ANNALES MATHÉMATIQUES BLAISE PASCAL

H. AIRAULT
P. MALLIAVIN

Some heat operators on $\mathbb{P}(\mathbb{R}^d)$

Annales mathématiques Blaise Pascal, tome 3, nº 1 (1996), p. 1-11
<http://www.numdam.org/item?id=AMBP_1996__3_1_1_0>

Ann. Math. Blaise Pascal, Vol. 3, N° 1, 1996, pp.1-11

SOME HEAT OPERATORS ON $\mathbb{P}(\mathbb{R}^d)$

H. AIRAULT AND P. MALLIAVIN

ABSTRACT. To a diffusion on R^n, we associate a heat equation on the path space $P(R^n)$ of continuous maps defined on $[0,1]$ with values in R^n. The heat operator is obtained by taking the sum of the square of twisted derivatives with respect to an orthonormal basis of the Cameron-Martin space. We give the expression of this heat operator when it acts on cylindrical functions defined on the Wiener space.

RÉSUMÉ . A une diffusion sur R^n, on associe une équation de la chaleur sur $P(R^n)$, l'espace des applications continues, définies sur $[0,1]$ à valeurs dans R^n. L'opérateur de la chaleur est construit en prenant la somme des carrés des dérivées amorties par rapport à une base de l'espace de Cameron-Martin. On exprime cet opérateur de la chaleur sur les fonctions cylindriques définies sur l'espace de Wiener.

§0: INTRODUCTION

Let $\Omega = P(R^n)$ be the Wiener space of continuous maps from $[0,1]$ with values in R^n and let $I : \omega \to x(\omega)$ be a map from Ω to itself. We assume that, for any $\tau \in [0,1]$, the map $\omega \to x_\tau(\omega)$ is differentiable on the Wiener space and that it is adapted. Given the heat operator A on the Wiener space $P(R^n)$ [See [2]], we construct a new operator $\tilde{A}$. The operator $\tilde{A}$ is the image of the operator A through the map I, and satisfy the identity

$$A(foI) = (\tilde{A}f)oI \tag{0.1}$$

This allows to obtain a heat equation associated to the map I. The operator $\tilde{A}$ is obtained by taking the sum of the square of twisted derivatives with respect to a basis $(e_{k,\alpha})_{k\geq 0, 1\leq\alpha\leq n}$ of the Cameron-Martin space of the Wiener space. We express the operator $\tilde{A}$ when it is applied to cylindrical functions defined on the Wiener space $P(R^n)$. The identity (0.1) extends to the Wiener space the elementary following computation: Let $A = \frac{d^2}{dx^2}$ be the derivative of order 2 on R, viewed as the infinitesimal generator of the brownian diffusion on R, and let ϕ be a differentiable homeomorphism of R; then $A(fo\phi) = (\tilde{A}f)o\phi$ holds where

$$\tilde{A} = (\phi'[\phi^{-1}(x)])^2 \frac{d^2}{dx^2} + \phi''(\phi^{-1}(x))\frac{d}{dx} \tag{0.2}$$

is the infinitesimal generator of a new diffusion on R. We explicit the computations when the map I is the Ito map associated to the diffusion on R^n

$$dx(\tau) = d\omega(\tau) + b(x(\tau))d\tau \tag{0.3}$$

This method extends when I is a map from $P(R^n)$ to $P(M)$ the path space of a Riemannian manifold M; it allows to obtain new diffusions on the space $P(M)$. See [3] for further developments related to this subject.

§1 Notations and definitions

Let ω be the brownian on R^n, and consider the diffusion given by the stochastic differential equation (0.3) where b is a differentiable map from R^n to R^n. We denote by

$$I : \omega \to x(\omega) \tag{1.1}$$

the Ito map and let

$$g_t : \omega \to \sqrt{t}\omega \tag{1.2}$$

be the dilation on $P(R^n)$. The evaluation map φ_τ at τ is given by

$$\varphi_\tau(\omega) = \omega_\tau$$

and we put

$$\tilde{\varphi}_\tau = \varphi_\tau o I \tag{1.3}$$

We denote by μ the Wiener measure on $\Omega = C([0,1], R^n)$ and let $\nu_t = (Iog_t) * \mu$ be the image of the Wiener measure μ by the map Iog_t. The Cameron-Martin space H is the set of differentiable functions h in $L^2([0,1]; R)$ such that $\int_0^1 h'(s)^2 ds < +\infty$. We consider for a basis of the Cameron-Martin space H, the functions defined by

