\ \alpha \in I^\infty\} = \overline{Sp}\{W_\alpha^* e\ ;\ \alpha \in I^\infty\}$,

i.e. the linear subspaces $Sp\{W_\alpha e\ ;\ \alpha \in I^\infty\}$ and $Sp\{W_\alpha^* e; \alpha \in I^\infty\}$ are dense in H.

We shall use in the sequel a special RSR of (T,Q), called by us a standard RSR of (T,Q), constructed by Robertson in [6] in the following manner.

Let U_i be the contraction on $L^2(X,\mu)$ defined by the formula

$$U_i f = \chi_{B_i} \cdot U_T f,\ f \in L^2(X,\mu),\ i \in I.$$

For $\alpha \in I^\infty$, $\alpha = i_1 \ldots i_n$ we put $U_\alpha = U_{i_1} \ldots U_{i_n}$ and for $\alpha = \emptyset$ U_α means the identity operator.

Let

$$\mathcal{P} = \bigvee_{k=1}^{\infty} T^k Q \text{ and } \mathcal{F} = \bigvee_{k=0}^{\infty} T^{-k} Q .$$

The following equalities are valid

$$L^2(\mathcal{P}) = \overline{Sp}\{U_\alpha^* 1,\ \alpha \in I^\infty \setminus \emptyset\} \text{ and } L^2(\mathcal{F}) = \overline{Sp}\{U_\alpha 1,\ \alpha \in I^\infty\}.$$

Let V_α and V_T be restrictions of U_α and U_T respectively to $L^2(\mathcal{F})$ and let $H = L^2(\mathcal{F}) \ominus (L^2(\mathcal{F}) \ominus L^2(\mathcal{P}))$ where for closed subspaces $H_1, H_2 \subset L^2(X,\mu)$, $H_1 \ominus H_2 = \{f \in H_1 \; ; \; f \perp H_2\}$ ..

It is proved in [6] that $H = \overline{Sp}\{V_\alpha^* 1 \; ; \; \alpha \in I^\infty\}$.
Let $W_\alpha^* = V_\alpha^*|_H$, $W_T^* = V_T^*|_H$, $W_\alpha = W_\alpha^{**}$ and $W_T = W_T^{**}$. It is clear that $W_\alpha = \pi_H \cdot V_\alpha$ where π_H is the orthogonal projection of $L^2(X,\mu)$ onto H.

It is shown in [6] that the triple $(H, 1, \{W_i, i \in I\})$ is a RSR of (T,Q) .

§2. Algebraic properties of finitary shifts

Let (T,Q) be a process, $Q = \{B_i, i \in I\}$ and A_I^c be the free associative algebra over the field C of complex numbers, generated by I.

We denote by P_I^c the complex linear functional on A_I^c defined by the formula

$$P_I^c(\xi) = P_I(\mathrm{re}\,\xi) + i \cdot P_I(\mathrm{im}\,\xi), \; \xi \in A_I^c.$$

It is easy to check that the largest left ideal N_I^c contained in the kernel of P_I^c has the form

$$N_I^c = N_I + iN_I = \{\xi \in A_I^c \; ; \; P_I^c(A_I^c \cdot \xi) = 0\} .$$

Hence we obtain at once the following

<u>Remark</u>. A process (T,Q) is finitary iff the vector space A_I^c/N_I^c is finite-dimensional over C.

Now, let $(H, e, \{W_i, i \in I\})$ be a RSR of (T,Q) and let $H_I = Sp\{W_\alpha e \; , \alpha \in I^\infty\}$.

<u>Lemma 1</u>. The vector spaces A_I^c/N_I^c and H_I are isomorphic.

<u>Proof</u>. Let $L_I = Sp\{W_\alpha, \alpha \in I^\infty\}$. It is clear that L_I is the linear subalgebra of the algebra of all linear operators of H. We denote by $\varphi : A_I^c \to L_I$ the mapping defined on I^∞ by the

formula $\varphi(\alpha) = W_\alpha$, $\alpha \in I^\infty$ and then linearly extended on A_I^c. It is an easy matter to see that φ is a homomorphism of A_I^c onto L_I.

