JOSEPH GLOVER

Markov processes and their last exit distributions

Annales scientifiques de l’Université de Clermont-Ferrand 2, tome 71, série Mathématiques, n° 20 (1982), p. 107

<http://www.numdam.org/item?id=ASCFM_1982__71_20_107_0>
In a forthcoming paper [1], we prove the following result.

**Theorem.** Let \( (X(t), P^X) \) and \( (Y(t), Q^X) \) be two (canonically defined) transient Hunt processes on \( E \). Assume for each compact set \( K \) contained in \( E \) that \( P^X(f(X_{L(K)}) ; L(K)>0) = Q^X(f(Y_{L(K)}) ; L(K)>0) \) for all bounded functions \( f \) on \( E \), where \( L(K) \) is the last time the process is in \( K \). Then \( Y \) is equivalent to a time change of \( X \) by the inverse of a strictly increasing continuous additive functional.

Let \( \{q_k\} \) be a collection of points dense in \( E \), and let \( B_r(q_k) \) be the open ball of radius \( r \) about \( q_k \). If we let \( L(r,k) \) be the last time the process is in \( B_r(q_k) \), and if we set \( A(t) = \sum 2^{-j} \int_0^1 1_{\{0<L(r,k)<t\}} dr \), then \( A(t) \) is a raw additive functional of \( X \) and \( Y \) and the hypothesis of the theorem implies that \( P^X \int f(X_{L(-}) \ dA(t) = Q^X \int f(Y_{L(-}) \ dA(t) \). Let \( B(t) \) (resp. \( C(t) \)) denote the dual predictable projection of \( A(t) \) for the process \( (X(t), P^X) \) (resp. \( (Y(t), Q^X) \). It is not difficult to show that \( B(t) \) (resp. \( C(t) \)) is a strictly increasing continuous additive functional of \( X \) (resp. \( Y \)), which implies that \( P^X \int f(X(t)) dB(t) = Q^X \int f(Y(t)) dC(t) \). Thus if we let \( T(t) \) (resp. \( S(t) \)) denote the right continuous inverse of \( B(t) \) (resp. \( C(t) \)), then \( P^X \int f(X(T(t))) dt = Q^X \int f(Y(S(t))) dt \). Therefore, the resolvents of the processes \( (X(T(t)), P^X) \) and \( (Y(S(t)), Q^X) \) agree, so the processes have the same joint distributions.

The main result follows.

This result can also be interpreted (with natural auxiliary hypotheses) as a statement in potential theory involving equilibrium measures.