$$e_{k,\alpha}(\tau) = \sqrt{2}\frac{\sin(k\pi\tau)}{k\pi} \otimes \varepsilon_\alpha \tag{1.4}$$

with $k \geq 1$ and

$$e_{0,\alpha}(\tau) = \tau \otimes \varepsilon_\alpha$$

where $(\varepsilon_\alpha)_{\alpha=1,\dots,n}$ is a basis of R^n. We shall write

$$e_k(\tau) = \sqrt{2}\frac{\sin(k\pi\tau)}{k\pi}$$

$$e_0(\tau) = \tau$$

Let h be an element of the Cameron-Martin space H and let $f : \Omega \to R^n$. We let

$$D_h f(\omega) = \frac{d}{d\varepsilon}_{|\varepsilon=0} f(\omega + \varepsilon h) \tag{1.5}$$

For $s \in [0,1]$, we define $D_s f(\omega)$ such that

$$D_h f(\omega) = \int_0^1 D_s f(\omega) h'(s) ds \tag{1.6}$$

Let

$$\nabla f(\omega)(s) = \int_0^s D_u f(\omega) du \tag{1.7}$$

On the Cameron-Martin space H, denote $(\ \ |\ \ )_H$ the scalar product given by $(h_1|h_2)_H = \int_0^1 h_1'(s)h_2'(s)ds$. We have

$$D_h(\omega) = (h|\nabla f(\omega)) \tag{1.8}$$

and for f_1 and f_2 defined on Ω with real values, we have

$$(\nabla f_1(\omega)|\nabla f_2(\omega)) = \int_0^1 D_s f_1(\omega) D_s f_2(\omega) ds \tag{1.9}$$

§2 Twisting and intertwining identities

Let b' be the Jacobian map of b and let h in the Cameron-Martin space; we put

$$\beta(\tau)(\omega) = \int_0^\tau exp[\int_s^\tau b'(\omega_u)du] h'(s) ds \tag{2.1}$$

Definition 2.1. *We call $\beta(\tau)$ the twisted vector field associated to the element h through the diffusion (0.3).*

We denote $\beta'(\tau) = \frac{d}{d\tau}\beta(\tau)$ the derivative of β as a function of τ. By (2.1), we have

$$\beta'(\tau)(\omega) = h'(\tau) + b'(\omega_\tau)\beta(\tau)(\omega) \tag{2.2}$$

and

$$\beta(0)(\omega) = 0$$

Lemma 2.1. *Assume that β and h are related by (2.1), then the derivative of the evaluation map (1.3) is*

$$D_h\tilde{\varphi}_\tau(\omega) = \beta(\tau)(I\omega) \tag{2.3}$$

proof. Let $h \in H$; from (1.2) and (0.3), the function

$$y^\epsilon(\tau)(\omega) = \tilde{\varphi}_\tau(\omega + \epsilon h)$$

is solution of the stochastic equation

$$dy^\epsilon(\tau)(\omega) = d\omega(\tau) + \epsilon h'(\tau)d\tau + b(y^\epsilon(\tau)(\omega))d\tau \tag{2.4}$$

Taking the derivative with respect to ϵ, we obtain that

$$z(\tau)(\omega) = \frac{d}{d\epsilon}_{|\epsilon=0} y^\epsilon(\tau)(\omega)$$

satisfies

$$dz(\tau)(\omega) = h'(\tau)d\tau + b'(x(\tau)(\omega))z(\tau)d\tau$$

and

$$z(0)(\omega) = 0$$

By (2.2), we obtain the identity (2.3).

Corollary. *We have*

$$D_s x(\tau)(\omega) = exp[\int_s^\tau b'(x(u)(\omega))]du \tag{2.5}$$

proof. $D_s x(\tau)(\omega)$ means $D_s\tilde{\varphi}_\tau(\omega)$ Thus, by (2.3) and (1.6), we have

$$\beta(\tau)(I\omega) = \int_0^\tau D_s x_\tau(\omega)h'(s)ds \tag{2.6}$$

Then, we use (2.1).