Now we define a mapping $f\colon A_I^c \to H_I$ by

$$f(\xi) = \varphi(\xi) e \ , \quad \xi \in A_I^c.$$

It is, of course, a homomorphism of A_I^c onto H_I. We shall show that $N_I^c = \ker f$.

From the property (e) of $(H, e, \{W_i, i \in I\})$ follows $P_I^c(\alpha) = (e, W_\alpha e) \geqslant 0$, $\alpha \in I^\infty$. Hence $P_I^c(\alpha) = (W_\alpha e, e)$ and so $P_I^c(\xi) = (\varphi(\xi) e, e)$, $\xi \in A_I^c$.

Since φ is a homomorphism we have

$$\begin{aligned} N_I^c &= \{\xi \in A_I^c \ ; \ P_I^c(\eta \xi) = 0 \ , \ \eta \in A_I^c\} = \\ &= \{\xi \in A_I^c \ ; \ (\varphi(\eta\xi) e, e) = 0, \ \eta \in A_I^c\} = \\ &= \{\xi \in A_I^c \ ; \ (\varphi(\xi) e, \ \varphi(\eta)^* e) = 0 \ , \ \eta \in A_I^c\} \ . \end{aligned}$$

Now, by (f) the subspace $\mathrm{Sp}\{W_\alpha^* e \ ; \ \alpha \in I^\infty\}$ is dense in H and so

$$N_I^c = \{\xi \in A_I^c \ ; \ \varphi(\xi) e = \sigma\} = \ker f.$$

This equality and the fact that $H_I = f(A_I^c)$ imply the vector spaces A_I^c/N_I^c and H_I are isomorphic, what completes the proof.

Since every finite-dimensional subspace of a linear normed space is closed we have at once the following

<u>Corollary 1</u>. The following conditions are equivalent

(i) A process (T, Q) is finitary,

(ii) for every RSR $(H, e, \{W_i, i \in I\})$ of (T, Q) holds $\dim H < \infty$,

(iii) there exists a RSR $(H, e, \{W_i, i \in I\})$ of (T, Q) with $\dim H < \infty$.

Corollary 2. A process (T,Q) is finitary iff there exists a spectral representation $(H, e, \{W_i, i\in I\})$ with $\dim H<\infty$.

Proof. In view of Corollary 1 it is enough to prove only the sufficiency.

Let $(H, e, \{W_i, i\in I\})$ be a spectral representation of (T,Q) with $\dim H<\infty$. We shall show that for any RSR $(H', e', \{W_i', i\in I\})$ of (T,Q) holds $\dim H_I' \leqslant \dim H$ where $H_I' = \mathrm{Sp}\ \{W_\alpha' e', \alpha\in I^\infty\}$.

First we shall prove that

$$\text{if } \sum_{i=1}^{s} a_i W_{\alpha_i} e = 0 \text{ then } \sum_{i=1}^{s} a_i W'_{\alpha_i} e' = 0, s\geqslant 1.$$

The first equality implies

$$0 = \sum_{i=1}^{s} a_i (W_{\alpha_i} e, W_\beta^* e) = \sum_{i=1}^{s} a_i (W_{\beta\alpha_i} e, e)$$

for every $\beta\in I^\infty$.

Since $(H, e, \{W_i, i\in I\})$ and $(H', e', \{W_i', i\in I\})$ are representations of the same process we have $(W_\alpha e, e) = (W_\alpha' e', e')$, $\alpha\in I^\infty$. Thus we obtain

$$0 = \sum_{i=1}^{s} a_i (W'_{\beta\alpha_i} e, e) = \Big(\sum_{i=1}^{s} a_i W'_{\alpha_i} e', W_\beta'^{*} e'\Big),\ \beta\in I^\infty.$$

The fact that $(H', e', \{W_i', i\in I\})$ is reduced implies the equality $H = \overline{\mathrm{Sp}}\, \{W_\alpha'^{*} e', \alpha\in I^\infty\}$ and so $\sum_{i=1}^{s} a_i W'_{\alpha_i} e' = 0$.

Now, let $\psi : H_I \to H_I'$ be the mapping defined as follows. For every $\alpha\in I^\infty$ we put $\psi(W_\alpha e) = W_\alpha' e'$ and then ψ is linearly extended on H. From the above remark follows that the mapping ψ is well defined. Since ψ is onto we obtain $\dim H \geqslant \dim H_I \geqslant \dim H_I'$. Now our assumption implies $\dim H_I' < \infty$ and so $\dim H' < \infty$. Using again Corollary 1 we obtain the desired result.

Now, let $Y\in\mathcal{B}$ be such that $TY = Y$ and $\mu(Y) > 0$.

We consider the dynamical system $(Y, \mathcal{B}_Y, \mu_Y, T_Y)$ where $\mathcal{B}_Y = \mathcal{B} \cap Y$, $\mu_Y = \mu(\cdot \mid Y)$ and $T_Y = T|_Y$. For a partition Q of X we put $Q_Y = Q \cap Y$.

<u>Theorem 1</u>. If T is finitary then T_Y is finitary.

<u>Proof</u>. Let $Q = \{B_i, i \in I\}$ be a finite generator for T such that the process (T,Q) is finitary and $(H, 1, \{W_i, i \in I\})$ be the standard RSR of (T,Q). The assumption and Corollary 1 yields $\dim H < \infty$. Since Q_Y is a generator for T_Y it is enough to prove that the process (T_Y, Q_Y) is finitary.

For $f, g \in L^2(X,\mu)$ we put

$$(f,g)_Y = \frac{1}{\mu(Y)} \int_X f(x) \cdot \overline{g(x)}\, \mu(dx), \quad \|f\|_Y = \sqrt{(f,f)_Y}$$

and $H_Y = (H, (\cdot,\cdot)_Y)$. Of course $\dim H_Y < \infty$.

We assert that the triple $(H_Y, \chi_Y, \{W_i, i \in I\})$ is a spectral representation of (T_Y, Q_Y), i.e. it satisfies the properties (a) - (e).

The properties (a) and (c) are trivial. Since $TY = Y$ we have $U_T \chi_Y = \chi_Y$. It follows from lemma (5.1) of [6] that $\chi_Y \in H$ and so $\chi_Y \in H_Y$. It is clear that $\|\chi_Y\|_Y = 1$ and thus (b) is fulfilled. The equality $TY = Y$ and the fact that $\chi_Y \in H \subset L^2(\mathcal{F})$ imply

$$W_T \chi_Y = \pi_H U_T \chi_Y = \pi_H \chi_Y = \chi_Y$$

and

$$W_T^* \chi_Y = V_T^* \chi_Y = E(\chi_Y \circ T^{-1} \mid \mathcal{F}) = E(\chi_Y \mid \mathcal{F}) = \chi_Y ,$$

where E denotes the conditional expectation operator. Hence (d) is satisfied. It remains to prove (e) . Let $\alpha = i_1 i_2 \ldots i_n \in I^\infty$. Since $W_\alpha = \pi_H U_\alpha$ we have

(1) $(\chi_Y, W_\alpha \chi_Y)_Y = (\chi_Y, U_\alpha \chi_Y)_Y$.

An easy computation and the equality $TY = Y$ give

(2) $U_\alpha \chi_Y = \chi_{B_{i_1} \cap T^{-1} B_{i_2} \cap \ldots \cap T^{-(n-1)} B_{i_n} \cap Y}$.

The equalities (1) and (2) imply

$$(\chi_Y, W_\alpha \chi_Y)_Y = \mu_Y(B_{i_1} \cap T^{-1}B_{i_2} \ \dots \ T^{-(n-1)}B_{i_n})$$

and so $(H_Y, \chi_Y, \{W_i, i \in I\})$ is a spectral representation of (T_Y, Q_Y) with $\dim H_Y < \infty$. Using Corollary 2 we obtain the result.

Theorem 2. If T is finitary then T^n is finitary, $n \geqslant 1$.