Remark: If we denote $\varphi_\tau(\omega) = \omega_\tau$ then (2.6) can be written

$$\beta(\tau)(\omega) = ((\nabla(\varphi_\tau oI)(I^{-1}(\omega)|h)_H \tag{2.7}$$

Definition 2.2. *We let*

$$D_\beta f(\omega) = \frac{d}{d\varepsilon}_{|\varepsilon=0} f(\omega + \varepsilon\beta(\omega)) \tag{2.8}$$

Lemma 2.2. *If β is the twisted vector field related to h through (2.1), the following intertwinning relation holds*

$$D_h(foI)(\omega) = (D_\beta f)(I\omega) \tag{2.9}$$

proof.

$$D_h(foI)(\omega) = \frac{d}{d\varepsilon}_{|\varepsilon=0}(foI)(\omega + \varepsilon h) \tag{2.10}$$

We verify (2.9) when $f = \psi o \varphi_\tau$ where $\varphi_\tau(\omega) = \omega_\tau$ and $\psi : R^n \to R$. For the solution $y^\epsilon(\tau)$ of (2.4), we have

$$(foI)(\omega + \epsilon h) = \psi(y^\epsilon(\tau)) \tag{2.11}$$

We deduce that

$$\frac{d}{d\varepsilon}_{|\varepsilon=0}\psi(y^\epsilon(\tau)) = \psi'(x_\tau)\beta(\tau)(I\omega) \tag{2.12}$$

On the other hand

$$(D_\beta f)(\omega) = \psi'(\omega(\tau))\beta(\tau)(\omega) \tag{2.13}$$

By comparison of (2.13) and (2.12), we get (2.9).

Remark that (2.3) is the particular case of (2.9) when $f = \varphi_\tau$.

Definition 2.3. *When h and β are related through (2.1), we define the twisted derivative $\tilde{D}_h f$ by*

$$\tilde{D}_h f(\omega) = D_\beta f(\omega) \tag{2.14}$$

Lemma 2.3. *We have*

$$(\tilde{D}_h^2 f)(I\omega) = D_h^2(foI)(\omega) \tag{2.15}$$

proof. From (2.14) and (2.9), we get

$$(\tilde{D}_h f)(I\omega) = D_h(foI)(\omega) \tag{2.16}$$

and

$$\tilde{D}_h(\tilde{D}_h f)(I\omega) = D_h((\tilde{D}_h f)oI)(\omega)$$

$$= D_h(D_h(foI))(\omega)$$

Thus, we obtain (2.15).

§3 Heat operators on the space $P(R^n)$

We shall construct the heat operator on $P(R^n)$ using the Ito map.

Definition 3.1. *Let $e_{k,\alpha}$ given by (1.4) and let $D_{e_{k,\alpha}}$ the derivation in the direction $e_{k,\alpha}$ (See (1.5)), we define the second order operator*

$$A = \sum_{k\geq 0} \sum_{1\leq\alpha\leq n} D^2_{e_{k,\alpha}} \tag{3.1}$$

The operator A on $P(R^n)$ does not depend on the basis of the Cameron-Martin space; See [2].

Definition 3.2. *Let $\tilde{D}_{e_{k,\alpha}}$ be the twisted derivation, we define the twisted operator $\tilde{A}$ by*

$$\tilde{A} = \sum_{k\geq 0} \sum_{1\leq\alpha\leq n} \tilde{D}^2_{e_{k,\alpha}} \tag{3.2}$$

We verify that the definition (3.2) for the operator $\tilde{A}$ on $P(R^n)$ does not depend on the basis of the Cameron-Martin space.

Lemma 3.1. *We have*

$$A(foI) = (\tilde{A}f)oI \tag{3.3}$$

proof. This is a consequence of (2.15), (3.1) and (3.2).

We shall see in §4 that $\tilde{A}$ corresponds to a change of variables on the Wiener space analoguous to the elementary one (0.2) on R.

Definition 3.3. *We denote by μ the Wiener measure on $\Omega = C([0,1], R^n)$ and let*

$$\nu_t = (Iog_t) * \mu \tag{3.4}$$

the image of the Wiener measure μ through the map Iog_t. See (1.2).