Proof. Let $Q = \{B_i, i \in I\}$ be a finite generator for T such that the process (T,Q) is finitary and $n \geqslant 1$ be an arbitrary natural number. The partition $Q_n = \bigvee_{k=0}^{n-1} T^{-k}Q$ is, of course, a generator for T^n. We shall show that the process (T^n, Q_n) is finitary.

Let (A_I, P_I) and (A^*, P^*) be the Heller representations of (T,Q) and (T^n, Q_n) respectively. It is clear that $A^* = A_{I^n}$. We define a map $\varphi : A^* \to A_I$ as follows. For every $\alpha^* \in (I^n)^\infty$, $\alpha^* = (\alpha_{11} \dots \alpha_{1n})(\alpha_{21} \dots \alpha_{2n}) \dots \dots (\alpha_{r1} \dots \alpha_{rn})$ we put $\varphi(\alpha^*) = \alpha_{11} \dots \alpha_{1n} \alpha_{21} \dots \alpha_{2n} \dots \dots \alpha_{r1} \dots \alpha_{rn}$ and then we extend φ on A^* linearly. It is easy to see that φ is an injection and

$$(3) \qquad P^*(\alpha^*) = P(\varphi(\alpha^*)), \quad \alpha^* \in (I^n)^\infty.$$

Let $N_I \subset A_I$ and $N^* \subset A^*$ be the ideals corresponding to P_I and P^* respectively. We shall check that

$$(4) \qquad \varphi(N^*) = \varphi(A^*) \cap N_I .$$

Let $\xi^* \in N^*$ and $\eta \in I^\infty$. Suppose the lenght of η is k. Then we may write $k = qn+l$ where q,l are nonnegative integers with $0 \leqslant l \leqslant n-1$. Thus $\sigma^{n-l}\eta \in \varphi(A^*)$ where $\sigma = \sum_{i \in I} i$. Let $\eta^* \in A^*$ be such that $\varphi(\eta^*) = \sigma^{n-l}\eta$. Since μ is T-invariant, φ is homomorphism and $\xi^* \in N^*$ we obtain by (3)

$$P_I(\eta\varphi(\xi^*)) = P_I(\sigma^{n-l}\eta\,\varphi(\xi^*)) = P_I(\varphi(\eta^*\xi^*)) = P^*(\eta^*\xi^*) = 0.$$

This means that $\varphi(\xi^*)\in N_I$ and so $\varphi(N^*)\subset\varphi(A^*)\cap N_I$.

To prove the converse inclusion let us suppose $\xi\in\varphi(A^*)\cap N_I$ and $\xi^*=\varphi^{-1}(\xi)$. It is enough to check that $\xi^*\in N^*$. Let $\eta^*\in A^*$. Using the same arguments as above and the fact that $\xi\in N_I$ we have

$$P^*(\eta^*\xi^*) = P_I(\varphi(\eta^*\xi^*)) = P_I(\varphi(\eta^*)\xi) = 0,$$

i.e. $\xi^*\in N^*$ and the equality (4) is proved.

Now we shall show that the vector space A^*/N^* is isomorphic to a subspace of A_I/N_I.

Let $\Phi : A^*/N^* \longrightarrow A_I/N_I$ be a map which to a coset $[\xi^*]\in A^*/N^*$ assigns the coset $[\varphi(\xi^*)]\in A_I/N_I$. From (4) follows that $[\xi^*]=[\eta^*]$ implies $[\varphi(\xi^*)]=[\varphi(\eta^*)]$, i.e. Φ is well defined. Φ is, of course, a homomorphism. Using again (4) and the fact that φ is one-to-one we see that Φ is one-to-one. Hence

$$\dim A^*/N^* = \dim \Phi(A^*/N^*) \leqslant \dim A_I/N_I .$$

Since T is finitary we have $\dim A_I/N_I<\infty$ and thus $\dim A^*/N^*<\infty$, i.e. the process (T^n,Q_n) is finitary.

§3. Isomorphism theorem

Lemma 2. If T is finitary then the number of eigenvalues of U_T is finite.