Theorem 3.1. *Let f be a regular function from $P(R^n)$ to R. We have*

$$\frac{\partial}{\partial t}\int f(\omega)d\nu_t(\omega) = \int \tilde{A}f(\omega)d\nu_t(\omega) \tag{3.5}$$

proof. We verify (3.4) when $f(\omega) = \psi(\omega_{\tau_1}, \omega_{\tau_2})$ and $\psi : R^n \to R$. In this case, we have

$$\int f(\omega)d\nu_t(\omega) = \int f(Iog_t(\omega))d\mu(\omega)$$
$$= \int \psi(x_{\tau_1}(\sqrt{t}\omega), x_{\tau_2}(\sqrt{t}\omega))d\mu(\omega)$$

From the heat equation related to the brownian motion on $P(R^n)$, we know (see [2]) that

$$\frac{\partial}{\partial t}\int (foI)(g_t(\omega))d\mu(\omega) = \int A(foI)(g_t(\omega))d\mu(\omega) \tag{3.6}$$

From (3.6) and (3.3), we deduce (3.5).

§4 Expression of the twisted Laplacian $\tilde{A}$ on cylindrical functions

Notation. *Let $p_i : R^n \to R$ be the projection on the i component; we denote*

$$x^i(\tau) = p_i o \tilde{\varphi}_\tau$$

and $x(\tau) = (x^1(\tau), x^2(\tau), ..., x^n(\tau))$. We put $\nabla x^i(\tau) = \nabla(p_i o \tilde{\varphi}_\tau)$.

From (1.9), we have

$$(\nabla x^i(\tau)|\nabla x^j(\tau))_H = \int_0^\tau D_s x^i(\tau) D_s x^j(\tau) ds \tag{4.1}$$

Theorem 4.1. *Let $\psi : R^n \to R$ and $\varphi_\tau(\omega) = \omega_\tau$. We have*

$$\tilde{A}(\psi o \varphi_\tau) = \sum_{1\leq i\leq n} \sum_{1\leq j\leq n} (\nabla x^i(\tau)|\nabla x^j(\tau))_H (I^{-1}\omega) \frac{\partial^2 \psi}{\partial x_i \partial x_j}(\omega_\tau) + A(x^i(\tau))(I^{-1}\omega) \frac{\partial \psi}{\partial x_i}(\omega_\tau) \tag{4.2}$$

The proof of (4.2) will result from the following lemmas and definitions.

Remark: If we take a $\Phi = (\Phi_1, \Phi_2, ..., \Phi_n)$ to be a differentiable homeomorphism of R^n and let

$$\Delta = \sum_{1\leq i\leq n} \frac{\partial^2}{\partial {x_i}^2}$$

to be the usual Laplacian on R^n, we have, for $F : R^n \to R$

$$\Delta(F o \Phi) = (\tilde{\Delta} F) o \Phi$$

with

$$\tilde{\Delta} = \sum_{i,j} (\nabla\Phi_i|\nabla\Phi_j)(\Phi^{-1}(x)) \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{1\leq i\leq n} (\Delta\Phi_i)(\Phi^{-1}(x)) \frac{\partial}{\partial x_i}$$

The theorem 4.1 is an extension of this remark to the Wiener space.

Definition 4.1. *Let*

$$M(s,\tau)(\omega) = \exp[\int_s^\tau b'(\omega_u) du] \tag{4.3}$$

From (2.5), we see that

$$M(s,\tau)(I\omega) = D_s x(\tau)(\omega) \tag{4.4}$$

Lemma 4.2. *We assume that we have a one dimensional diffusion, i.e. $n = 1$ in (0.3). For $k \geq 1$, let*

$$\beta_k(\tau)(\omega) = \int_0^\tau M(s,\tau)(\omega)\sqrt{2}\cos(k\pi s)ds \tag{4.5}$$

and

$$\beta_o(\tau)(\omega) = \int_0^\tau M(s,\tau)(\omega)ds \tag{4.6}$$

We have

$$\sum_{k\geq 0} \beta_k(\tau)^2(\omega) = \int_0^\tau \exp[2\int_s^\tau b'(\omega_u)du]ds \tag{4.7}$$

proof. For fixed τ, let g be the even function which is periodic, of period 2 and given by

$$g(s) = 1_{s\leq\tau} \exp[\int_s^\tau b'(\omega_u)du] \tag{4.8}$$

Its development in Fourier series, for $s \leq \tau$ is equal to

$$\beta_o(\tau) + \sum_{k\geq 1} 2\beta_k(\tau)cos(k\pi s) = g(s) \tag{4.9}$$

From Parseval's identities, we obtain

$$2\int_0^1 g(s)^2 ds = 2\sum_{k\geq 0} \beta_k(\tau)^2 \tag{4.10}$$

This proves (4.7).