Proof. Let T be finitary, $Q=\{B_i, i\in I\}$ be a finite generator for T such that the process (T,Q) is finitary and $(H,1,\{W_i, i\in I\})$ be the standard RSR of (T,Q). Corollary 1 implies $\dim H<\infty$ and so the set of eigenvalues of the contraction W_T is finite. By theorem (5.2) of [6] every eigenvalue λ of W_T with $|\lambda|=1$ is an eigenvalue of U_T. Thus the number of eigenvalues of U_T is finite and the lemma is proved.

Corollary 3. Every totally ergodic finitary shift is weak mixing.

Proof. It is enough to show that 1 is the only eigenvalue of U_T . Let us suppose , on the contrary , that there exists $\lambda \neq 1$ which is also an eigenvalue of U_T. Since T is totally ergodic the numbers λ^n, $n \in Z$ form an infinite subset of the set of all eigenvalues of U_T. But this is a contradiction to lemma 2.

Since every weak mixing finitary shift is isomorphic to a Bernoulli shift (cf. [2]) we have at once the following

Corollary 4. Every totally ergodic finitary shift is isomorphic to a Bernoulli shift.

Definition. We say that an automorphism T admits a n-stack if there exists a set $B \in \mathcal{B}$ such that T^iB , $0 \leq i \leq n-1$ are disjoint and $\bigcup_{i=0}^{n-1} T^iB = X$ a.e.

We shall need in the sequel the following

Lemma 3. [3] T admits a n-stack iff the nth roots of unity are proper values of U_T.

Now, let T be an ergodic finitary shift and let $d = d(T)$ denote the number of eigenvalues of U_T.

If p is a natural number, then Z_p will denote the additive group of integers mod p equipped with the measure m_p defined by $m_p(i) = \frac{1}{p}$, $0 \leq i \leq p-1$. Let $\varrho = \varrho_p$ be the rotation on Z_p defined by the formula $\varrho(z) = z + 1$, mod p.

Theorem 3. If T is an ergodic finitary shift with $d > 1$ then T is isomorphic to $\varrho_d \times \beta$ where β is a Bernoulli shift.

Proof. The ergodicity of T implies the set of eigenvalues of U_T is a cyclic group of rank d. Thus from lemma 3 follows that T admits a d-stack, i.e. there exists a set $Y \in \mathcal{B}$ such that T^iY, $0 \leq i \leq d-1$ are disjoint and $\bigcup_{i=0}^{d-1} T^iY = X$ a.e. It is clear that $T^dY = Y$. Let $T_Y^d = T^d|_Y$.

The ergodicity of T implies T_Y^d is ergodic. We shall show that, in fact, T_Y^d is totally ergodic.

Let us suppose, on the contrary, that T_Y^d is not totally ergodic. Hence there exists a natural number $k \geq 2$ and an eigenvalue $\lambda \neq 1$ of $U_{T_Y^d}$ such that $\lambda^k = 1$. From lemma 3 applied to Y and T_Y^d follows that there exists a set $B \in \mathcal{B}$, $B \subset Y$ such that $(T_Y^d)^i B = T^{id}B$, $0 \leq i \leq k-1$ are disjoint and $\bigcup_{i=0}^{k-1} T^{id}B = Y$ a.e. Hence $(T^iB,\ 0 \leq i \leq kd-1)$ is a kd-stack. Applying again lemma 3 we see that kdth roots of unity are eigenvalues of U_T. But this, in view of the inequality $k \geq 2$, is impossible. Thus T_Y^d is totally ergodic.

Since T is finitary, theorem 2 implies T^d is also finitary. Using theorem 1 to T^d we see that T_Y^d is finitary. Combining this with the fact that T_Y^d is totally ergodic and using Corollary 4 we conclude T_Y^d is isomorphic to a Bernoulli shift.

Now, proceeding in the same way as in the proof of Theorem 2 [1], we obtain the desired result.

From theorem 3 easily follows

<u>Corollary 5</u>. Two ergodic finitary shifts are isomorphic iff they have the same number of eigenvalues and the same entropy.

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