Lemma 4.3. *The line vectors of the matrix $M(s,\tau)(I\omega)$ are the vectors $D_s x^i(\tau)(\omega)$. See (4.4). For $n = 1$, we get*

$$\sum_{k\geq 0} \beta_k(\tau)^2(I\omega) = |\nabla x(\tau)(\omega)|_H^2 \tag{4.11}$$

proof.

By (3.1), we have

$$D_h\tilde{\varphi}_\tau(\omega) = \int_0^\tau \exp[\int_s^\tau b'(x_u)du]h'(s)ds \tag{4.12}$$

thus, from (2.5), we get

$$D_s\tilde{\varphi}_\tau(\omega) = \exp[\int_s^\tau b'(x_u)du]1_{s\leq\tau} \tag{4.13}$$

This proves the first assertion. Then, we deduce (4.11) from (4.7) and (1.9).

Proposition 4.4. *The second order term in* $\tilde{A}(\psi o \varphi_\tau)$ *is given by*

$$(\nabla x^i(\tau)|\nabla x^j(\tau))_H(I^{-1}\omega)\frac{\partial^2\varphi}{\partial x_i \partial x_j}(\omega_\tau) \tag{4.14}$$

proof. We have to calculate $D^2_\beta(\psi o \varphi_\tau)$, taking care that

$$\beta(\tau)(\omega) = \int_0^\tau exp[\int_s^\tau b'(\omega_u)du]h'(s)ds$$

depends on ω when the gradient of b is not constant. We have

$$D_\beta(\psi o \varphi_\tau)(\omega) = \psi'(\omega_\tau)\beta(\tau)(\omega) \tag{4.15}$$

and

$$D^2_\beta(\psi o \varphi_\tau)(\omega) = \psi''(\omega_\tau)[\beta(\tau)(\omega), \beta(\tau)(\omega)] + \psi'(\omega_\tau)D_\beta[\beta(\tau)(\omega)] \tag{4.16}$$

We obtain (4.14) from (4.11), (4.16) and (3.2) as follows: Let

$$\beta_{k,\alpha}(\tau)(\omega) = \int_0^\tau exp[\int_s^\tau b'(\omega_u)du]e'_{k,\alpha}(s)ds$$

$$= \int_0^\tau e'_k(s)\exp[\int_s^\tau b'(\omega_u)du](\epsilon_\alpha)ds$$

$$= \int_0^\tau e'_k(s)M(s,\tau)(\epsilon_\alpha)ds$$

See (4.3). We put

$$M(s,\tau)(\epsilon_\alpha) = \sum_j A^j_\alpha(s,\tau)(\epsilon_j)$$

We obtain

$$\beta_{k,\alpha}(\tau)(\omega) = \sum_{1\leq j\leq n}\int_0^\tau e'_k(s)A^j_\alpha(s,\tau)ds(\epsilon_j)$$

We denote

$${}_kB^j_\alpha(\tau) = \int_0^\tau e'_k(s)A^j_\alpha(s,\tau)ds \tag{4.17}$$

We have

$$\sum_{\alpha=1}^n \psi''(\omega_\tau)[\beta_{k,\alpha}(\tau)(\omega), \beta_{k,\alpha}(\tau)(\omega)]$$

$$= \sum_{\alpha=1}^n \sum_{j_1=1,j_2=1}^n [{}_kB^{j_1}_\alpha(\tau)\,{}_kB^{j_2}_\alpha(\tau)]\psi''(\omega_\tau)(\epsilon_{j_1}, \epsilon_{j_2})$$

$$= \sum_{j_1,j_2}[\sum_{\alpha=1}^{n}[{}_kB^{j_1}_\alpha(\tau){}_kB^{j_2}_\alpha(\tau)]\frac{\partial^2\psi}{\partial x_{j_1}\partial x_{j_2}}(\omega_\tau)$$

On the other hand,

$$(\nabla x^{j_1}(\tau)|\nabla x^{j_2}(\tau))_H = \sum_{\alpha=1}^{n}\sum_{k\geq 0}[{}_kB^{j_1}_\alpha(\tau){}_kB^{j_2}_\alpha(\tau)]$$

This proves (4.14).

We shall now evaluate the first order term on cylindrical functions.

Lemma 4.5. *Let $\beta_k(\tau)(\omega)$ and $\beta_o(\tau)(\omega)$ given by (4.5)-(4.6) and $n = 1$, we have*

$$\sum_{k\geq 0} D_{\beta_k}[\beta_k(\tau)(\omega)] = \int_0^\tau M(s,\tau)\int_s^\tau M(s,\alpha)b''(\omega_\alpha)d\alpha \tag{4.18}$$

proof. By (2.2), we have

$$\beta(\tau)(\omega+\varepsilon\beta(\omega)) = \int_0^\tau exp[\int_s^\tau b'(\omega_u+\epsilon\beta(u)(\omega))du]h'(s)ds \tag{4.19}$$

We deduce

$$\frac{d}{d\varepsilon}_{|\varepsilon=0}\beta(\tau)(\omega+\varepsilon\beta(\omega))$$

$$= \int_0^\tau M(s,\tau)(\omega)\int_s^\tau b''(\omega_\alpha)\beta(\alpha)(\omega)h'(s)d\alpha ds$$

$$= \int_0^\tau M(s,\tau)(\omega)\int_s^\tau b''(\omega_\alpha)\int_0^\alpha M(u,\alpha)h'(u)h'(s)duds$$

$$= \int_0^\tau M(s,\tau)(\omega)h'(s)ds\int_0^\tau h'(u)\int_{sup(u,s)}^\tau b''(\omega_\alpha)M(u,\alpha)d\alpha du \tag{4.20}$$

where, at the second step, we have replaced $\beta(\alpha)$ by its expression (2.1). We have to evaluate the sum

$$J = \sum_{k\geq 0} e'_k(s)\int_0^\tau e'_k(v)g_s(v)dv \tag{4.21}$$

where

$$g_s(v) = \int_{sup(s,v)}^\tau M(v,\alpha)b''(\omega_\alpha)d\alpha \tag{4.22}$$

J is the sum of the Fourier series of g at the point $v = s$. We deduce (4.18).

Proposition 4.6. *Let A be the Laplacian (3.1). The first order term in (4.2) is given by*

$$A(x^i(\tau))(I^{-1}\omega)\frac{\partial\psi}{\partial x_i}(\omega_\tau) \tag{4.23}$$

proof. We do the proof when $n = 1$. We calculate

$$\sum_k D^2_{e_k} x(\tau)(\omega) \tag{4.24}$$

We have

$$D_h x(\tau)(\omega) = \int_0^\tau exp[\int_s^\tau b'(x_u(\omega))du]h'(s)ds$$

and

$$D^2_h x(\tau)(\omega) = \frac{d}{d\varepsilon}_{|\varepsilon=0} D_h x(\tau)(\omega + \varepsilon h)$$

$$= \int_0^\tau ds \qquad M(s,\tau)(I\omega)h'(s)\int_s^\tau du \qquad b''(x_u(\omega))\int_0^\tau d\gamma \qquad M(\gamma,u)(I\omega)h'(\gamma) \tag{4.25}$$

After changing the order of integration in (4.25), we calculate the sum (4.24) as the sum of a Fourier series. We obtain that the sum (4.24) is equal to

$$\int_0^\tau M(s,\tau)(I\omega)\int_s^\tau M(s,u)(I\omega)b''(x_u(\omega))duds \tag{4.26}$$

We compare with (4.18) and it yields (4.23).

REFERENCES

1. H. AIRAULT, *Projection of the infinitesimal generator of a diffusion.*, J. of Funct. Anal., Vol.85 , Aug., 2, (1989).
2. H. AIRAULT and P. MALLIAVIN ., *Integration on loop groups II.*, J. of Funct. Anal., Vol.104, Feb.15, 1, (1992).
3. H. AIRAULT and P. MALLIAVIN ., *Integration by parts formulas and dilatation vector fields on elliptic probability spaces.*, To appear Probab. Theor. and Rel. Fields.
4. J.M. BISMUT ., *Large deviations and the Malliavin calculus.*, Progress in Math. Vol45. Birkhauser; (1984).
5. S. FANG and P. MALLIAVIN ., *Stochastic Analysis on the path space of a Riemannian manifold.*, J. of Funct. Anal. Vol.118 , Nov. 15, 1, (1983).

H.AIRAULT, INSSET, UNIVERSITÉ DE PICARDIE JULES VERNE, 48, RUE RASPAIL, SAINT-QUENTIN (AISNE) 02100 FRANCE.

P.MALLIAVIN, 10 RUE SAINT LOUIS EN L'ISLE, 75004 PARIS, FRANCE.